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DIRECT EQUALIZATION OF MULTIUSER DOUBLY-SELECTIVE CHANNELS BASED ON SUPERIMPOSED TRAINING

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ABSTRACT

Design of doubly-selective linear equalizers for multiuser frequency-selective time-varying communications channels is considered using superimposed training and without first estimating the underlying channel response. Both the time-varying channel as well as the linear equalizers are assumed to be described by a complex exponential basis expansion model (CE-BEM). User-specific periodic (non-random) training sequences are arithmetically added (superimposed) to the respective information sequences at the transmitter before modulation and transmission. There is no loss in information rate. Knowledge of the superimposed training specific to the desired user and properties of the other training sequences are exploited to design the equalizers. An illustrative simulation example is presented.

1. INTRODUCTION

Multiple access schemes allow multiple users to share a common channel. Random access methods provide each user a flexible way of gaining access to the channel whenever the user has information (packets) to be sent. In random access, typically when two packets collide, they are discarded and then have to be retransmitted. In wireless *ad hoc* networks (also called mobile *ad hoc* networks – MANETs), absence of base stations limits the use of traditional MAC protocols [3]. In *ad hoc* networks one needs some sort of distributed MAC requiring some form of random access which makes avoiding collisions difficult. Collisions arising from uncoordinated users decrease system throughput and worsen delay performance. Multiple packet reception (MPR) capability (or signal separation) is one way to resolve packet collisions and thereby enhance throughput, by using signal processing to separate multiple received signals [9]. Recently wireless *ad hoc* networks with asynchronous transmissions have been considered in [2], [7] and [8]. The approaches of [7] and [2] use user-specific modulation induced cyclostationarity coupled with receive antenna array to achieve MPR for frequency-selective time-invariant channels. In [8] user-specific superimposed training signals (also called hidden pilots or implicit training) have been used for MPR for frequency-selective time-invariant channels. The objective of this paper is to investigate approaches using user-specific superimposed training signals for MPR in MANETs for transmissions over doubly selective (frequency- and time-selective) channels, with emphasis on asynchronous networks.

Consider a time-varying MIMO (multiple-input multiple-output) FIR (finite impulse response) linear channel with M inputs (users) and N outputs (receiver array with N elements at the destination node). Let $\{s_m(k)\}$ denote the m-th user's information sequence which is input to the MIMO doubly-selective channel with the m-th user's discrete-time impulse response $\{\mathbf{h}_m(k;l)\}$ (N-vector channel response at time k to a unit input at time $k-l$). Consider a typical (one-hop) MANET structure in an asynchronous mode. Assume M active users with a packet length of S symbols, in the coverage area of the node under evaluation. Each node is equipped with N (≥ 1) receive antennas and receiver node processes a data record block of size T $(\geq S)$ symbols. Various packets can be located anywhere within this observation block. Using a sliding block approach (as in [7] and [2]), we assume that the packet of interest is totally within the observation block. [An energy detector or related approaches can be used to ensure this [7],[2].] The noisy received (baseband-equivalent, symbol-rate) signal at the node-of-interest at time k is an N-column vector $\mathbf{y}(k)$, $k_1 \leq k \leq k_1+T-1$, given by (k_1 is the "initial" time of the observation block)

$$\mathbf{y}(k) := \sum_{m=1}^{M}\sum_{l=0}^{L}\mathbf{h}_m(k;l)s_m(k-l)+\mathbf{v}(k), \tag{1}$$

In **superimposed training**-based approaches, for user m, one takes

$$s_m(k) = b_m(k) + c_m(k) \tag{2}$$

in (1) where $\{b_m(k)\}$ is the information sequence and $\{c_m(k)\}$ is a user-specific non-random periodic training (pilot) sequence. Exploitation of the periodicity of $\{c_m(k)\}$ allows identification (and equalization) of the multiuser channel without allocating any explicit time slots for training. Note that there is no bandwidth expansion or information rate reduction as a consequence of superimposed training; there is a reduction in effective SNR since power allocated to training could otherwise have been allocated to information sequence.

In a complex exponential basis expansion representation it is assumed that [5]

$$\mathbf{h}_m(n;l) = \sum_{q=-K/2}^{K/2}\mathbf{h}_{mq}(l)e^{j\omega_q n} \tag{3}$$

where the N-column vectors $\mathbf{h}_{mq}(l)$ are time-invariant. Such models have been used in [1] and [5], among others; see Sec. 2 for details.

Objectives and Contributions: The main problem considered here is: how to design an equalizer to estimate $\{b_1(n)\}$, the information sequence of user 1 (the desired user), when one knows only $\{c_1(n)\}$ but not (obviously) $\{b_1(n)\}$ and one does not also have (frame) synchronization with $\{c_1(n)\}$ at the receiver. We will design an equalizer to estimate $\{c_1(n)\}$ with a delay d. We will then show that this equalizer is a scaled version of the corresponding equalizer designed to estimate $\{b_1(n)\}$ with a delay d provided that $\{c_m(n)\}$ satisfy certain properties.

Notation: Superscripts H, $*$ and T denote the complex conjugate transpose, the complex conjugation and the transpose operations, respectively. $\delta(\tau)$ is the Kronecker delta and I_N is the $N \times N$ identity matrix.

2. SYSTEM MODEL

Consider a time-varying channel with continuous-time, baseband received signal $x(t)$ and transmitted signal $s(t)$ (with symbol interval T_s sec.) related by impulse response $h(t;\tau)$ (response at time t to a unit impulse at time $t-\tau$).

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Let τ_d denote the (multipath) delay-spread of the channel and let f_d denote the Doppler spread of the channel. If $x(t)$ is sampled once every T_s sec. (symbol rate), then by [5] (see also [1]), for $t = nT_s+t_0 \in [t_0, t_0+TT_s)$, the sampled signal $x(n) := x(t)|_{t=nT_s+t_0}$ has the representation

$$x(n) = \sum_{l=0}^{L} h(n;l)s(n-l) \tag{4}$$

where

$$h(n;l) = \sum_{q=-K/2}^{K/2} h_q(l)e^{j\omega_q n}, \;\; L := \lfloor \tau_d/T_s \rfloor, \tag{5}$$

$$w_q = \frac{2\pi q}{T}, \;\; K := 2\lceil f_d T T_s \rceil. \tag{6}$$

The above representation is valid over a duration of TT_s sec. (T samples). Eqn. (1) arises if we follow (4) and consider an SIMO model per user arising due to multiple antennas at the receiver.

Each user is assigned (or selects) a user-specific training sequence. Ref. [7] uses (at the risk of some confusion we use m as the user index as well as the training sequence index)

$$c_m(k) = \sigma_{cm} e^{j2\pi\left[fk^2+\alpha_m k\right]}, \tag{7}$$

$$f = T_f^{-1}, \; \alpha_m = \frac{m-1}{D}, \; m = 1,2,\cdots,D \geq M \tag{8}$$

and [8] uses the same set. The above sequence is periodic with period $P = DT_f$ where D, T_f and σ_{cm} are design parameters (D and T_f are coprime). Different users are characterized by different α_m's and distinct sequences are mutually orthogonal and individually periodic white. [There is a common codebook at each node of size D containing the possible values of α_m. During the "initial contact" period, a given node searches for all possible D signals.] In a different context, in [6] and [10], we have proposed the following choice. Let

$$c_0(k) = c_0(k+n\tilde{P}), \;\; \forall k, n \tag{9}$$

be a maximal length pseudo-random binary sequence (m-sequence). Then we design

$$c_m(k) := e^{j2\pi(m-1)k/P} c_0(k) = c_m(k+nP), \tag{10}$$

$$m = 1,2,\cdots,D \geq M, \;\; P := D\tilde{P}. \tag{11}$$

The above sequences are periodic with period P, mutually orthogonal and individually "nearly" periodic-white with period $\tilde{P}$.

Given the knowledge of the time-varying channel described by CE-BEM, design of (serial) time-varying FIR equalizers has been discussed in [1]. Direct design of time-invariant FIR equalizers based on superimposed training, for time-invariant channels, has been investigated in [8]. In this paper we investigate direct design of time-varying FIR linear equalizers for doubly selective channels using superimposed training and without first estimating the underlying channel response. We exploit the prior results of [1] and [8].

3. TIME-VARYING FIR EQUALIZERS

We will restrict ourselves to serial linear equalizers instead of block linear equalizers, since as shown in [1], the latter are computationally prohibitive (compared with the former). We look for a time-varying linear equalizer $\mathbf{g}(n;l)$ ($l = 0,1,\cdots,L_e$) over the same time-block as the received data with channel model (3). We note that for an arbitrary time-varying impulse response $\tilde{\mathbf{g}}(n;l)$, the following is always true

$$\tilde{\mathbf{g}}(n;l) = \sum_{q=-(T-1)/2}^{(T-1)/2} \tilde{\mathbf{g}}_q(l)e^{j\omega_q n}, \;\; n = 0,1,\cdots,T-1. \tag{12}$$

We would like to use a more parsimonious (but approximate) representation for $\tilde{\mathbf{g}}(n;l)$, denoted by $\mathbf{g}(n;l)$, given by

$$\mathbf{g}(n;l) = \sum_{q=-Q/2}^{Q/2} \mathbf{g}_q(l)e^{j\omega_q n}, \;\; n = 0,1,\cdots,T-1, \tag{13}$$

where $Q \ll (T-1)$. In order to estimate the input sequence of the desired user (user 1, with no loss of generality) $\{s_1(n)\}$ (see (1)), we may seek a linear time-varying FIR estimator to yield an estimate with equalization delay d

$$\hat{s}(n-d) = \sum_{i=0}^{L_e} \mathbf{g}^H(n;i)\mathbf{y}(n-i). \tag{14}$$

Existence of a zero-forcing linear equalizer (for $M = 1$) has been discussed in [1]. Their conclusion is that if N is at least 2, then with probability one, one has a zero-forcing solution for sufficiently large L_e and Q. For linear MMSE solution, existence is not an issue, although MMSE equalizer performance can be expected to be "good" if zero-forcing equalizers exist [1]. In this paper we will seek a least squares solution $\mathbf{g}(n;l)$ to minimize a cost such as

$$\frac{1}{T}\sum_{n=0}^{T-1} \left| s_1(n-d) - \hat{s}_1(n-d) \right|^2. \tag{15}$$

4. LINEAR LEAST-SQUARES FIR CE-BEM EQUALIZERS

We first state the underlying model assumptions.

(H1) The information sequence $\{b_m(n)\}$ is zero-mean, i.i.d. (independent and identically distributed), with $E\{|b_m(n)|^2\} = \sigma_{bm}^2$. They are also independent across users ($m = 1,2,\cdots,M$).

(H2) The measurement noise $\{\mathbf{v}(n)\}$ is **zero-mean** ($E\{\mathbf{v}(n)\} = \mathbf{0}$), white, independent of $\{b_m(n)\}$, with $E\{[\mathbf{v}(n+\tau)][\mathbf{v}(n)]^H\} = \sigma_v^2 I_N \delta(\tau)$.

(H3) The superimposed training sequence $c_m(n) = c_m(n+kP)$ $\forall k, n$ is a non-random periodic sequence with period P. Let $\sigma_{cm}^2 := (1/P)\sum_{n=1}^{P} |c_m(n)|^2$. The sequences are chosen as (7)-(8) or (9)-(11).

(H4) Record length T and period P satisfy $TP^{-1} > K$ and TP^{-1} is an integer. Moreover, $\tilde{P} > L + L_e - d$ where d (≥ 0) is the desired equalization delay and $\tilde{P} = T_f$ in (8), or as in (9).

It then follows that ([8], [10])

$$P^{-1}\sum_{n=0}^{P-1} c_m(n)c_k^*(n-\tau) = \gamma_m(\tau)\delta(\tau \bmod \tilde{P})\delta(m-k) \tag{16}$$

where

$$\gamma_m(\tau) = \begin{cases} \sigma_{cm}^2 e^{j2\pi m\tau/D} & \text{for } (7)-(8) \\ \sigma_{cm}^2 e^{j2\pi\tau(m-1)/P} & \text{for } (9)-(11). \end{cases} \tag{17}$$

4.1. Equalizer for Training Estimation

The periodic training sequence can be written as

$$c_m(n) = \sum_{p=0}^{P-1} c_{mp} e^{j\alpha_p n} \tag{18}$$

where $\alpha_p := \frac{2\pi p}{P}$. To design the time-varying linear equalizer to estimate a delayed version of the desired user's training sequence $c_1(n-d)$ $(0 \le d \le L_e)$:

$$\hat{c}_1(n-d) = \sum_{i=0}^{L_e} \mathbf{g}_d^H(n;i)\mathbf{y}(n-i) \tag{19}$$

where we assume that

$$\mathbf{g}_d(n;i) = \sum_{q=-Q/2}^{Q/2} \mathbf{g}_q(i) e^{j\omega_q n}. \tag{20}$$

Choose $\mathbf{g}_q(i)$'s to minimize the time-averaged cost

$$J_c := \frac{1}{T}\sum_{n=0}^{T-1} |c_1(n-d) - \hat{c}_1(n-d)|^2 \tag{21}$$

$$= \frac{1}{T}\sum_{n=0}^{T-1}\left| c_1(n-d) - \sum_{i=0}^{L_e}\sum_{q=-Q/2}^{Q/2} \mathbf{g}_q^H(i) e^{-j\omega_q n}\mathbf{y}(n-i)\right|^2. \tag{22}$$

By taking the derivative and setting it to be zero, we have

$$0 = \frac{\partial J_c}{\partial \mathbf{g}_{q_1}^*(i_1)} = -\frac{1}{T}\sum_{n=0}^{T-1} e^{-j\omega_{q_1} n}\mathbf{y}(n-i_1)$$
$$\times \left[c_1^*(n-d) - \sum_{i=0}^{L_e}\sum_{q=-Q/2}^{Q/2} e^{j\omega_q n}\mathbf{y}^H(n-i)\mathbf{g}_q(i)\right] \tag{23}$$

for $i_1 = 0, 1, \cdots, L_e$ and $q_1 = -Q/2, 1-Q/2, \cdots, Q/2$. This leads to

$$\sum_{i=0}^{L_e}\sum_{q=-Q/2}^{Q/2}\left[\frac{1}{T}\sum_{n=0}^{T-1} e^{j(\omega_q-\omega_{q_1})n}\mathbf{y}(n-i_1)\mathbf{y}^H(n-i)\right]\mathbf{g}_q(i) =$$

$$\frac{1}{T}\sum_{n=0}^{T-1} c_1^*(n-d)e^{-j\omega_{q_1}n}\mathbf{y}(n-i_1) =: \mathbf{R}_c(q_1, i_1). \tag{24}$$

4.2. Equalizer for Data Estimation

To design the time-varying linear equalizer to estimate the desired user's information sequence $b_1(n-d)$ $(0 \le d \le L_e)$,

$$\hat{b}_1(n-d) = \sum_{i=0}^{L_e} \bar{\mathbf{g}}_d^H(n;i)\mathbf{y}(n-i) \tag{25}$$

where we assume that

$$\bar{\mathbf{g}}_d(n;i) = \sum_{q=-Q/2}^{Q/2} \bar{\mathbf{g}}_q(i) e^{j\omega_q n}. \tag{26}$$

Choose $\bar{\mathbf{g}}_q(i)$'s to minimize

$$J_b := \frac{1}{T}\sum_{n=0}^{T-1}\left| b_1(n-d) - \hat{b}_1(n-d)\right|^2. \tag{27}$$

Mimicking the results for the superimposed training sequence-based equalization, we have

$$\sum_{i=0}^{L_e}\sum_{q=-Q/2}^{Q/2}\left[\frac{1}{T}\sum_{n=0}^{T-1} e^{j(\omega_q-\omega_{q_1})n}\mathbf{y}(n-i_1)\mathbf{y}^H(n-i)\right]\bar{\mathbf{g}}_q(i)$$

$$= \frac{1}{T}\sum_{n=0}^{T-1} b_1^*(n-d)e^{-j\omega_{q_1}n}\mathbf{y}(n-i_1) =: \mathbf{R}_b(q_1, i_1). \tag{28}$$

4.3. When Are The Two Equalizers Equal?

Comparing (24) and (28), we see that (ignoring the equalizer coefficients) the left-sides of the two are identical whereas the right-sides are different. We now seek to establish that for large T, $\mathbf{R}_c(q_1, i_1) = \beta\mathbf{R}_b(q_1, i_1)$ $\forall\, q_1, i_1$, for some scalar β, so that $\mathbf{g}_q(i) = \beta\bar{\mathbf{g}}_q(i)$ $\forall\, i$.

It is shown in the Appendix that ($\tilde{\gamma}_1 := \sum_{l=0}^{L} \gamma_1(d-i_1-l)\delta((d-i_1-l) \bmod \tilde{P})$)

$$\lim_{T\to\infty} \mathbf{R}_c(q_1, i_1) \overset{\text{m.s.}}{=}$$

$$\begin{cases} \tilde{\gamma}_1^* e^{-j\omega_{q_1} i_1}\mathbf{h}_{1q_1}((d-i_1) \bmod \tilde{P}) & \text{if } |q_1| \le K/2 \\ 0 & \text{otherwise} \end{cases} \tag{29}$$

for $i_1 = 0, 1, \cdots, L_e$ and $q_1 = -Q/2, 1-Q/2, \cdots, Q/2$. It is also shown that for $i_1 = 0, 1, \cdots, L_e$ and $q_1 = -Q/2, 1-Q/2, \cdots, Q/2$ but $|q_1| \le K/2$,

$$\lim_{T\to\infty} \mathbf{R}_b(q_1, i_1) \overset{\text{m.s.}}{=} \mathbf{h}_{1q_1}(d-i_1)e^{-j\omega_{q_1} i_1}\sigma_{b1}^2. \tag{30}$$

If $\tilde{P} > L + L_e - d$, then (29) equals (30) (within a scale factor) and $\tilde{\gamma}_1 = \sigma_{c1}^2$. Therefore, for "large" T, $\mathbf{R}_c(q_1, i_1) = \beta\mathbf{R}_b(q_1, i_1)$ $\forall\, q_1, i_1$ with $\beta = \sigma_{c1}^2/\sigma_{b1}^2$; hence $\mathbf{g}_q(i) = \beta\bar{\mathbf{g}}_q(i)$ $\forall\, i$.

4.4. Desired Equalizer Design

We execute the following steps:

(i) Pick L_e and d ($= \frac{L_e}{2}$ in Sec. 5). Pick $Q \ge K$, $\tilde{P} > L + L_e - d$.

(ii) Solve (24), given data $\mathbf{y}(n)$, for $\mathbf{g}_q(i)$ where $0 \le i \le L_e$ and $-\frac{Q}{2} \le q \le \frac{Q}{2}$. Then

$$\mathbf{g}_d(n;i) = \sum_{q=-Q/2}^{Q/2} \mathbf{g}_q(i) e^{j\omega_q n}. \tag{31}$$

(iii) The equalized output is then given by

$$e_1(n) = \sum_{i=0}^{L_e} \mathbf{g}_d^H(n;i)\mathbf{y}(n-i) \approx \alpha_1 c_1(n-d) + \alpha_2 b_1(n-d) + \tilde{v}(n) \tag{32}$$

where $\tilde{v}(n)$ is the equalized noise plus multiuser interference. Estimate α_1 as

$$\hat{\alpha}_1 = \frac{\frac{1}{T}\sum_{n=0}^{T-1} e_1(n)c_1^*(n-d)}{\frac{1}{T}\sum_{n=0}^{T-1} |c_1(n-d)|^2} = \frac{\frac{1}{T}\sum_{n=0}^{T-1} e_1(n)c_1^*(n-d)}{\sigma_{c1}^2}. \tag{33}$$

(iv) Define

$$e_2(n) = e_1(n) - \hat{\alpha}_1 c_1(n-d) \approx \alpha_2 b_1(n-d) + \tilde{v}(n). \tag{34}$$

Then we hard-quantize $e_2(n)$ to estimate $b_1(n-d)$.

5. SIMULATION EXAMPLE

We consider a random frequency-selective Rayleigh fading channel. We took $N = 1, 2, 3$ or 4 (receiver antennas), M=3 users, and $L = 2$ in (1) with $\mathbf{h}_m(n;l)$ mutually independent for all m, l and all components, zero-mean complex-Gaussian with equal variance, following Jakes' model with specified Doppler spread for each tap component. We consider a system with carrier frequency of 2GHz, data rate of 40kB (kB= kilo-Bauds), therefore, $T_s = 25 \times 10^{-6}$ sec., and a varying Doppler spread f_d. Additive noise was zero-mean complex white Gaussian. The SNR refers to the energy per bit over one-sided noise spectral density with both information and superimposed training sequence counting toward the bit energy. Information sequences for each user were BPSK (binary). We took the superimposed training sequences' period $P = 15$ ($D = 3$, $T_f = 5$) or $P = 52$ ($D = 4$, $T_f = 13$). The average transmitted power in $c_m(n)$ was equal to the power in $b_m(n)$, leading to a training-to-information power ratio (TIR) of 1.0 . All simulations results presented herein are based on 500 Monte Carlo runs.

Fig. 1 shows the BER results vs Doppler spread for $P = 15$ and Fig. 2 shows the same for $P = 52$, both for an SNR of 25dB. In Fig. 1 we had a fixed $Q = 4$ whereas in Fig. 2 we picked $Q = K$, as given by (6) as a function of f_d. Fig. 3 shows the BER results vs equalizer length. It is seen that performance improves with N. In Figs. 1-3 we considered the asynchronous case where the observation window fully contains the desired user's signal and the other two interfering signals ($m = 2, 3$) occupy window $[t_m, t_m + T - 1]$ where t_m is uniformly distributed in $[-(T-1), T-1]$; t_m changes from run-to-run. It is seen that while BER deteriorates with increasing Doppler spread f_d, it is "gradual."

Finally we also simulated "oversampled" (in the Doppler domain) equalizers ([4]) where we take

$$\mathbf{g}_o(n;l) = \sum_{q=-\bar{Q}/2}^{\bar{Q}/2} \mathbf{g}_{oq}(l) e^{j\omega_q n/2}, \quad n = 0, 1, \cdots, T-1, \tag{35}$$

$\bar{Q} = 2Q$ and Doppler frequency resolution is now $1/(2T)$ instead of $1/T$. [It is known that (critically sampled) CE-BEM has significant modeling errors which can be alleviated via oversampled CE-BEM [4].] A distinct improvement can be seen in Fig. 4. An analysis of this case has yet to be done.

The BER in all these figures is rather high. This can be alleviated by error-correction coding and by nonlinear equalizers such as decision-feedback equalizers where both forward and feedback parts are approximated by CE-BEM. Such extensions are currently underway.

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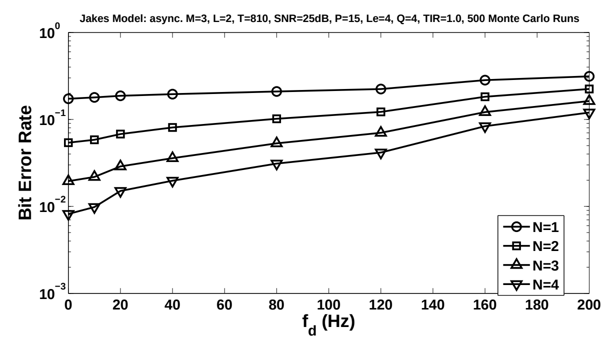

Figure 1. BER for varying number N of receive antennas and varying Doppler spreads. No. of users $M = 3$, asynchronous case, $T = 810$. Equalizer length L_e=4, $d = 2$, $Q = 4$ and $P = 15$. Each channel tap component follows Jakes' model (not CE-BEM).

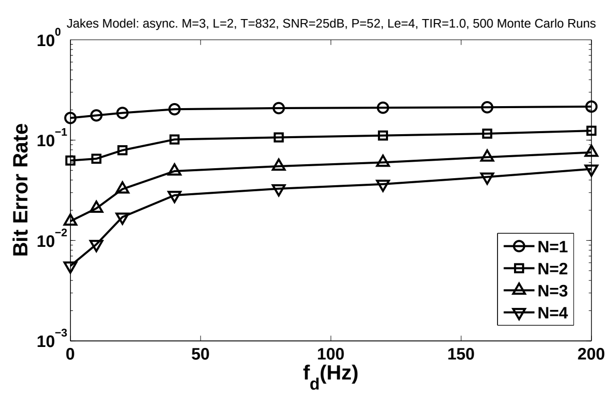

Figure 2. As in Fig. 1 except $P = 52$ and $Q = K$, as in (6).

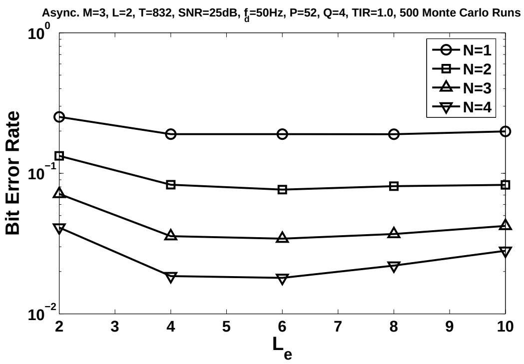

Figure 3. BER vs equalizer length L_e; $f_d = 50$Hz, P =52, Q =4, T =832.

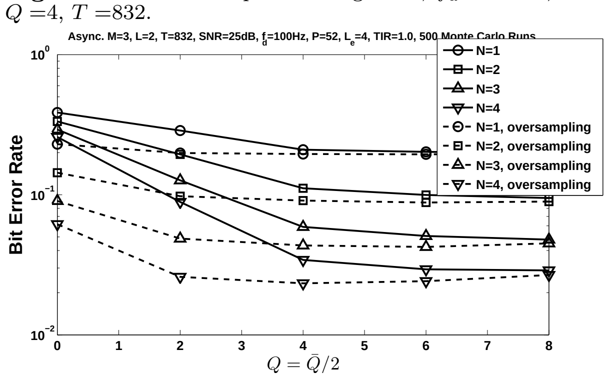

Figure 4. BER vs Q (see (31)) or $\bar{Q}$ (see (35)); $P = 52$, $L_e = 4$, SNR=25dB, T=832, f_d=100Hz.

6. APPENDIX

We have

$$\mathbf{R}_c(q_1, i_1) = \frac{1}{T}\sum_{n=0}^{T-1} c_1^*(n-d)e^{-j\omega_{q_1}n}$$

$$\times \left\{ \sum_{m=1}^{M}\sum_{l=0}^{L} \mathbf{h}_m(n-i_1;l)s_m(n-i_1-l) + \mathbf{v}(n-i_1) \right\}$$

$$= \sum_{m=1}^{M}\sum_{k=-K/2}^{K/2}\sum_{l=0}^{L}\sum_{p_1=0}^{P-1}\sum_{p_2=0}^{P-1} c_{1p_1}^* c_{mp_2} e^{j\alpha_{p_1}d}e^{-j\alpha_{p_2}(i_1+l)}$$

$$\times e^{-j\omega_k i_1}\mathbf{h}_{mk}(l)A_0 + \sum_{m=1}^{M}\sum_{k=-K/2}^{K/2}\sum_{l=0}^{L}\sum_{p_1=0}^{P-1} c_{1p_1}^* e^{j\alpha_{p_1}d}e^{-j\omega_k i_1}$$

$$\times \mathbf{h}_{mk}(l)A_{1m} + \sum_{p_1=0}^{P-1} c_{1p_1}^* e^{j\alpha_{p_1}d}\mathbf{A}_2 \quad (36)$$

where

$$A_0 := \frac{1}{T}\sum_{n=0}^{T-1} e^{j(-\alpha_{p_1}+\alpha_{p_2}-\omega_{q_1}+\omega_k)n}$$

$$A_{1m} := \frac{1}{T}\sum_{n=0}^{T-1} e^{j(-\alpha_{p_1}-\omega_{q_1}+\omega_k)n} b_m(n-i_1-l)$$

$$\mathbf{A}_2 := \frac{1}{T}\sum_{n=0}^{T-1} e^{-j(\alpha_{p_1}+\omega_{q_1})n}\mathbf{v}(n-i_1).$$

Under the condition $TP^{-1} > K$ (then $(\alpha_m + \omega_q) = (\alpha_n + \omega_k)$ iff $m = n$ and $q = k$), we have

$$A_0 = \delta(p_1 - p_2)\delta(q_1 - k). \quad (37)$$

Furthermore we have

$$E\left\{|A_{1m}|^2\right\}$$

$$= \frac{1}{T^2}\sum_{n_1=0}^{T-1}\sum_{n_2=0}^{T-1} e^{j(-\alpha_{p_1}-\omega_{q_1}+\omega_k)(n_1-n_2)}\sigma_{bm}^2\delta(n_1-n_2) = \frac{\sigma_{bm}^2}{T}. \quad (38)$$

Similarly, it follows that

$$E\left\{\|\mathbf{A}_2\|^2\right\} = \frac{N\sigma_v^2}{T}. \quad (39)$$

In the mean-square sense (and thus in probability), we then have the following two limits

$$\lim_{T\to\infty} A_{1m} \overset{\text{m.s.}}{=} 0 \text{ and } \lim_{T\to\infty}\mathbf{A}_2 \overset{\text{m.s.}}{=} \mathbf{0}. \quad (40)$$

Thus for "large" T, we have (after some manipulations)

$$\lim_{T\to\infty}\mathbf{R}_c(q_1,i_1) \overset{\text{m.s.}}{=} \sum_{m=1}^{M}\sum_{k=-K/2}^{K/2}\sum_{l=0}^{L}\sum_{p=0}^{P-1} c_{1p}^* c_{mp}$$

$$\times e^{j\alpha_p(d-i_1-l)}e^{-j\omega_k i_1}\mathbf{h}_{mk}(l)\delta(q_1-k). \quad (41)$$

Using (16) and (18), it follows that

$$\sum_{p=0}^{P-1} c_{1p}c_{mp}^* e^{j\alpha_p\tau} = \gamma_m(\tau)\delta(\tau \bmod \tilde{P})\delta(m-1). \quad (42)$$

We then have (29).

Turning to (28), we have

$$\mathbf{R}_b(q_1,i_1) = \sum_{m=1}^{M}\sum_{k=-K/2}^{K/2}\sum_{l=0}^{L}\sum_{p=0}^{P-1} c_{mp}\mathbf{h}_{mk}(l)e^{-j\alpha_p(i_1+l)}$$

$$\times e^{-j\omega_k i_1}A_3 + \sum_{m=1}^{M}\sum_{k=-K/2}^{K/2}\sum_{l=0}^{L}\mathbf{h}_{mk}(l)e^{-j\omega_k i_1}A_{4m} + \mathbf{A}_5 \quad (43)$$

where

$$A_3 := \frac{1}{T}\sum_{n=0}^{T-1} e^{j(\alpha_p-\omega_{q_1}+\omega_k)n} b_1^*(n-d)$$

$$A_{4m} := \frac{1}{T}\sum_{n=0}^{T-1} e^{j(\omega_k-\omega_{q_1})n} b_m(n-i_1-l)b_1^*(n-d)$$

$$\mathbf{A}_5 := \frac{1}{T}\sum_{n=0}^{T-1} e^{-j\omega_{q_1}n}\mathbf{v}(n-i_1)b_1^*(n-d).$$

We can show (as before) that

$$\lim_{T\to\infty} A_3 \overset{\text{m.s.}}{=} 0 \text{ and } \lim_{T\to\infty}\mathbf{A}_5 \overset{\text{m.s.}}{=} \mathbf{0}. \quad (44)$$

Consider

$$A_6 := \frac{1}{T}\sum_{n=0}^{T-1} e^{j(\omega_k-\omega_{q_1})n}\left[b_1(n-i_1-l)b_1^*(n-d) - \sigma_{b1}^2\delta(d-i_1-l)\right]. \quad (45)$$

It then follows (after some manipulations) that

$$E\left\{|A_6|^2\right\} = \frac{1}{T}\left[E\left\{|b_1(n)|^4\right\} - \sigma_{b1}^4\right]\delta(d-i_1-l). \quad (46)$$

Therefore, we have $\lim_{T\to\infty} A_6 \overset{\text{m.s.}}{=} 0$, and consequently

$$\lim_{T\to\infty} A_{4m} \overset{\text{m.s.}}{=} \frac{1}{T}\sum_{n=0}^{T-1} e^{j(\omega_k-\omega_{q_1})n}\sigma_{b1}^2\delta(d-i_1-l)\delta(m-1)$$

$$= \sigma_{bm}^2\delta(d-i_1-l)\delta(q_1-k)\delta(m-1). \quad (47)$$

Hence, for "large" T, we have $\lim_{T\to\infty}\mathbf{R}_b(q_1,i_1) \overset{\text{m.s.}}{=}$

$$\sum_{k=-K/2}^{K/2}\sum_{l=0}^{L}\mathbf{h}_{1k}(l)e^{-j\omega_k i_1}\sigma_{b1}^2\delta(d-i_1-l)\delta(q_1-k). \quad (48)$$

We therefore have (30).