14th European Signal Processing Conference (EUSIPCO 2006), Florence, Italy, September 4-8, 2006, copyright by EURASIP

COMPARISON OF DIFFERENT APPROACHES FOR ROBUST IDENTIFICATION OF A LIGHTLY DAMPED FLEXIBLE BEAM

Allahyar Montazeri, Hadi Esmaeilsabzali, Javad Poshtan, and MohammadReza JahedMotlagh

Electrical Engineering Faculty, Iran University of Science & Technology
Narmak, 16844, Tehran, Iran
phone: + (98)-21-73912675, fax: +(98)-21-77240490, email: amontazeri@iust.ac.ir

ABSTRACT

The aim of this paper is robust identification of a lightly damped flexible beam model with parametric and non-parametric uncertainties. We examined two main approaches for robust identification which are based on deterministic and stochastic assumptions on uncertainties. In the first case uncertainties are assumed to be unknown but bounded which is known as "Set Membership" method (SM), while stochastic assumptions lead to the so-called "Stochastic Embedding" method (NSSE). In order to proper handling with the high magnitude non-parametric uncertainties the proposed methods are compared and it is shown that the combination of set membership approach with model error modelling techniques will result in superior results.

1. INTRODUCTION

Robust control theory plays an important role in the application of control theory in practical problems. The main concept is to consider a physical system as an uncertain model which may be represented as a family of mathematical models. Using robust control techniques, all models in this family will be stabilized in an appropriate manner. This family is described by a nominal model and a bounded uncertainty. Thus it is customary to identify not only a nominal model, but also an uncertainty bound associated to this nominal model. Identification methods producing a nominal model and its associated uncertainty are known as "*Robust Identification*" or *"(Robust) Control-Oriented Identification*" methods. Because of the outspread use of robust control techniques in practical problems, robust identification is an area which has received a growing interest of researchers since beginning of 1990's due to the weakness of classical identification methods to produce suitable models for robust control theory. Robust identification algorithms use *a priori information* on system and its input-output data (posteriori information) to produce a nominal model and its associated uncertainty.

Two main philosophies for description of model's uncertainties have been used. The first one is based on statistical assumptions and produces so-called "soft bound" on model's uncertainty. Second approach is based on deterministic hypothesizes and gives "hard bound" on uncertainty. Indeed in this approach, uncertainties are assumed to be "Unknown but Bounded" (UBB) [1]. Deterministic hypothesis on model's uncertainties, leads to set membership identification methodologies.

In all system identification problems, perturbation are potentially arise form two main sources: a variance error due to the measurement noises and a bias term due to effect of unmodeled dynamics (dynamics that have not been included by nominal estimated model- also known as model error). The nature of these two error types is quite different. Variance error generally uncorrelated with the input signal (in open loop data collection case), but bias error is strongly depends on nominal model's structure and identification experiment input signal [1].

Three main approaches for robust identification have been addressed in the literature, namely:

1. Stochastic Embedding (SE)
2. Model Error Modeling (MEM)
3. Set Membership (SM)

SE is a frequency domain method based on statistical hypothesizes about uncertainties. This method potentially has the ability of handling both variance and bias errors but is mostly used for the aim of non-parametric uncertainties modeling [2, 14, 15]. This approach to robust identification was first introduced by Goodwin in [14]. Later in [15] this method was modified by using maximum likelihood technique for the estimating of parameters. To alleviate the problems associated with the identification procedure in [15], in [2] unmodeled dynamics relevant uncertainties are represented by a non stationary stochastic process whose variance increases with frequency. This method which is known as "*Non-Stationary Stochastic Embedding*", has a high ability of capturing typical cases of non-parametric uncertainties, including systems with unmodeled lightly damped modes [2].

MEM is indeed a model validation tool but is also used for the purpose of robust identification [3, 4]. In comparison with other methods, MEM is more general in the sense that it is a time domain method that can handle both statistic and deterministic assumptions on uncertainties. It can be shown that the advantage of MEM method is its ability to making separation between noise and unmodeled dynamics.

Finally SM is a time/frequency domain method, based on deterministic assumptions on system's perturbations. In fact uncertainties deem to be unknown but bounded by a suitable norm. In the first works the idea is used for state estimation [5, 6]. Later, SM theory is employed for the aim of system identification [7, 8]. Because of its deterministic framework, this approach to robust identification is more popular than SE and other statistical based approaches. Both parametric and non-parametric uncertainties can be accounted in SM identification problem. In [7], [8], and [9] just parametric uncertainties are considered while [1], [4], [10], and [11] deal with parametric and non-parametric uncertainties.

Fundamentally lightly damped flexible structures are distributed parameter systems and thus have infinite dimensional analytic models. In order to design a controller one has to have a finite dimensional model. Using truncated or reduced order model, "*spill over effect*" is a possible phenomenon. To fulfill this problem robust controller is a beneficial tool. So, robust identification of lightly damped flexible structures is an evident necessity.

Because of lightly damped nature of the model and its considered uncertainties, having a good identification of in-bandwidth modes as well as including high amplitude modes uncertainties in the identified model is not a straight forward task. In this paper this problem is addressed and the above approaches are applied and compared.

In the next section we introduce the main concepts of SM, MEM and NSSE identification methods with respect to our problem. Section 3 presents the robust identification results for a lightly damped flexible beam and section 4 concludes the paper.

2. ROBUST IDENTIFICATION PROBLEM FORMULATION

2.1. The Set Membership Approach

Suppose that N samples of input-output data that have been generated by real system *G(q)* are available:

$$y_m(k) = G(q)u_m(k) + v(k) \tag{1}$$

where *v(k)* is the measurement noise and is bounded by a suitable norm:

$$\|v(k)\|_\beta \le \delta(k) \tag{2}$$

It is possible to represent the real system as follow:

$$G(q) = G(q,\theta) + \Delta G(q) \tag{3}$$

where $G(q,\theta)$ is the parameterized nominal model and $\Delta G(q)$ stands for possible unmodeled dynamics and is also bounded by suitable norm in the space of transfer functions. More details on deriving bound of $\Delta G(q)$ can be found in [11]. For our identification problem we choose ∞-norm. Using this, the effect of the frequency response amplitude of unmodeled dynamics can be considered effectively.

Regarding (3), the input-output relationship (1) can be presented as:

$$y_m(k) = [G(q,\theta) + \Delta G(q)]u_m(k) + v(k) \tag{4}$$

$$y_m(k) - G(q,\theta)u_m(k) = \Delta G(q)u_m(k) + v(k) \tag{5}$$

As it has been addressed earlier, we choose *L*∞ and *H*∞ norms for noise and unmodeled dynamics respectively, so:

$$\|y_m(k) - G(q,\theta)u_m(k)\|_\infty = \|\Delta G(q)u_m(k) + v(k)\|_\infty \tag{6}$$

$$\|y_m(k) - G(q,\theta)u_m(k)\|_\infty \le \|\Delta G(q)u_m(k)\|_\infty + \|v(k)\|_\infty \tag{7}$$

$$\|y_m(k) - G(q,\theta)u_m(k)\|_\infty \le \|\Delta G(q)\|_1 \|u(k)\|_\infty + \|v(k)\|_\infty \tag{8}$$

where $\|\Delta G(q)\|_1$ and $\|v(k)\|_\infty$ are nonparametric and parametric perturbation bounds respectively and come from a priori information on system to be identified. Let:

$$\|\Delta G(q)\|_1 \le \gamma \;;\quad \|v(k)\|_\infty \le v_k \;;\quad \|u(k)\|_\infty = u_k$$

$$w_k = \gamma u_k + v_k$$

Thus (8) can be expressed as:

$$\|y_m(k) - G(q,\theta)u(k)\|_\infty \le w_k \tag{9}$$

Another way in determination of perturbation bound for set membership problem is to use a constant upper bound instead of variable bound. In order to do this, we can choose the maximum value of the variable perturbation bound over all N samples and consider it in (9) for all data samples.

Now we have to determine structure of $G(q,\theta)$ in order to complete the set membership inequality in (9). Different model structures are available for nominal model. Among them, output error (OE) structure is a popular model structure. To avoid high computational complexity due to nonlinear optimization in the process of parameter estimation and to obtain linear in model structure, we use the linear combination of orthonormal basis functions for OE model structure. This choice has another advantage in the way that much more a priori information can be imported to the identification algorithm by proper choice of basis functions. In other words by selecting basis functions whose dynamics are close to the dynamics of the real system, it will be conceivable to estimate the nominal model by minimum number of parameters [12]. Because of resonant nature of our system, we use so-called *"Kautz"* or two-parameter basis functions [13]:

$$G(q,\theta) = \sum_{i=1}^{n} \theta_i \psi_i(q) \tag{10}$$

where n is the order of nominal model and $\psi_i(q)$ is Kautz basis function [13]. Now by (9) and (10):

$$\|y_m(k) - \theta^T \mathbf{x}_m(q,k)\|_\infty \le w_k \tag{11}$$

where $\mathbf{x}_m(q,k)$ is the regression(information) vector and computed as:

$$\mathbf{x}_m(q,k) = [\psi_1(q)u_m(k) \quad \psi_2(q)u_m(k) \ldots \psi_n(q)u_m(k)] \tag{12}$$

And $\theta = [\theta_1 \quad \theta_2 \ldots \theta_n]^T$ is the vector of parameters. For each time stamp (k=1, 2,…, N), (11) produces a so-called strip in the space of parameters. By intersecting these strips, "*Feasible Parameter Set*" (FPS) will be obtained as follow:

$$\Theta = \{\theta : \bigcap_{k=1}^{N} \left\| y_m(k) - \theta^T \mathbf{x}_m(q,k) \right\|_\infty \leq w_k\} \tag{13}$$

In fact, Θ is the set of all parameters compatible with input-output data, a priori information on system and the uncertainty bounds. For the case that inequalities are linear in parameters, as (13), the FPS is a convex polytope in the space of nominal model's parameters. The aim of set membership robust identification problem is to compute the FPS and determine an optimal point in FPS (in some sense) as the nominal model's parameters. It is possible to outbound the FPS by simple geometrical shapes like "*Ellipsoid*" and "*Parallelotope*" and consider their center as the parameters of nominal model [7, 8, 11]. Because of the greater DOF (Degree of Freedom) of the parallelotopes, they can outbound the FPS more tighten than ellipsoids.

2.2. Model Error Modelling Technique

"Model Error Modelling" (MEM) is a time domain technique with various applications in the area of system identification such as model validation and direct model error modelling (i.e. combination simple models to obtain a suitable model of system and its uncertainties), which will be used in this paper. More details about various respects of MEM can be found in Ljung's survey paper [3].

Consider (1) and let $G(q,\theta^*)$ be the nominal estimated model of the system in (1). Although it is possible to obtain this model by several identification methods, but in this paper $G(q,\theta^*)$ is estimated using SM Method as mentioned in previous subsection.

Let "*residual*" sequence to be computed as follow:

$$\varepsilon(k) = y_m(k) - G(q,\theta^*)u_m(k) \tag{14}$$

It is possible to consider the "*Error System*" whose input and output are respectively u_m and ε :

$$\varepsilon(k) = G_e(q)u_m(k) + v(k) \tag{15}$$

where $v(k)$ is the measurement noise as in (1). G_e is also known as Model Error Model and is the estimate of unmodeled dynamics relevant error. As for nominal model, G_e can be identified by any identification method. Here, in this paper, G_e is identified again by SM technique. In other words, using a priori information of system, it is possible to define a suitable parametric model structure for G_e. Then by input-output data in (14), the model error model and its associated uncertainty will be determined. It is a simple fact that if the uncertainty region of G_e contains zero element, the nominal estimated model $G(q,\theta^*)$ will be unfalsified. Having system's nominal model $G(q,\theta^*)$ and G_e along with its uncertainty, it is possible to obtain a complete model of the system in (1). This can be done by adding up the nominal model and the uncertainty bound of G_e. This complete model can demonstrate the system in (1) and its uncertainties in a suitable manner. It is easy to verify that MEM technique has the ability of separating bias and variance errors completely. Due to this fact, recently this approach to robust identification attracts some interests [1, 4].

2.3. Non-Stationary Stochastic Embedding Technique

Our approach in this section is similar to [2], which can be present as follow:

Suppose that the true system's frequency response is given as:

$$G(j\omega) = G_0(j\omega) + \Delta G(j\omega) \tag{16}$$

where $G_0(j\omega)$ is the nominal model that we want to estimate and $\Delta G(j\omega)$ is a stochastic process independent of nominal model whose variance increases with frequency and is stand for model errors. Let $\hat{G}_k$ to be the noisy observations of the true system at certain frequency:

$$\hat{G}_k = g(j\omega_k) + v_k \; ; \quad k = 1,2,..,m \tag{17}$$

where v_k is the measurement noise. One way to estimating the nominal model is to parameterize it using some orthonormal basis as [12]:

$$G_0(q) = \sum_{i=1}^{n} \theta_i b_i(q) = \mathbf{B}^T \theta \tag{18}$$

where $\mathbf{B} = [b_1 \; b_2 \ldots b_n]^T$ is the vector of basis functions and $\boldsymbol{\theta}$ is the vector of parameters. It is also possible to use this structure to determine $\Delta G(j\omega)$. So (16) can be represented as follow:

$$G = \mathbf{B}^T \theta + \mathbf{B}^T \overline{\theta} \Lambda \tag{19}$$

where Λ is the (integrated) random walk process over frequency. Now by (16) and (17):

$$\hat{G} = \mathbf{B}^T \theta + \mathbf{B}^T \overline{\theta} \Lambda + v_k \tag{20}$$

So the non-stationary stochastic embedding robust identification process cab be stated as follow:

1. Point-wise least square estimation of the transfer function at certain frequencies. The input u for this purpose must be sum of sinusoids. This step delivers a raw estimation of the real transfer function at certain frequencies which is considered as $\hat{G}_k$. Additionally, statistical properties of the noise are calculated assuming Gaussian white noise.
2. Choice of basis functions **B**.

3. Estimation of the parameter $\boldsymbol{\theta}$ and the (integrated) random walk process Λ in (20) based on the frequency function point estimation $\hat{G}_k$ according to following procedure:

a) Least square estimation of $\boldsymbol{\theta}$ based on frequency point estimation $\hat{G}_k$.

b) Using this estimate for model error parameterization as shown in (19).
c) Computation of an unbiased estimate of the variance of the (integrated) random walk process.
d) Quantification of the model error for any frequency (calculation of its statistical properties).

3. SIMULATION RESULTS

This section presents the identification results for a lightly damped simply-supported flexible beam. The flexible beam which is considered in this work is assumed to be out of steal. The identification experiment has been simulated using a "*Finite Element*" model of the beam. The input and output time domain signals for identification are force and displacement, respectively. The input signal is the combination of 180 sinusoidal with excitation frequencies that have been picked according to the a priori information of system which in this case is the rough estimate of beam's FRF (Frequency Response Function) (fig. 1). Distribution of these frequencies is a key point in the experiment design. The output signal has been corrupted by a normally distributed Gaussian random signal with the variance of 1%.
Our aim is to identify the two first modes of system and consider the two last modes as non-parametric uncertainty. First we examine the SM algorithm that has been introduced in the last section. We use Kautz basis functions for this purpose, which their parameters have been tuned with respect to the system's FRF. We also use two different outbounding algorithms for approximation of FPS. The result for parallelotopic approximation is shown in fig. 2 which is better than ellipsoidal approximation. Secondly, NSSE algorithm described in section 2 with both random walk and integrated random walk processes has been utilized. For the identification of the first two modes, two continuous-time Kautz basis functions are selected whose parameters are tuned based on the FRF of the beam. Fig. 3 shows the point estimation of the FRF of the beam used in NSSE algorithm. The estimated model and its uncertainty cloud using random walk process and with 99.99% confidence level are plotted in fig. 4. The same results for integrated random walk are shown in fig. 5. Athough the results are improved rather than the previous method, but it has not offered a good compromise between robust stability and performance of the controller. The quality of estimation is increased by using the MEM method for better handling of non-parametric uncertainties. This method has been introduced in section 2. In order to identify of the non-parametric uncertainties, we use SM method and Kautz basis functions again. This method can be known as combination of SM and MEM. The result of this approach is shown in fig. 6. As it is evident the uncertainty bound has a good tightness and also the real system is included by this bound in

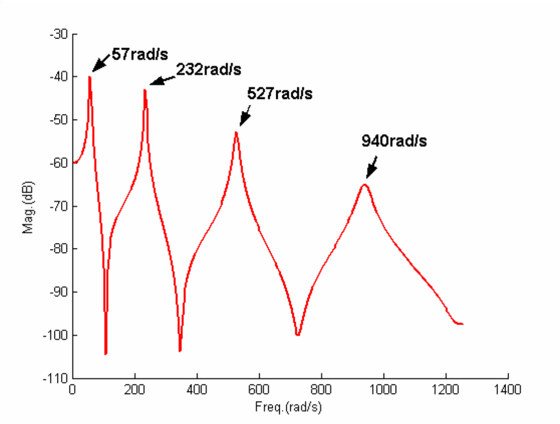

Figure 1 – FRF of the beam under study

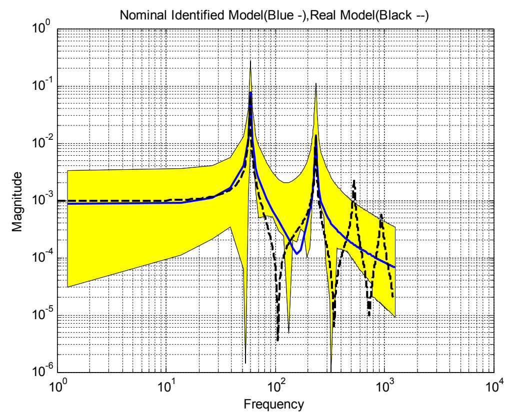

Figure 2 – Nominal identified model and its associated uncertainty (parallelotopic approximation)

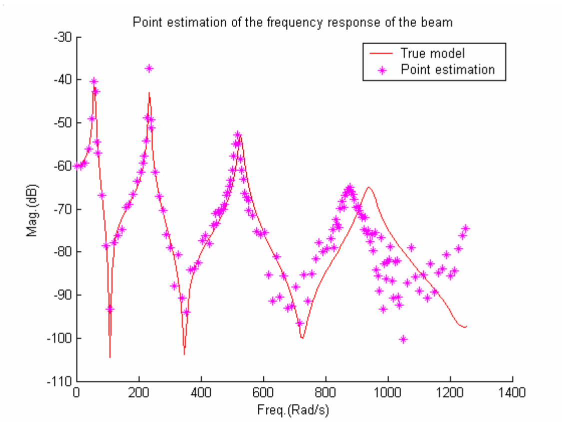

Figure 3 –Point estimation of the FRF of the beam

all frequencies. Using such a model for robust controller design will result in a good performance/stability results.

4. CONCLUSION

In order to design robust controllers one has to have a suitable model which consists of the nominal model and some measure of its uncertainties. Robust identification methods provide such models that are indicate the real uncertainties of the system. SM method is one of these techniques that is based on deterministic assumptions on uncertainties. This type of uncertainty representation is greatly adopted by various robust control methods. In this paper this method is used for the purpose of robust identification of a lightly damped flexible beam.

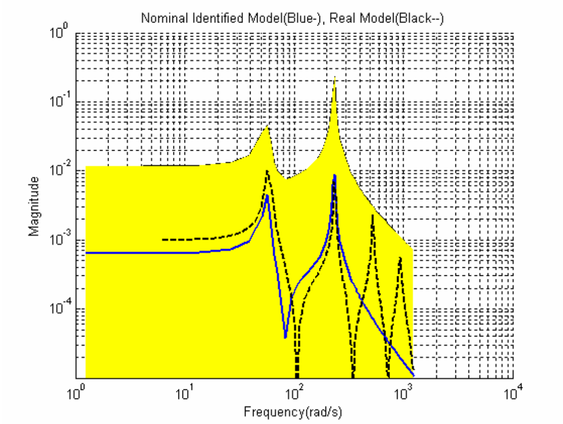

Figure 4 –Estimated model (dashed line) and its uncertainty band for random walk process (yellow cloud)

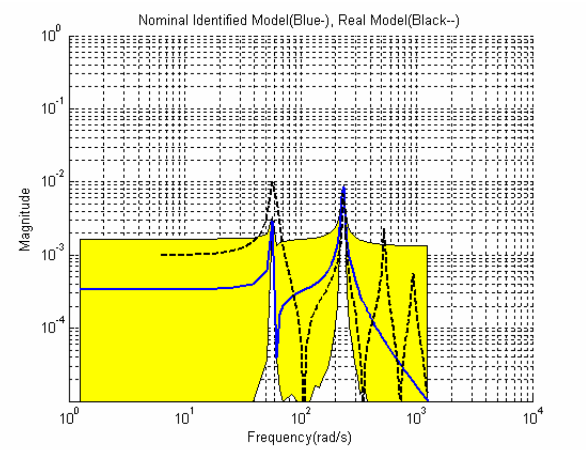

Figure 5 –Estimated model (dashed line) and its uncertainty band for integrated random walk process (yellow cloud)

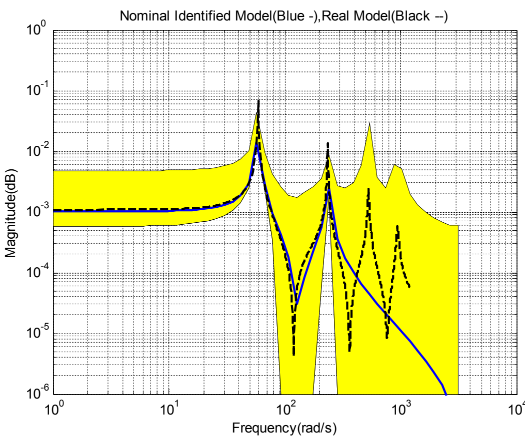

Figure 6– Nominal identified model and its associated uncertainty (SM/MEM method)

REFERENCES

[1] Wolfgang Reinelt, Andrea Garulli, and Lennart Ljung, “Comparing Different Approaches to Model Error Modeling in Robust Identification”, *Automatica,* vol. 38, pp. 787-803, 2002.

[2] Graham C. Goodwin, Julio H. Braslavsky and Maria M. Seron, “Non-Stationary Stochastic Embedding for Transfer Function Estimation”, *Automatica,* vol. 38, pp. 47-62, 2002.

[3] Lennart Ljung, “Model validation and Model Error Modeling”, *Proceeding of Astrom Symposium on Control,* pp. 15-42, Lund, Sweden, August 1999.

[4] Andrea Garulli and Wolfgang Reinelt, “On Model Error Modeling in Set Membership Identification”, *Proceeding of The IFAC Symposium on System Identification,* Santa Barbara, USA, pp. 169-174, 21-23 June 2000.

[5] Fred C. Schweppe, “Recursive State Estimation: Unknown but Bounded Errors and System Inputs”, *IEEE Trans. Automat. Contr.* AC. 13, No.1, pp. 22-28, Feb. 1968.

[6] Dimitri P. Bertsekas and Ian B. Rhodes, “Recursive State Estimation for a Set Membership Description of Uncertainty”, *IEEE Trans. Automat. Contr.,* vol. 16, No. 2, Apr. 1971.

[7] Eli Fogel, “System Identification via Membership Set Constraints with Energy Constrained Noise”, *IEEE Trans. Automat. Contr.,* vol. 24, No. 5, Oct. 1979.

[8] Eli Fogel and Y. F. Huang, “On the Value of Information in System Identification-Bounded Noise Case”, *Automatica,* vol. 18, No. 2, pp. 229-238, 1982.

[9] Mitchell M. Livstone and Munther A. Dahleh, “A Framework for Robust Parametric Set Membership Identification”, *IEEE Trans. Automat. Contr.,* vol. 40, No. 11, Nov. 1995.

[10] Robert L. Kosut, Ming K. Lau, and Stephen P. Boyd, “Set Membership Identification of Systems with Parametric and Nonparametric Uncertainty”, *IEEE Trans. Automat. Contr.,* vol. 37, No. 7, pp. 929-941, July 1992.

[11] Antonio Vicino and Giovanni Zappa, “Sequential Approximation of Parameter Sets for Identification with Parametric and Nonparametric Uncertainty”, *Proceeding of the 32nd Conference on Decision and Control,* San Antonio, Texas, Dec. 1993.

[12] Paul M. J. Van Den Hof, Peter S. C. Heuberger, and Jozsef Bokor “System Identification with Generalized Orthonormal Basis Functions”, *Automatica,* vol. 31, No.12, pp. 1821-1834, 1995.

[13] Bo Wahlberg, “System Identification Using Kautz Models”, *IEEE Trans. Automat. Contr.*, vol. 39, No. 6, pp. 1276-1282, June. 1994.

[14] G. C. Goodwin and M. Salgado, “A Stochastic Embedding Approach for Quantifying Uncertainty in the Estimation of Restricted Complexity Models”, *International Journal of Adaptive Control and Signal Processing*, 3(4), pp. 333-356, 1989.

[15] G. C. Goodwin, M. Gevers, and B. Ninness, “Quantifying the Error in Estimated Transfer Functions with Application to Model Order Selection”, *IEEE Trans. Automat. Contr.*, vol. 37, pp. 913-928, July 1992.