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SPACE-TIME BLOCK CODING FOR NONCOHERENTLY DETECTED CPFSK

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ABSTRACT

In this paper the problem of unitary rate space-time block coding for multiple-input multiple-output communication systems employing continuous phase frequency shift keying is investigated. First, the problem of optimal codeword by codeword noncoherent detection is analysed; then, design criteria for optimal space-time block codes are proposed and some novel coding schemes are devised. Simulation results evidence that the proposed schemes can efficiently exploit spatial diversity and that their use can entail a limited energy loss with respect to other solutions available in the technical literature for coherent systems, with the substantial advantage, however, of a simple detection algorithm.

1. GENERAL INFORMATION

Noncoherent detection of *frequency shift keying* (FSK) signals over frequency-flat fading channels has been deeply investigated in the technical literature. Most of the available research work in this area refers to *single-input single-output* (SISO) communication systems (see [1] and references therein).

In recent times, the interest in *multiple-input multiple-output* (MIMO) communication systems has resulted in a number of proposals for *space-time* (ST) coding schemes, i.e. forcoded signalling techniques suitable for multiple transmit antennas. Most of the these techniques have been explicitly devised for linear modulations combined with coherent or noncoherent detection [2], whereas a very limited number of results are available for nonlinear modulation formats, like *continuous phase modulations* (CPMs). More specifically, ST trellis coding schemes for coherently detected CPM signals have been proposed in [3], [4], but, as far as we know, the design of *ST block codes* (STBCs) for these signals remains an open problems.

This paper illustrates some novel research results about unitary rate ST block coding techniques for a specific class of CPMs, namely *continuous phase* FSK (CPFSK) signals, transmitted over a $2\times N_R$ (i.e., over a MIMO) frequency-flat fading channel. In this scenario, an algorithm for optimal codeword by codeword noncoherent detection is derived and design criteria for optimal design of STBCs are inferred from the analysis of its pairwise error probability.

This paper is organized as follows. Channel and signal models are described in Section 2. Optimal noncoherent detection of STBCs is analysed in Section 3 and design criteria for such codes are derived in Section 4. In Section 5 error performance results of various ST coding schemes are compared. Finally, some conclusions are offered in Section 6.

2. CHANNEL AND SIGNAL MODELS

In the following we focus on a $2\times N_R$ communication system employing CPFSK and unitary rate ST block coding. In our encoder each couple of information bits is mapped into one ST block codeword belonging to a quaternary alphabet $\Omega \doteq \{\mathbf{B}_k, k=1,...,4\}$, where

$$\mathbf{B}_k = \begin{bmatrix} b_{11}^{(k)} & b_{12}^{(k)} \\ b_{21}^{(k)} & b_{22}^{(k)} \end{bmatrix}$$

and $b_{il}^{(k)}$ denotes the symbol transmitted by the i-th antenna over the l-th interval of the k-th ST codeword. In the following, it is assumed that the symbols $\{b_{il}^{(k)}\}$ belong to the M-ary alphabet $\Sigma=\{\pm1,\pm3,...,\pm M-1\}$, with $M=2^z$. As discussed in Section 5, the parameter M represents a degree of freedom useful in the design problem of ST block codes for CPFSK.

The complex envelope of the signal transmitted by the i-th antenna in the time interval $[nT_s,(n+2)T_s)$, where T_s is the symbol interval, with n even, can be expressed as

$$s_i(t,\mathbf{B}[n],\theta_i[n]) \doteq \tilde{s}_i(t-nT_s, b_{i1}[n])\exp(j\theta_i[n]) \quad (1)$$

for $nT_s \le t < (n+1)T_s$ and

$$s_i(t,\mathbf{B}[n],\theta_i[n]) \doteq \tilde{s}_i(t-(n+1)T_s, b_{i2}[n]) \exp(j\pi h b_{i1}[n] + j\theta_i[n]) \quad (2)$$

for $(n+1)T_s \le t < (n+2)T_s$. Here

$$\mathbf{B}[n] \doteq \begin{bmatrix} b_{11}[n] & b_{12}[n] \\ b_{21}[n] & b_{22}[n] \end{bmatrix} \quad (3)$$

denotes the n-th transmitted codeword, $b_{il}[n]$ is the symbol transmitted by the i-th antenna in the $(n+l-1)$-th symbol interval,

$$\tilde{s}_i(t,b_{il}[n]) \doteq \sqrt{2E_s/T_s}\exp(j\psi(t,b_{il}[n]))$$

for $0\le t<T_s$,

$$\psi(t,b_{il}[n]) \doteq b_{il}[n]\pi h t/T_s$$

for $0\le t<T_s$ is the information bearing function, E_s is the transmitted signal energy per symbol, h is the modulation index and $\theta_i[n]$ is the phase state of the i-th CPFSK modulator in the n-th signalling interval. It is worth pointing out that $\theta_i[n]$ can be computed recursively, as [5]

$$\begin{aligned}\theta_i[n] &\doteq \{\theta_i[n-1]+\pi h b_{i2}[n]\} \bmod 2\pi \\ &= \{\theta_i[n-2]+\pi h (b_{i1}[n]+b_{i2}[n])\} \bmod 2\pi\end{aligned}$$

with $\theta_i[0] = \theta_0$, where θ_0 is the initial phase of the CPFSK modulator.

The signals $\{s_i(t, \mathbf{B}[n], \theta_i[n]),\ i = 1,2\}$, (2)-(3) are transmitted over $2N_R$ distinct frequency flat fading channels. The complex envelope of the received signal can be expressed as

$$\mathbf{r}(t) = \mathbf{a}(t)\,\mathbf{s}(t, \mathbf{B}[n], \boldsymbol{\theta}[n]) + \mathbf{w}(t) \tag{4}$$

for $nT_s \le t < (n+2)T_s$, where $\mathbf{r}(t) \doteq [r_1(t), ..., r_{N_R}(t)]^T$, $r_m(t)$ is the signal received by the m-th antenna (with $m = 1,2,...,N_R$), $\mathbf{a}(t) \doteq [a_{mi}(t)]$ with $m = 1,...,N_R$ and $i = 1,2$, $a_{mi}(t)$ is the fading distortion affecting the channel between the m-th receive antenna and the i-th transmit antenna, $\mathbf{s}(t, \mathbf{B}[n], \theta[n]) \doteq [s_1(t, \mathbf{B}[n], \theta_1[n]), s_2(t, \mathbf{B}[n], \theta_2[n])]$, $[n] \doteq [\theta_1[n], \theta_2[n]]^T$, $\mathbf{w}(t) = [w_1(t), ..., w_{N_R}(t)]^T$, $w_m(t)$ is *additive white Gaussian noise* (AWGN) with two-sided spectral density $2N_0$ (see [6], p. 60). In the following it is assumed that: (a) the noise processes $\{w_m(t),\ l = 1,...,N_R\}$ are mutually independent and are independent of channel fading; (b) $\{a_{mi}(t),\ m = 1,...,N_R,\ i = 1,2\}$ are complex *independent* wide-sense stationary Gaussian processes, each having zero-mean (Rayleigh fading) and average statistical power σ_a^2; (c) the channel distortion can be deemed constant over the duration of each codeword (*quasi-static channel*). Under the last assumption, eq. (4) can be simplified as

$$\mathbf{r}(t) = \mathbf{a}[n]\,\mathbf{s}(t, \mathbf{B}[n], \boldsymbol{\theta}[n]) + \mathbf{w}(t) \tag{5}$$

for $nT_s < t \le (n+2)T_s$, where

$$\mathbf{a}[n] \doteq [a_{mi}[n]] = \mathbf{a}((n+1)T_s) \tag{6}$$

is the vector of channel gains in the n-th codeword interval. To simplify the derivation of the optimal noncoherent detector for $\mathbf{r}(t)$ (5), phase states $\theta_1[n]$ and $\theta_2[n]$ can be absorbed in the channel gains $\{a_{m1}[n]\}$ and $\{a_{m2}[n]\}$, respectively, with $m = 1,...,N_R$. Then, if we define the vector

$$\tilde{\mathbf{a}}[n] \doteq [\tilde{a}_{mi}[n]] = [a_{mi}[n]\exp(j\theta_i[n])] \tag{7}$$

$\mathbf{r}(t)$ (5) can be rewritten as

$$\mathbf{r}(t) = \tilde{\mathbf{a}}[n]\,\mathbf{v}(t - nT_s, \mathbf{B}[n]) + \mathbf{w}(t) \tag{8}$$

for $nT_s < t \le (n+2)T_s$, where $\mathbf{v}(t, \mathbf{B}[n]) \doteq [v_1(t, \mathbf{B}[n]), v_2(t, \mathbf{B}[n])]^T$, with

$$v_i(t, \mathbf{B}[n]) \doteq \tilde{s}_i(t, b_{i1}[n]) \tag{9}$$

for $nT_s < t \le (n+1)T_s$ and

$$v_i(t, \mathbf{B}[n]) \doteq \tilde{s}_i(t, b_{i2}[n])\exp(j\pi h b_{i1}[n]) \tag{10}$$

for $(n+1)T_s < t \le (n+2)T_s$ and $i = 1,2$. It is not difficult to verify that, because of the joint statistical properties of the random variables $\{a_{mi}(t),\ m = 1,...,N_r,\ i = 1,2\}$, the vector $\tilde{\mathbf{a}}[n]$ (7) is statistically equivalent to $\mathbf{a}[n]$ (6).

3. OPTIMAL NONCOHERENT DETECTION

In this Section the optimal strategy for codeword-by-codeword noncoherent detection of $\mathbf{r}(t)$ (8) is derived. Following [7], the *maximum a posteriori* (MAP) strategy for $\mathbf{B}[n]$ (3) can be expressed as

$$\hat{\mathbf{B}}[n] = \max_{\tilde{\mathbf{B}}\in} \Pr(\mathbf{r}(t) \mid \mathbf{B}[n] = \tilde{\mathbf{B}}) \tag{11}$$

where $\Pr(\mathbf{r}(t) \mid \mathbf{B}[n] = \tilde{\mathbf{B}})$ denotes the probability of observing $\mathbf{r}(t)$ in the interval $nT_s \le t < (n+2)T_s$, when $\tilde{\mathbf{B}} = [\tilde{b}_{il}] \in \Omega$ has been transmitted. The probability $\Pr(\mathbf{r}(t) \mid \mathbf{B}[n] = \tilde{\mathbf{B}})$ can be evaluated as

$$\Pr(\mathbf{r}(t) \mid \mathbf{B}[n] = \tilde{\mathbf{B}}) = \int_{\mathbf{u}} \Pr(\mathbf{r}(t) \mid \mathbf{B}[n] = \tilde{\mathbf{B}}, \mathbf{u})\, f(\mathbf{u})\, d\mathbf{u} \tag{12}$$

where $\mathbf{u} = [u_{mi}] \doteq \tilde{\mathbf{a}}[n]$ (see (7)),

$$f(\mathbf{u}) = \prod_{m=1}^{N_R}\prod_{i=1}^{2} \frac{1}{\sqrt{2\pi\sigma_a^2}} \exp(-|u_{mi}|^2/2\sigma_a^2) \tag{13}$$

is the joint *probability density function* (pdf) of $\mathbf{u}$, and (see [7], eq. (44))

$$\Pr(\mathbf{r}(t) \mid \mathbf{B}[n] = \tilde{\mathbf{B}}, \mathbf{u}) \propto \exp\left(-\int_{t=nT_s}^{(n+2)T_s}\int_{s=nT_s}^{(n+2)T_s} \big(\mathbf{r}(t) - \mathbf{u}\,\mathbf{v}(t - nT_s, \tilde{\mathbf{B}})\big)\mathbf{Q}(t,s)\big(\mathbf{r}(t) - \mathbf{u}\,\mathbf{v}(t - nT_s, \tilde{\mathbf{B}})\big)\, dt\, ds \right) \tag{14}$$

Here $\mathbf{Q}(t,s)$ is the inverse of the covariance matrix of the noise vector $\mathbf{w}(t)$ in (8), i.e.

$$\mathbf{Q}(t,s) = \mathbf{I}_{N_R}/(2N_0) \tag{15}$$

where $\mathbf{I}_{N_R}$ is the $N_R \times N_R$ identity matrix, and $(\cdot)^H$ denotes the Hermitian adjoint operator. From (15) it is easily inferred that (14) can be rewritten as

$$\Pr(\mathbf{r}(t) \mid \mathbf{B}[n] = \tilde{\mathbf{B}}, \mathbf{u}) \propto \exp\left(-\int_{t=nT_s}^{(n+2)T_s} \left|\mathbf{r}(t) - \mathbf{u}\, v(t - nT_s, \tilde{\mathbf{B}})\right|^2 dt \,/ 2N_0 \right) \tag{16}$$

Substituting (16) and (13) into (12), carrying out integration with respect to all the components of $\mathbf{u}$ and then substituting (12) in (11) yields, after some manipulation, the strategy

$$\hat{\mathbf{B}}[n] = \max_{\tilde{\mathbf{B}}\in\Omega} \left[\exp\left(\frac{\sigma_a^2}{2N_0^2} \sum_{m=1}^{N_R}\sum_{i=1}^{2} \left|X_{m,i,n}(\tilde{\mathbf{B}})\right|^2 \right) + 2P(\tilde{\mathbf{B}})^{-1/2} + 2Q(\tilde{\mathbf{B}})^{-1/2} \right] \tag{17}$$

where

$$X_{m,i,n}(\tilde{\mathbf{B}}) \doteq \int_{t=0}^{2T_s} r_m(t + nT_s)\, v_i^*(t, \tilde{\mathbf{B}})\, dt \tag{18}$$

with $m = 1,...,N_R$ and $i = 1,2$,

$$P(\tilde{\mathbf{B}}) \doteq 1 - \sigma_a^4 \big(\mathrm{Re}\{S_{12}(\tilde{\mathbf{B}}[n])\}\big)^2/(2N_0^2)$$

$$Q(\tilde{\mathbf{B}}) \doteq 1 - \sigma_a^4 \big(\mathrm{Im}\{S_{12}(\tilde{\mathbf{B}}[n])\}\big)^2/(2N_0^2)$$

$$S_{12}(\tilde{\mathbf{B}}) \doteq \int_{t=0}^{2T_s} v_1(t, \tilde{\mathbf{B}})\, v_2^*(t, \tilde{\mathbf{B}})\, dt \tag{19}$$

and $(\cdot)^*$ denotes complex conjugation.

It is important to point out that: (a) the quantity $X_{m,i,n}(\tilde{\mathbf{B}})$ (18), with $m = 1,...,N_R$ and $i = 1,2$, represents the *correlation* between the received signal $\mathbf{r}(t)$ (8) and the signal $\nu_i(t,\tilde{\mathbf{B}})$ (see (9)-(10)) evaluated in the interval $[nT_s,(n+2)T_s]$; (b) the quantities $\{X_{m,i,n}(\tilde{\mathbf{B}})$, $m = 1,...,N_r$, $i = 1,2\}$ (18) can be also computed sampling the output of proper matched filters, like in the SISO case [1]; (c) $S_{12}(\tilde{\mathbf{B}})$ (19) expresses the *cross-correlation* between $\nu_1(t,\tilde{\mathbf{B}})$ and $\nu_2(t,\tilde{\mathbf{B}})$, and is *independent* of the received signal $\mathbf{r}(t)$; (d) the strategy (17) has been derived under the assumptions that

$$P(\tilde{\mathbf{B}}) > 0 \tag{20}$$

$$Q(\tilde{\mathbf{B}}) > 0 \tag{21}$$

since, if the last inequalities do not hold, the integral of (12) does not lead to a *finite* result.

The detection strategy (17) lends itself to a simple interpretation. In fact, it looks for a codeword producing a significant value of the energy $\sum_{m=1}^{N_R}\sum_{i=1}^{2}\left|X_{m,i,n}(\tilde{\mathbf{B}})\right|^2$ in (17), keeping into account, at the same time, the cross-correlation $S_{12}(\tilde{\mathbf{B}})$ of the signals sent by the two transmit antennas when the trial codeword $\tilde{\mathbf{B}}$ is selected. It is also important to note that the strategy (17) depends on the *average received signal-to-noise ratio* $\bar{E}_s/N_0 = \sigma_a^2 E_s/N_0$. In particular, a threshold for $\bar{E}_s/N_0$ exists, beyond which the inequalities (20) and (21) are not satisfied and the detection strategy (17) is no more optimal. This results in an *error floor*, whose existence is due to the disturbance generated by a non zero cross-correlation $S_{12}(\tilde{\mathbf{B}})$ and that can be avoided *if and only if*

$$S_{12}(\tilde{\mathbf{B}}) = 0 \tag{22}$$

for any codeword $\tilde{\mathbf{B}}$ belonging to the alphabet Ω. Equation (22) introduces some well defined constraints on the codewords of Ω. In fact, substituting (19) in (22) gives

$$\delta_2\left(\exp(j\pi h\delta_1)-1\right)+\delta_1\exp(j\pi h\delta_1)(\exp(j\pi h\delta_2)-1) = 0 \tag{23}$$

where the parameter

$$\delta_m \doteq \tilde{b}_{2m} - \tilde{b}_{1m}$$

must be different from zero for $m = 1,2$, if the equality (22) holds. It can be shown that the equality (23) with the just mentioned constraints holds *if and only if* [8]

$$h\delta_1 = h\delta_2 = s \tag{24}$$

where s is a relative integer different from 0. The last result establishes that the *channel symbols sent by the couple of transmit antennas in the same symbol interval must be different.*

If (22) holds, eq. (17) can be easily simplified as

$$\hat{\mathbf{B}}[n] = \max_{\tilde{\mathbf{B}}\in}\left(\sum_{m=1}^{N_R}\sum_{i=1}^{2}\left|X_{m,i,n}(\tilde{\mathbf{B}})\right|^2\right) \tag{25}$$

Finally, it is worth pointing out that the well known optimal decision strategy for FSK signals exploiting multipath diversity [9] is in perfect agreement with the strategy (25), the only difference being that the former takes advantage of frequency diversity, whereas the latter of space diversity.

4. OPTIMAL SPACE-TIME BLOCK CODING

Further useful criteria for the design of unitary rate STBCs for CPFSK signals can be derived from the optimal detection strategy (25), under the assumption that Ω consists of *orthogonal* codewords, i.e. that (22) is always satisfied. In this case, the *average codeword error probability* P_e can be expressed as

$$P_e = \Pr\left(\bigcup_{\substack{k=0\\k\neq l}}^{3} E_{q,k}\right) \tag{26}$$

where $E_{q,k}$ denotes an error event in a binary decision problem in which the detector selects the wrong codeword $\mathbf{B}_k$ in place of the transmitted (and arbitrary) one $\mathbf{B}_q$. Since (26) cannot be easily evaluated, the corresponding *union bound* (see [5], p. 261)

$$P_e \le \sum_{\substack{k=0\\k\neq q}}^{3} P_{q,k} \tag{27}$$

is considered in the following, where $P_{q,k} \doteq \Pr\{E_{q,k}\}$ denotes the *pairwise error probability* (PEP) associated with $E_{q,k}$.

The optimality criterion we adopt in devising novel ST block coding schemes consists of the *joint minimization* of the PEPs $P_{q,k}$, for $q,k = 0,...,3$, with $q \neq k$. It is important to note that the PEP $P_{q,k}$ can be expressed as (see (27))

$$P_{q,k} = \Pr\left\{\sum_{m=1}^{N_R}\sum_{i=1}^{2}\left|X_{m,i,n}(\mathbf{B}_q)\right|^2 < \sum_{m=1}^{N_R}\sum_{i=1}^{2}\left|X_{m,i,n}(\mathbf{B}_k)\right|^2\right\} \tag{28}$$

The minimization of (28) can be accomplished by jointly maximizing the *left hand side* (LHS) and minimizing the *right hand side* (RHS) of the inequality

$$\sum_{m=1}^{N_R}\sum_{i=1}^{2}\left|X_{m,i,n}(\mathbf{B}_q)\right|^2 < \sum_{m=1}^{N_R}\sum_{i=1}^{2}\left|X_{m,i,n}(\mathbf{B}_k)\right|^2 \tag{29}$$

It can be shown that, on one hand, the minimization of the LHS in the inequality (29) leads to the relationship

$$h\left(b_{2l}^{(q)} - b_{1l}^{(q)}\right) = 2z+1 \tag{30}$$

for $l = 1,2$, where z is an integer. On the other hand, the RHS of (29) cannot be put in a matrix form and for this reason, its minimization is accomplished via an exhaustive computer search.

Note that (30) expresses a couple of constraints on the elements of a *single* ST codeword, whereas the RHS of (29) involves the elements of two *distinct* ST codewords. The bound (27) is minimized if, for each ST codeword, the equalities (30) hold and, for each couple of ST codewords, the RHS of (29) is minimized.

It can be proved that the RHS of (29) is equal to zero, and, consequently, takes on its minimum value, if

$$v_i(t, \mathbf{B}_q) \cdot v_j(t, \mathbf{B}_k) = 0$$

This relationship expresses an orthogonality condition for the signals associated with *different* ST codewords.

In the following we consider a set Ω_O, dubbed *orthogonal set* and consisting of four ST codewords for which: (a) the relation (22) holds; (b) the constraints (30) are satisfied; (c) the RHS of (29) takes on its minimum value, i.e. zero[1].

5. NUMERICAL RESULTS

The error performance of the noncoherent detection algorithm (25) has been assessed in the communication system described in Section 2. The constraints (24) and (30) admit a simple formulation and can be easily satisfied in the code costruction, whereas the minimisation of the RHS in (29) requires an exhaustive search over the set of codes satisfying both constraints (24) and (30). In particular, it has been found out that, if the modulation index $h = 0.5$ is selected, $M = 8$ is the *minimum* value of the parameter M ensuring the existence of *optimal* STBCs, i.e. ensuring the existence of an *orthogonal set* Ω_O. In this case, we have found out that an *optimal* choice for the STBC alphabet Ω is

$$\begin{bmatrix} -5 & -5 \\ -7 & -7 \end{bmatrix}, \begin{bmatrix} -1 & -1 \\ -3 & -3 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 7 & 7 \\ 5 & 5 \end{bmatrix} \qquad (31)$$

It is important to note that the choice $M = 8$ entails a significant bandwidth expansion with respect to a SISO system employing noncoherently detected binary orthogonal FSK [1]. The bandwidth occupancy can be reduced relaxing some of the constraints (24), (29) and (30). It is important to point out that these constraints have different impact on the performance of the receiver. In fact, as it has been explained in Section 3, relaxing the condition (24) leads to an error floor, since this affects the decision strategy (17); on the other hand, relaxing the conditions (29) and (30) does not modify the detectability of the transmitted signals, but affects the diversity of the different codewords. In other words, (24) cannot be relaxed in order to avoid an error floor, whereas neglecting the constraints (29) and (30) allows to reduce the bandwidth occupancy at the price of worsening the error performance. For these reasons, the bandwidth occupancy can be reduced neglecting the constraints (29), i.e. giving up the orthogonality of signals associated with *distinct* codewords; this means that in the optimal noncoherent receiver the response of the filters matched to wrong codewords (in the absence of channel noise) is no more zero. Computer search has enabled us to identify a suboptimal STBC characterized by $h = 0.5$ and $M = 4$ minimising the RHS of (29) (but not cancelling it out); this is characterized by the alphabet

$$\begin{bmatrix} -1 & -1 \\ -3 & -3 \end{bmatrix}, \begin{bmatrix} -3 & -1 \\ 1 & 3 \end{bmatrix}, \begin{bmatrix} -1 & -3 \\ 3 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix} \qquad (32)$$

In all the following performance results it is assumed that: (1) the *signal-to-noise ratio* (SNR) is defined as E_b/N_0, where E_b is the average received energy per information bit and

[1]In Section 5, it will be given one example of *orthogonal set*.

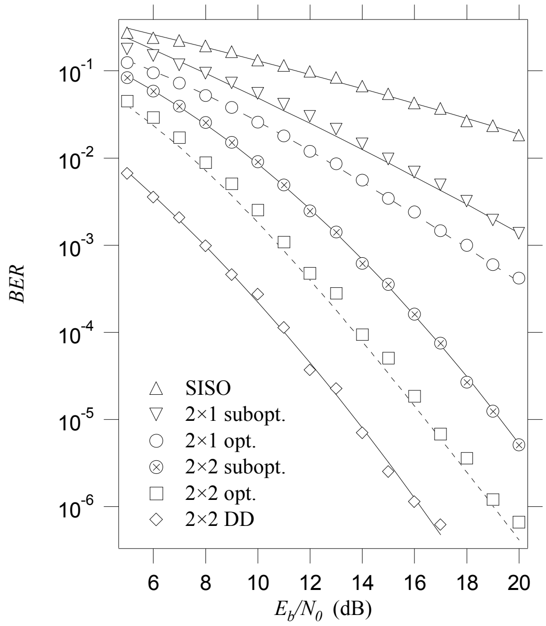

Figure 1 - BER performance of various uncoded systems.

per receive antenna; (2) the quasi-static channel is characterized by a Jakes Doppler spectrum with normalized bandwidth $B_D T_b = 10^{-3}$.

In Fig. 1 the following uncoded systems are compared: a SISO noncoherent system using binary CPFSK ($M = 2$) with $h = 1$ [5] (this choice ensures that the maximum frequency deviation in the SISO system is close to that of the suboptimal MIMO system), MISO 2×1 noncoherent systems with both suboptimal (32) and optimal (31) STBCs, MIMO noncoherent 2×2 systems with both suboptimal (32) and optimal (31) STBCs. The performance of a MIMO 2×2 *coherent* system using binary CPFSK ($M = 2$) with $h = 0.5$ and *delay diversity* (DD) [3] is also shown for comparison. The delay diversity space-time encoder is characterized by the generator matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and achieves *full diversity* [3].

Numerical results evidence that:

- the BER curves for noncoherent systems get steeper as the available spatial diversity increase;
- the energy gain offered by the optimal code (31) respect with the subptimal one (32) is about 3 dB, but is attained at the price of a larger frequency deviation, i.e. of a larger bandwidth;
- the coherent MIMO 2×2 system performs 3 (6) dB better than the optimal (suboptimal) noncoherent MIMO 2×2 and has a smaller (the same) frequency deviation, at the price, however, of a substantiall larger computational complexity;
- the BER curves referring to systems equipped with the same number of antennas have the same slope, since all achieve *full diversity* [8];
- the energy gap among the coherent system and the noncoherent systems gets higher when the spatial di-

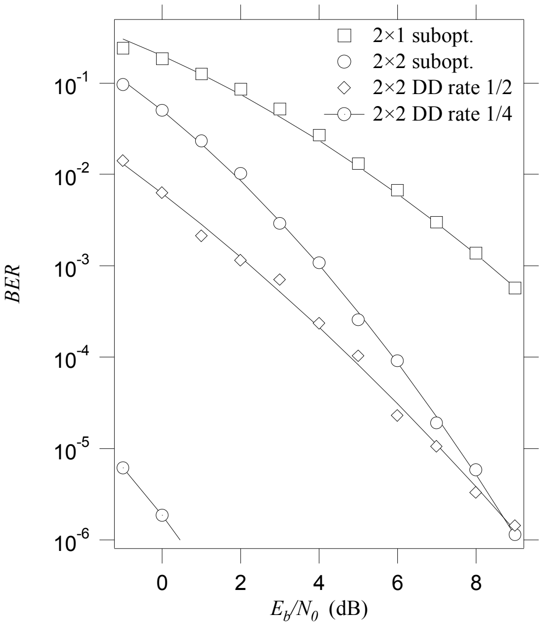

Figure 2 - BER performance of various coded MIMO system.

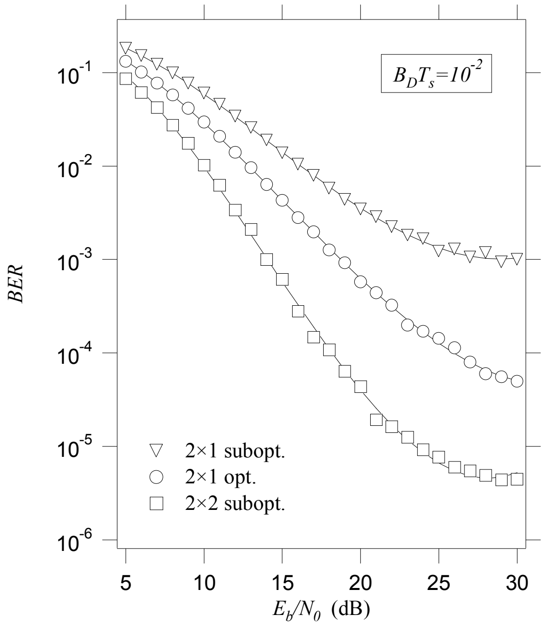

Figure 3 - BER performance of various noncoherent uncoded systems for $B_DT_b = 10^{-2}$.

versity increases, because the coherent system can rely on the channel knowledge and, thus, can exploit more effectively the available spatial diversity.

In Fig. 2 coded systems characterized by different bandwidth occupancies are compared. Both the noncoherent systems employ a rate $1/2$ 4-state convolutional channel code, whereas the two coherent systems adopt different codes: one is a rate $1/2$ 4-state channel code, the other is a rate $1/4$ 4-state code. Note that the coherent system with code rate $1/4$ has a bandwidth occupancy comparable to that of the noncoherent systems, whereas the coherent system with code rate $1/2$ has a smaller bandwidth. Numerical results show that the MIMO 2×2 coded noncoherent system outperforms the MIMO 2×2 coherent system with the same code rate, but requires a larger bandwidth occupancy; on the other hand, the MIMO 2×2 coherent system with code rate $1/4$ can provide an energy saving of 6 dB with respect to the MIMO 2×2 coded noncoherent system, at the price of a substantial larger computational complexity and with a comparable bandwidth occupancy.

The robustness of the proposed noncoherent ST coded systems with respect to the time selectivity of the channel is evidenced by the results shown in Fig. 3. Here $B_DT_b = 10^{-2}$ has been considered, and the error performance of both optimal and suboptimal 2×1 MISO noncoherent systems and suboptimal 2×2 MIMO noncoherent system is illustrated. We note that the spatial diversity offered by the channel lowers the error floor in the considered noncoherent systems.

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