14th European Signal Processing Conference (EUSIPCO 2006), Florence, Italy, September 4-8, 2006, copyright by EURASIP

New Results in Iterative Frequency-Domain Decision-Feedback Equalization

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ABSTRACT

Single-carrier transmission with frequency-domain equalization (SCT/FDE) is today recognized as an attractive alternative to orthogonal frequency-division multiplexing (OFDM) for wireless applications with large channel dispersions. In this paper, we investigate iterative frequency-domain decision-feedback equalization (FD/DFE), which significantly improves performance compared to minimum mean-square error (MMSE) and zero-forcing (ZF) linear equalizers. We introduce a new FD/DFE and compare it to previously proposed equalizers.

1. INTRODUCTION

Despite the strong popularity of orthogonal frequency-division multiplexing (OFDM) and its adoption in many wireless communications standards such as the IEEE 802.11 standard for wireless local area networks (LANs) [1] and the IEEE 802.16 standard for broadband wireless access (BWA) [2], single-carrier transmission with frequency-domain equalization (SCT/FDE) is today recognized as an attractive alternative. This is particularly true for the uplink, because the transmit power of user terminals must be used as efficiently as possible. Indeed, the peak-to-average power ratio (PAPR) of OFDM signals is very high, and the transmit power amplifier must be substantially backed off from its saturation point to limit nonlinear signal distortion. This is one of the major problems associated to OFDM. Frequency-domain equalization of single-carrier systems was originally proposed in [3] and [4], and the concept was further developed in [5] by incorporating a time-domain feedback filter.

The original papers on SCT/FDE ([3], [4] and several other papers by the same authors) considered linear equalizers optimized under the minimum mean-square error (MMSE) or the zero-forcing (ZF) criterion. The purpose there was to introduce the concept and point out that this technique offers similar performance to OFDM while avoiding its PAPR and synchronization problems. Obviously, linear equalizers have serious performance limitations on highly distorted channels, and it is desirable to use a nonlinear equalizer structure, such as a decision-feedback equalizer (DFE), instead. Such attempts were made in [5] and [6], but the feedback part of the DFE was kept in the time domain. The number of feedback coefficients in this approach must be kept small, and this limits the performance improvement with respect to linear equalizers. Further work on the subject considered a DFE structure that is fully in the frequency domain and the iterative block DFE structure first introduced in [7] was extended to the frequency domain in [8].

In a recent paper, the present authors introduced an iterative frequency-domain DFE structure in which the feedforward filter shifts linearly from a linear MMSE filter at the first pass to a matched filter at the last iteration [9]. That DFE was shown to achieve a performance that is close to the iterative DFE performance of [8] while significantly reducing implementation complexity. In the present paper, we introduce another optimization criterion for frequency-domain iterative DFEs. The feedforward and feedback coefficients are optimized to perfectly equalize the channel (zero-forcing equalization) and also minimize the sum of filtered noise and decision error powers at the threshold detector input. This equalizer is compared to the iterative DFE optimized under the MMSE criterion [8] and to the MF-based iterative DFE presented in [9].

The paper is organized as follows: In Section 2, we give a brief review of frequency-domain equalization and describe the previously proposed iterative frequency-domain DFEs. In Section 3, we describe the new iterative DFE structure. Section 4 reports some computer simulation results to assess the performance of the proposed receiver and compare it to the scheme described in [8]. Finally, we give our conclusions in Section 5.

2. A REVIEW OF FREQUENCY-DOMAIN EQUALIZATION

Single-carrier transmission is the conventional approach to digital communications. With time-domain equalization (TDE), this technique has been used for decades on time-dispersive channels. Despite this, there was a widely shared perception within the digital broadcasting community in the early 1980' that single-carrier transmission would not work for mobile reception, and OFDM was viewed as the only realistic transmission technique for this application.

Then, in [3], [4] and some subsequent papers, H. Sari *et al.* proposed SCT/FDE as an alternative to OFDM and showed that this technique can achieve the performance of OFDM while avoiding its main drawbacks which are its high PAPR and the necessity of local oscillators with significantly reduced phase noise for carrier synchronization. Subsequent work by other authors led to similar conclusions, and SCT/FDE was recently adopted in the IEEE 802.16 specifications as one of the modes of operations of broadband wireless access (BWA) systems operating at frequencies between 2 and 11 GHz. Note that since standards do not specify receivers, the IEEE specifications do not explicitly mention FDE, but a cyclic prefix is provisioned so that FDE structures can be used as described in [3] and [4].

In order to improve the performance of SCT/FDE, it was proposed in [5] and [6] to use a DFE with time-domain feedback. Although this approach gives some performance enhancement compared to linear equalization, the resulting improvement is limited, because the feedback part has only a small number of coefficients and can only compensate for causal interference.

More recently, other SCT/DFE schemes were proposed where both the feedforward and the feedback parts of the equalizer are implemented in the frequency domain [8]. Furthermore, the DFE was made iterative by using the decision block of the previous iteration to compute a new equalizer output. To describe this DFE structure, suppose that (a_1, a_2, …., a_N) is a symbol block and that (x_1, x_2, …., x_N) is the corresponding received signal block. The received block is fed to the DFT operator, whose output block is denoted (X_1, X_2, …., X_N). The equalizer multiplies this signal block with its feedforward coefficients (F_1, F_2, …., F_N), and the resulting signal block enters an inverse DFT, which yields the output block (y_1, y_2, …., y_N) on which the threshold detector bases its first decisions for the transmitted signal block.

Once the receiver makes a first set of decisions, the decision block is fed to a feedback filter with coefficients (B_1, B_2, …., B_N), and an iterative DFE is implemented. At the *k*th iteration, the feedforward and feedback filter block supplies

$$Y_n(k) = F_n(k) X_n + B_n(k) \hat{A}_n(k-1) \tag{1}$$

for the *n*th frequency bin, where $F_n(k)$ and $B_n(k)$ are respectively the corresponding feedforward and feedback filter coefficients at the *k*th iteration, and $\hat{A}_n(k-1)$ is the *n*th frequency bin content of the equalizer decision block at the previous iteration. A general block diagram of an iterative frequency-domain DFE is shown in Fig. 1.

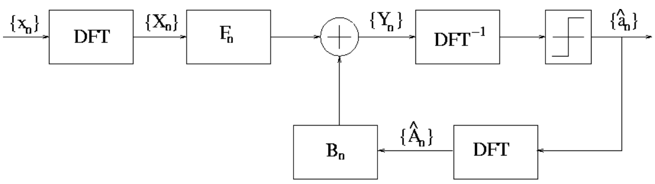

Fig. 1: General block diagram of an iterative frequency-domain DFE.

The first pass decisions of all iterative DFEs are obtained using a linear equalizer optimized under the MMSE criterion. This corresponds to the equalizer coefficients:

$$F_n(0) = \frac{H_n^*}{|H_n|^2 + \sigma_w^2 / \sigma_a^2} \tag{2}$$

and

$$B_n(0) = 0 \tag{3}$$

where H_n is *n*th bin frequency response of the channel, σ_w^2 is the additive noise power, and σ_a^2 is the power of the transmitted data symbols.

2.1. Iterative MMSE DFE

In the iterative DFE of [8], the coefficients are optimized under the MMSE criterion. Derivation of the feedforward and feedback filter coefficients requires the computation of the correlation between the transmitted data vector and the decisions from the previous iteration. Denoting this correlation at the kth iteration by ρ_k, the equalizer coefficients are given by

$$F_n(k) = \frac{H_n^*}{\sigma_a^2\left[1-\left(\rho_{k-1}^2\right)\right]|H_n|^2 + \sigma_w^2} \tag{4}$$

and

$$B_n(k) = \rho_{k-1}\left[F_n(k) H_n - \frac{1}{N}\sum_{m=1}^{N} F_m(k) H_m\right] \tag{5}$$

Computation of the ρ_k coefficients in this equalizer is quite involved, and therefore, it is of interest to look for simpler iterative DFEs.

2.2. Iterative Matched Filter Based DFE

In the iterative DFE proposed in [9], the feedforward filter is a matched filter (MF) at the final iteration. The idea is to maximize the SNR by the feedforward filter, and then to restore the ideal channel frequency response by the feedback filter. Obviously, the MF cannot be used as the feedforward filter until a set of decisions is available, because it increases ISI, and compensation of this requires a feedback filter. Therefore, the feedforward filter in that DFE shifts linearly from a linear MMSE filter at the first pass to the MF at the last iteration.

At the *k*th iteration, the feedforward and the feedback filter coefficients are respectively given by

$$F_n(k) = \alpha_k \frac{H_n^*}{|H_n|^2 + \sigma_w^2 / \sigma_a^2} + (1-\alpha_k) H_n^* \tag{6}$$

and

$$B_n(k) = 1 - F_n(k) H_n \tag{7}$$

The first equalizer decisions are obtained using $\alpha_0 = 1$, i.e., clearly the equalizer is a linear MMSE equalizer. Then, the α_k parameter decreases linearly as

$$\alpha_k = 1 - k / K, \tag{8}$$

where K is the number of iterations. At the last iteration, $\alpha_K = 0$, and the feedforward filter is a matched filter.

At all iterations, the feedback filter is computed such that the combined channel and equalizer response is ideal (flat frequency response and linear phase) when the decisions from the previous iteration are all correct. In the present paper, we use this MF-based iterative DFE in a slightly different manner. After making the first pass decisions using an MMSE LE, we switch to the coefficients

$$F_n(k) = H_n^* \tag{9}$$

$$\begin{aligned} B_n(k) &= 1 - F_n(k) H_n \\ &= 1 - |H_n|^2 \end{aligned} \tag{10}$$

for all iterations. That is, unlike the DFE of [9], where the switch from the MMSE to the MF filter is performed linearly over a number of iterations, the switch is done here abruptly, and the MF is used as the feedforward filter at all iterations.

3. THE NEW EQUALIZER

As in the previously proposed iterative frequency-domain DFEs, the first pass decisions are obtained using a simple linear MMSE filter. For iterating the decisions, the equalizer output at the kth iteration is written as:

$$Y_n(k) = F_n(k) X_n + B_n(k) \hat{A}_n(k-1) \tag{11}$$

where $\hat{A}_n$ represents the symbol decision block transformed to the frequency domain. This equation can be developed as follows:

$$\begin{aligned} Y_n(k) &= F_n(k)(H_n A_n + W_n) + B_n(k)(A_n + E_n(k)) \\ &= [F_n(k) H_n + B_n(k)] A_n + F_n(k) W_n + B_n(k) E_n(k) \end{aligned}$$

In this equation, H_n is the channel frequency response at the nth frequency bin, W_n is the noise term at the same bin, and $E_n(k)$ is the nth bin component of the decision error block at the kth iteration.

The first requirement for the equalizer coefficients is that the coefficient of A_n in the first term of $Y_n(k)$ must be equal to 1. This ensures that the channel frequency response is perfectly equalized. This reads:

$$B_n(k) = 1 - F_n(k) H_n$$

The second requirement is to minimize the sum of noise and decision error powers at the threshold detector input.

$$MinJ = Min\left\{ \sigma_w^2 \sum_n |F_n(k)|^2 + \sigma_e^2(k) \sum_n |B_n(k)|^2 \right\}$$

where σ_w^2 is the noise power and σ_e^2 is the power of the equalized decision errors. The derivative of this function with respect to the feedforward coefficients is:

$$\frac{\partial J}{\partial F_n(k)} = 2\left\{ F_n(k)\sigma_w^2 - H_n^*[1 - F_n(k) H_n]\sigma_e^2(k) \right\}$$

The optimum values of the feedforward coefficients are obtained by setting this derivative to zero. We get:

$$F_n(k) = \frac{H_n^*}{|H_n|^2 + \sigma_w^2 / \sigma_e^2(k)}$$

Focusing on QPSK, an estimate of the decision errors power can be determined as follows. At low bit error rate (BER) values, most decision errors will be toward one of the adjacent symbol values, which means that only one of the components (in-phase or quadrature) will be in error. Then we have

$$\sigma_e^2(k) = 2\sigma_a^2 P_e(k-1)$$

where σ_e^2 is the power of the transmitted symbols and $P_e(k-1)$ designates the decision error probability at the previous iteration.

4. PERFORMANCE ANALYSIS

Performance of the presented frequency-domain iterative DFEs was investigated using the quaternary phase-shift keying (QPSK) modulation and two channel models. The first channel is the Stanford University Interim 4 (SUI 4) channel model used by the IEEE 802.16 Group to assess performance of broadband wireless access systems for fixed and nomadic services. SUI channel models correspond to propagation along two delayed paths in addition to the main signal path, each of these paths being characterized by its attenuation and delay relative to the main path.

In the SUI 4 model, the second path is delayed by 1.5 µs and attenuated by 4 dB, and the third path is delayed by 4 µs and attenuated by 8 dB relative to the first path. Each of these signal paths is also subjected to Rayleigh fading, but this was not included in our simulations. The second channel model is the UMTS Vehicular A channel model given by five delayed paths with respective delays of [310, 710, 1090, 1730, 2510] ns and [-1.0, -9.0, -10.0, -15.0, -20.0] dB attenuation relative to the main path. The equalizer was implemented using 256-point DFTs, and in all of the simulations, the channel was assumed perfectly known from the receiver.

The results obtained using the *SUI 4* channel model are depicted in Fig. 2. The figure shows the results corresponding to the linear equalizer (MMSE-LE), which is used in the first pass, and those corresponding to the first iteration in each DFE. This was deemed sufficient to plot, as further iterations only gave very slight improvements.

We can see that the new DFE performs indeed better than the MMSE DFE and MF-based DFE at bit error rate (BER) values lower than 10^{-4}.

Next, the simulation results using the UMTS Vehicular A channel are reported in Fig. 3. Here too, only the results corresponding to the first iteration are plotted, because further iterations bring little improvement.

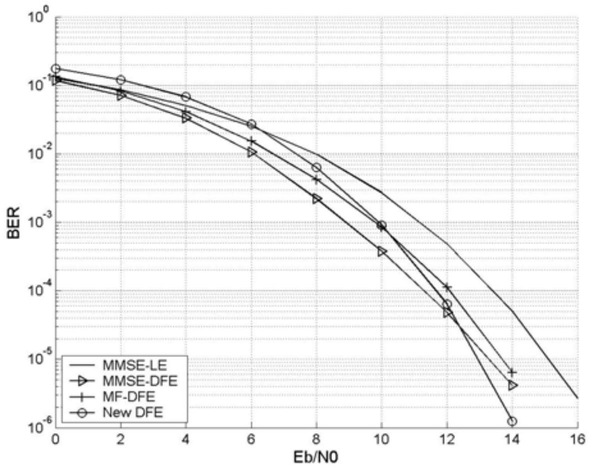

Fig. 2: BER performance of the 3 iterative DFEs on the SUI 4 channel model.

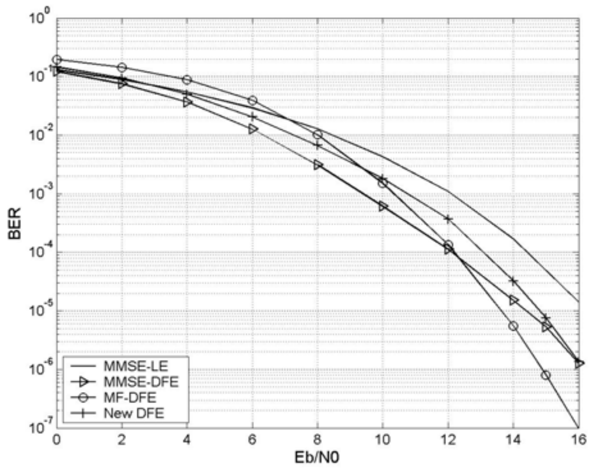

Fig. 3: BER performance of the 3 iterative DFEs on the UMTS Vehicular A channel model.

The results show the same type of behaviour as on the SUI 4 channel. Indeed, the new DFE lead to a lower performance at BER higher than 10^{-4}, but it outperforms the other two DFEs at lower BER values.

5. CONCLUSIONS

We have presented and compared three different iterative frequency-domain decision-feedback equalizers for single-carrier systems. The first one is an iterative DFE introduced in [8] whose coefficients are optimized under the MMSE criterion. The second DFE is a particular version of the equalizer introduced by the present authors in [9], which consists of using a matched filter as the feedforward filter and a feedback filter which restores the signal spectrum after matched filtering, assuming that the equalizer decisions at the previous iteration are all correct. Finally, the third DFE is a new DFE structure, which takes into account decision errors and minimizes the sum of noise power and decision errors power at the threshold detector input while perfectly equalizing the channel frequency response. Using two channel models, the three iterative DFE structures were compared, and the results indicated that the iterative DFE proposed in this paper leads to the best performance at high SNR values.

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