

# COUPLING CHANNEL IDENTIFICATION FOR OFDM RELAY STATION IN SFN

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## ABSTRACT

The paper is concerned with identification of coupling channel and multi-path channel for SFN relay station in the OFDM transmission systems. Channel identification is important and necessary to design a stable coupling canceller. Nevertheless, it is a difficult issue to identify an overall relay transfer function in full-band since the transmitted OFDM signals are band-limited. The purpose of this paper is to propose a new identification scheme of the relay transfer function from the transmitted signals of a key station to the re-transmitted signal of a relay station, by efficiently making use of property of the OFDM signals with CP. Moreover, an upper bound on the transfer function estimation error is also evaluated.

## 1. INTRODUCTION

The OFDM systems are recently considered to be a reliable choice for high rate transmissions and are widely adopted in digital audio and video broadcasting, and broadband wireless local area networks. In digital broadcasting, the OFDM signal is usually transmitted through single frequency networks (SFN), where the transmission efficiency is higher than that in multi-frequency networks. Nevertheless, since the carrier frequency for transmission is the same as that of the received signals at a relay station, the radio wave from the transmitter antenna couples into the receiver antenna. The coupling wave deteriorates the quality of the signal transmission, even causes the serious oscillation problem. Hence, signal processing schemes for the coupling cancellation should be developed to reduce the coupling effects.

Several methods have been proposed to deal with the coupling cancellation issue. In the time-domain algorithms, the coupling channel  $C(z)$  is estimated, then the coupling wave canceller  $W(z)$ , which is often an FIR filter as illustrated in Figure 1, is designed to cancel the coupling wave [1]. In the space-based approaches, the antenna arrays are used for beamforming and direction-of-arrival estimations [2]. Space-time joint algorithms have also been proposed [3]. Zero-forcing receiver design has also been proposed to compensate for coupling interferences [4]. In both the time-domain and space-domain algorithms, the FIR models of the coupling channel  $C(z)$  and multi-path transmission channel  $G(z)$  are necessary for stable canceller design. Channel identification for OFDM transmission has also been studied via subspace-based blind identification approach [5] and cyclostationarity-based blind identification [6]. However, in order not to interfere the adjacent transmission bands, the transmitted signal in the OFDM systems is generally band limited, and it leads to that it is very difficult to estimate the frequency response of the channels in the whole frequency range.

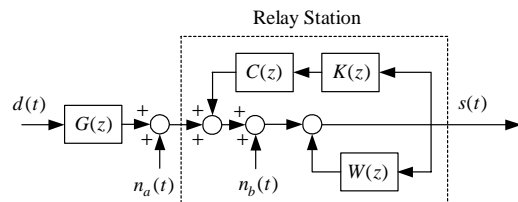


Figure 1: Schematic diagram of SFN relay station.  $G(z)$ : multi-path of transmission channel;  $C(z)$ : Model of coupling effect;  $K(z)$ : Model of amplifier;  $n_a(t)$ : Noise in transmission channel;  $n_b(t)$ : Noise in relay station.

The purpose of this paper is to propose a novel identification method to obtain transfer functions of the coupling and multi-path channels as well as an overall transfer function from a key station to relay station by efficiently using the property of the OFDM signals with CP. Moreover, an upper bound of the estimation error is also given, so the obtained results contribute to stable design of the coupling canceller.

## 2. MODEL OF OFDM RELAY STATION

The desired transmitted base band signal  $d(t)$  from a key station in one symbol period is given by

$$d(t) = \sum_{n=-\frac{M}{2}+1}^{\frac{M}{2}} d_n e^{jn\omega_0 t} \quad (1)$$

where  $\omega_0$  is the frequency interval,  $M$  the size of Fourier transform, and  $d_n$  is the source symbol given as

$$d_n = \begin{cases} \text{information} (\neq 0) & |n| \leq \frac{N-1}{2} \\ 0 & |n| > \frac{N-1}{2} \end{cases} \quad (2)$$

where  $N$  is the number of carriers and  $N < M$ . One symbol period is denoted by  $T_{\text{inf}} = 2\pi/\omega_0$ , and the power spectrum of  $d(t)$  is limited within the frequency band  $|n| \leq (N-1)/2$ .

In the OFDM systems, a guard interval, i.e., a cyclic prefix (CP), whose length is commonly larger than the channel memory at the transmitter, is inserted to the head of symbol, enables OFDM to avoid multi-path interferences.

As illustrated in Figure 2, the CP is inserted before the original symbol by copying the last part of the original signal within time  $T_{\text{cp}}$ . Then the total period of transmitted symbol, which is denoted as  $T_{\text{sig}}$ , becomes  $T_{\text{inf}} + T_{\text{cp}}$ .

Let the coupling channel  $C(z)$  in the relay station and the multi-path channel  $G(z)$  from the key station be expressed respectively by

$$\begin{aligned} C(z) &= c_0 + c_1 z^{-1} + \dots + c_{L_c} z^{-L_c} \\ G(z) &= g_0 + g_1 z^{-1} + \dots + g_{L_g} z^{-L_g} \end{aligned} \quad (3)$$



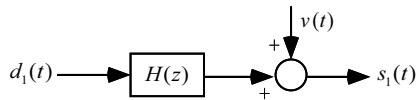


Figure 3: Non-parametric identification problem

### 4.3 Non-parametric Estimation Method

Consider an estimation problem depicted in Figure 3. Let  $d_1(t)$  and  $s_1(t)$  be defined by (8) and (9) respectively, and let  $v(t)$  be noise. We consider how to obtain the impulse response of  $H(z)$  from  $d_1(t)$  and  $s_1(t)$  in  $t \in [-T_{cp}, T_{cp}]$ .

From the property of the impulse response, any initial condition of the impulse response should be zero. The problem is how to remove the influence by non-zero initial condition from  $s_1(t)$ . Here we pre-filter  $d_1(t)$  in two consecutive signal periods by two filters  $Q_1(z)$  and  $Q_2(z)$  respectively to generate the input signal as  $0, \dots, 0, 1, 0, \dots, 0$ , and pad sufficiently large zeros before the impulse. The obtained signal is input to the system, thus we can obtain the impulse response with deleted influence of non-zero initial condition.

#### 4.3.1 Generation of estimated input

A specific estimated input can be generated using  $d_1(t)$ . We treat with the signal  $d_1(t)$  in two signal periods  $2T_{sig}$ . Then we take  $x_1(t) = d_1(t)$  and  $x_2(t) = d_1(t + T_{sig})$ . As shown in Figure 4, by designing the filters  $Q_1(z)$  and  $Q_2(z)$ , we can give a desirable estimated input sequence  $q(t)$  as

$$q(t) = x_1(t)Q_1(z) + x_2(t)Q_2(z) \quad (12)$$

For instance, let the sequences  $x_1(t) = 1, 2, 1, 0, \dots, 0$  and  $x_2(t) = 1, 2, 1, 0, \dots, 0$  in  $-\frac{2}{M}T_{cp} \leq t < T_{cp}$ , then we can specify second-order filters  $Q_1(z)$  and  $Q_2(z)$  as

$$\begin{aligned} Q_1(z) &= a + (0.25 - 2a)z^{-1} + az^{-2} \\ Q_2(z) &= -a + (-0.25 - 2a)z^{-1} + az^{-2} \end{aligned}$$

Thus we obtain an impulse signal as  $q(t) = 0, 0, 1, 0, \dots, 0$ , where  $a \neq 0$ .  $Q_1(z)$  and  $Q_2(z)$  can be decided uniquely by, for example, choosing  $a = 1$ .

Next, we consider the generation of the estimated input sequence  $q(t)$  in the interval  $t < -\frac{L_2}{M}T_{inf}$ . Let  $X_i(z)$  be an  $L_2$ -th order polynomial, and let the coefficients be a signal sequence  $x_i(t)$  in the interval  $-\frac{L_2}{M}T_{inf} \leq t \leq 0$ . If  $X_1(z)$  and  $X_2(z)$  are coprime polynomials, the next polynomial equation

$$Q(z) = X_1(z)Q_1(z) + X_2(z)Q_2(z) \quad (13)$$

has the solutions  $Q_1(z)$  and  $Q_2(z)$  for arbitrary  $2L_2$ -th order polynomial  $Q(z) \neq 0$ . The coefficients of  $Q(z)$  are the sequence of  $q(t)$  in the interval  $-\frac{L_2}{M}T_{inf} \leq t \leq \frac{L_2}{M}T_{inf}$ . Thus, even if any initial condition for  $t < -\frac{L_2}{M}T_{inf}$  takes non-zero values, we can obtain the impulse response  $\gamma(t)$  of  $H(z)$  by setting  $q(t) = 0, \dots, 0, 1, 0, \dots$  and giving  $Q_1(z)$  and  $Q_2(z)$ , as shown in Figure 4. Thus, we can obtain the overall transfer function outside the signal frequency band.

#### 4.3.2 Non-parametric model estimation

Let  $x_1(t) = d_1(t)$  and  $x_2(t) = d_1(t + T_{sig})$ . By filtering the signals  $x_1(t)$  and  $x_2(t)$  for the interval  $-\frac{L_2}{M}T_{inf} \leq t \leq \frac{L_2}{M}T_{inf}$

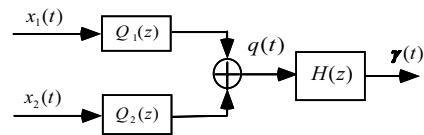


Figure 4: Generation of identification input

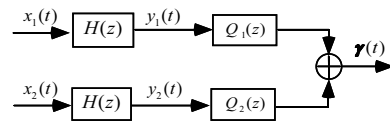


Figure 5: Non-parametric estimation model

via the filters  $Q_1(z)$  and  $Q_2(z)$  with  $L_2$  order, we obtain the impulse signal  $q(t)$  in (12) which is given by

$$\underbrace{0 \ \dots \ 0}_{L_1} \quad 1 \quad \underbrace{0 \ \dots \ 0}_{L_2 + \frac{T_{cp}}{T_{inf}}M - L_1} \quad (14)$$

As already stated, we can obtain the filters  $Q_1(z)$  and  $Q_2(z)$ . If  $L_1$  is chosen sufficiently large such that the initial condition of the impulse response  $H(z)$  becomes zero, we can obtain the impulse response of  $H(z)$  from the response  $\gamma(t)$  by inputting  $q(t)$  to  $H(z)$ .

However, in actual communication systems, we cannot input  $q(t)$  to  $H(z)$ . Therefore, we exchange  $Q_1(z)$  and  $Q_2(z)$  with  $H(z)$  in Figure 4 and transform the structure to Figure 5. Then we can consider a similar problem in which  $y_1(t)$  and  $y_2(t)$  are filtered by  $Q_1(z)$  and  $Q_2(z)$  respectively. Thus, we can obtain the output sequence as

$$Q_1(z)y_1(t) + Q_2(z)y_2(t) \quad \text{for} \quad -\frac{L_2}{M}T_{inf} \leq t \leq T_{cp} \quad (15)$$

Then, by deleting  $L_1$  zeros of the initial condition, we can obtain the observed sequence of the impulse response corrupted by noise. Thus let the sequence be denoted by  $\hat{h}(k)$ ,  $k = 0, 1, \dots$ , where  $y_1(t) = s_1(t)$ ,  $y_2(t) = s_1(t + T_{sig})$ .

On the other hand, the solution of  $Q_1(z)$  and  $Q_2(z)$  is obtained so that  $q(t)$  can satisfy that

$$\underbrace{0 \ \dots \ 0}_{L_1} \quad \overbrace{\hat{g}_{-1,0}, \hat{g}_{-1,1}, \dots}^{2L_2+1} \quad (16)$$

Thus, we obtain the non-parametric estimate of the sensitivity function  $\Gamma(z)$ , and denote it by  $\hat{\gamma}(k)$ , where  $\hat{g}_{-1,l}$  is an impulse response of  $1/\hat{G}(z)$ . Transient response of  $\Gamma(z)$  attenuate sufficiently, a true response becomes zero and  $\hat{\gamma}(k)$  involves almost noise components.

### 4.4 Estimation of Channel $G(z)$ and $C(z)$ in Full Frequency Band

By using the total transfer function (4) and the estimated sensitivity function, we calculate  $s_2(t)$  by filtering  $s_1(t)$  as  $s_2(t) = s_1(t)/\hat{\Gamma}(z)$ . Then we have

$$s_2(t) = G(z)d_1(t), \quad \text{for} \quad -T_{cp} \leq t \leq T_{cp} \quad (17)$$

Thus, using the signal data  $s_2(t)$  and  $d_1(t)$ , we can obtain the estimate of the multi-path channel  $\hat{G}(z)$ . Moreover, from the definition of the sensitivity function  $\Gamma(z)$ , it follows that the cascade connection of the coupling channel  $C(z)$  and amplifier  $K(z)$  in Figure 1 can be estimated by

$$C(e^{j\omega})\widehat{K}(e^{j\omega}) = 1 + W(e^{j\omega}) - \frac{1}{\hat{\Gamma}(e^{j\omega})} \quad (18)$$

## 5. ESTIMATE OF SENSITIVITY FUNCTION

In order to design a stable adaptive coupling canceller by using estimated transfer functions of the relay station, it is important to evaluate the uncertainty bound on the channel estimates.

### 5.1 Evaluation of Uncertainty Due to Noise

Let  $\hat{\Gamma}(e^{j\omega})$  be a frequency response of  $\hat{\gamma}(k)$ . By introducing the  $m$ -point rectangular window  $\psi_{m,k}$ ,  $k = 0, \dots, m-1$ , the approximately estimated model  $\hat{\Gamma}_{\text{rectw}}(z)$  of  $\hat{\Gamma}(z)$  is given by

$$\hat{\Gamma}_{\text{rectw}}(z) = \sum_{k=0}^{m-1} \psi_{m,k} \hat{\gamma}(k) z^{-k}. \quad (19)$$

Let the frequency responses of  $\hat{\Gamma}(z)$  and  $\hat{\Gamma}_{\text{rectw}}(z)$  be denoted by

$$\hat{\Gamma}(e^{j\omega}) = \Gamma(e^{j\omega}) + V(e^{j\omega}) \quad (20)$$

$$\hat{\Gamma}_{\text{rectw}}(e^{j\omega}) = \Gamma_{m,M}(e^{j\omega}) + V_{m,M}(e^{j\omega}) \quad (21)$$

where  $\Gamma(e^{j\omega})$  and  $V(e^{j\omega})$  are the frequency response of the sensitivity function  $\Gamma(z)$  and noise. Let  $\Gamma_{m,M}(e^{j\omega})$  and  $V_{m,M}(e^{j\omega})$  be denoted as

$$\Gamma_{m,M}(e^{j\omega}) = \Psi(e^{j\omega}) \otimes \Gamma(e^{j\omega}) \quad (22)$$

$$V_{m,M}(e^{j\omega}) = \Psi(e^{j\omega}) \otimes V(e^{j\omega}) \quad (23)$$

where  $\Psi(e^{j\omega})$  is a frequency response of the window function, and  $\otimes$  is a convolution operator in the frequency domain. It holds for sufficiently large  $m$ , that  $\Gamma_{m,M}(e^{j\omega}) \approx \Gamma(e^{j\omega})$ , then we have

$$\begin{aligned} & \left\| \hat{\Gamma}_{\text{rectw}}(e^{j\omega}) - \hat{\Gamma}(e^{j\omega}) \right\| \approx \left\| V_{m,M}(e^{j\omega}) - V(e^{j\omega}) \right\| \\ & \geq \left| \left\| V_{m,M}(e^{j\omega}) \right\| - \left\| V(e^{j\omega}) \right\| \right| \end{aligned} \quad (24)$$

where  $\|X(e^{j\omega})\| = \max_{0 \leq \omega < 2\pi} \{|X(e^{j\omega})|\}$ . By using the definition of  $\|V(e^{j\omega})\| = \varepsilon$ ,  $\|V_{m,M}(e^{j\omega})\|$  can be evaluated [7] as

$$\frac{Q_w}{\sqrt{2}} \varepsilon \leq \sup_{|V(e^{j\omega})| < \varepsilon} \|V_{m,M}(e^{j\omega})\| \leq Q_w \varepsilon \quad (25)$$

where  $Q_w$  ifs defined by

$$Q_w = \frac{1}{M} \sum_{l=0}^{M-1} \left| \sum_{k=0}^{m-1} \psi_{m,k} e^{-j\frac{2\pi l}{M}k} \right| \quad (26)$$

Here  $Q_w / \sqrt{2} - 1$ . Then it follows from (24) and (25),

$$\sup_{|V(e^{j\omega})| < \varepsilon} \left\| \hat{\Gamma}_{\text{rectw}}(e^{j\omega}) - \hat{\Gamma}(e^{j\omega}) \right\| \geq \left( \frac{Q_w}{\sqrt{2}} - 1 \right) \varepsilon \quad (27)$$

holds where  $\sup \|\cdot\|$  implies a supreme on all possible noise realizations, but it is impossible to strictly evaluate it. Hence, by using obtained samples, we evaluate  $\varepsilon$  as

$$\hat{\varepsilon} \approx \frac{1}{\frac{1}{\sqrt{2}} Q_w - 1} \left\| \hat{\Gamma}_{\text{rectw}}(e^{j\omega}) - \hat{\Gamma}(e^{j\omega}) \right\| \quad (28)$$

Thus, the estimation error  $\|\Delta\Gamma_{\text{noise}}(e^{j\omega})\|$  due to noise is given by

$$\begin{aligned} \|\Delta\Gamma_{\text{noise}}(e^{j\omega})\| &= \left\| \Gamma(e^{j\omega}) - \hat{\Gamma}_{\text{rectw}}(e^{j\omega}) \right\| \\ &= \left\| \Gamma(e^{j\omega}) - \hat{\Gamma}(e^{j\omega}) + \hat{\Gamma}(e^{j\omega}) - \hat{\Gamma}_{\text{rectw}}(e^{j\omega}) \right\| \\ &\leq \left\| \Gamma(e^{j\omega}) - \hat{\Gamma}(e^{j\omega}) \right\| + \left\| \hat{\Gamma}(e^{j\omega}) - \hat{\Gamma}_{\text{rectw}}(e^{j\omega}) \right\| \\ &\leq (Q_w + 1) \varepsilon \\ &\approx \frac{Q_w + 1}{\frac{1}{\sqrt{2}} Q_w - 1} \left\| \hat{\Gamma}(e^{j\omega}) - \hat{\Gamma}_{\text{rectw}}(e^{j\omega}) \right\| \end{aligned} \quad (29)$$

### 5.2 Evaluation of Uncertainty Due to Signal Band Limitation and Model Reduction

Estimation error due to the signal band limitation and model reduction  $|\Delta\Gamma_{\text{band+low}}(e^{j\omega})|$  is approximately evaluated by

$$|\Delta\Gamma_{\text{band+low}}(e^{j\omega})| \leq \left| \hat{\Gamma}(e^{j\omega}) - \hat{\Gamma}_{\text{rectw}}(e^{j\omega}) \right| \quad (30)$$

Thus the error bound of the estimate  $\hat{\Gamma}(e^{j\omega})$  is given by

$$\begin{aligned} |\Delta\Gamma(e^{j\omega})| &= |\Gamma(e^{j\omega}) - \hat{\Gamma}(e^{j\omega})| \\ &\leq \|\Delta\Gamma_{\text{noise}}(e^{j\omega})\| + |\Delta\Gamma_{\text{band+low}}(e^{j\omega})| \\ &\leq \frac{Q_w + 1}{\frac{1}{\sqrt{2}} Q_w - 1} \left\| \hat{\Gamma}(e^{j\omega}) - \hat{\Gamma}_{\text{rectw}}(e^{j\omega}) \right\| \\ &\quad + \left| \hat{\Gamma}(e^{j\omega}) - \hat{\Gamma}_{\text{rectw}}(e^{j\omega}) \right| \end{aligned} \quad (31)$$

### 5.3 Proposed Identification Algorithm

The proposed algorithm for estimating the overall relay transfer function, sensitivity function and its uncertainty bound is summarized below:

**Step 1:** Calculate  $S(e^{j\omega})$  from one symbol period of  $s(t)$ .

**Step 2:** Estimate transfer functions.

**2.a:** Estimate the overall relay transfer function  $H(e^{j\omega_{p,k}})$  at pilot symbol frequencies using (10).

**2.b:** Estimate the overall relay transfer function  $H(e^{j\omega})$  in the signal band using the linear interpolation.

**2.c:** Estimate the sensitivity function  $\Gamma(e^{j\omega})$  by using the parametric or nonparametric identification method.

**2.d:** Estimation of  $G(e^{j\omega})$  in (17) and  $C(e^{j\omega})$  in (18).

**Step 3:** Evaluation of uncertainty bound

**3.a:** By generating the estimated input by (16), calculate the impulse response  $\hat{\gamma}(k)$  of sensitivity function using (15).

**3.b:** Obtain a reduced-order model of the sensitivity function by using a windowing procedure by (19).

**3.c:** Evaluate the uncertainty bound on  $\hat{\Gamma}(e^{j\omega})$  by (31).

## 6. SIMULATION RESULTS

Let the transmitted symbols  $d_n$  be 64QAM signals. The number of carriers is  $N = 1405$ , and the pilot symbols are inserted

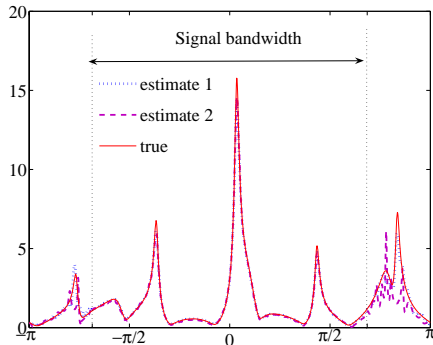


Figure 6: Relay frequency property. Solid line: True value of  $|H(e^{j\omega})|$ ; Dotted line:  $|\hat{H}(e^{j\omega})|$  under SNR= 35dB; Dashed line:  $|\hat{H}(e^{j\omega})|$  under SNR= 20dB

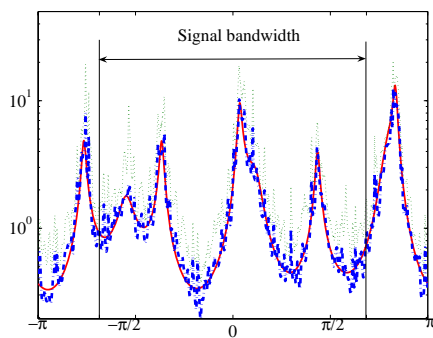


Figure 7: Frequency property of sensitivity function  $|\Gamma(e^{j\omega})|$  under SNR= 35dB. Solid line: True value. Thick dotted line:  $|\hat{\Gamma}(e^{j\omega})|$ ; Thin dotted line:  $|\hat{\Gamma}(e^{j\omega})| + |\Delta\Gamma(e^{j\omega})|$

at 118 frequencies. The frequency interval is  $\omega_0 = 2\pi f_0$ ,  $f_0 = 4$  kHz, and the size of FFT/IFFT is given by  $M = 2048$ . The length of the guard interval is  $T_{cp} = T_{inf}/4$ . Let  $K(z)$  be a positive constant, and the simulations are performed under two noise levels: SNR= 20dB and 35dB.

Let the number of taps in the canceller  $W(z)$  be 100. It is assumed that the relay station antenna receives five coupling waves with DU ratios (and tapped delay) 7.7dB(4), 3.25dB(6), 61dB(7), 10.9dB(9) and 65dB(12). It is also assumed that the unknown coupling and multi-path channel

$$C(z) = 0.6817e^{j0.3243\pi}z^{-3} + 0.85e^{j\frac{\pi}{6}}z^{-5} + 0.0457e^{-j\frac{11\pi}{12}}z^{-6} + 0.5795e^{-j0.5091\pi}z^{-8} + 0.0388e^{j\frac{\pi}{4}}z^{-11}$$

$$G(z) = 1 + 0.64e^{j\frac{\pi}{3}}z^{-9}$$

Figure 6 gives the estimate of the overall relay transfer function  $H(e^{j\omega})$ , while Figure 7 and 8 show the estimate of the sensitivity function  $|\Gamma(e^{j\omega})|$  and its uncertainty bound. All the figures demonstrate that the proposed method can give the very nice estimate of the transfer function outside the signal band as well as in the band, and the upper bound of uncertainty can also be appropriately given.

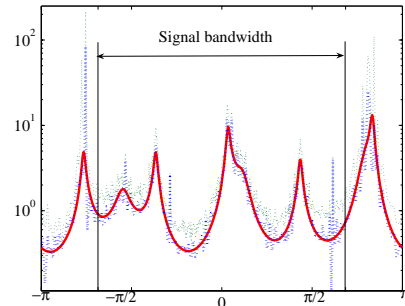


Figure 8: Frequency property of sensitivity function  $|\Gamma(e^{j\omega})|$  under SNR= 20dB. Solid line: True value. Thick dotted line:  $|\hat{\Gamma}(e^{j\omega})|$ ; Thin dotted line:  $|\hat{\Gamma}(e^{j\omega})| + |\Delta\Gamma(e^{j\omega})|$

## 7. CONCLUSION

The issue for estimating the overall relay transfer function and sensitivity function of the SFN OFDM relay station has been discussed, and the new approach has been proposed to estimate the transfer functions outside the signal band by using the property of the guard interval of OFDM signals. The upper bound of uncertainty of the estimate is also given, and it will be very useful to realize stable coupling cancellation at relay stations. Stability of adaptive algorithm for coupling cancellers can be attained by using the estimate of the sensitivity function and its estimation error bound, which will be reported in future.

## REFERENCES

- [1] H. Hamazumi, K. Imamura, N. Iai and K. Shibuya, "A study on coupling wave canceller for relay station in digital terrestrial broadcasting SFN," *IEICE Technical Report*, EMCJ98-111, pp.49-56, 1999
- [2] L.C. Godara, "Applications of antenna arrays to mobile communications, Part II: Beam-forming and direction-of-arrival considerations," *Proc. of the IEEE*, Vol.85, no.8, pp.1195-1245, 1997
- [3] K. Yang, Y. Zhang and Y. Mizuguchi, "A signal subspace-based subband approach to space-time adaptive processing for mobile communications," *IEEE Trans. on Signal Processing*, vol.49, no.2, pp.401-413, 2001
- [4] Y. Ding, T. Davidson, Z. Luo and K. Wong, "Minimum BER block precoders for zero-forcing equalization," *IEEE Trans. Signal Processing*, vol.51, no.9, pp.2410-2423, 2003
- [5] B. Muquet, M. Courville and P. Duhamel, "Subspace-based blind and semi-blind channel estimation for OFDM systems," *IEEE Trans. Signal processing*, vol.50, no.7, pp.1699-1712, 2002
- [6] R. Heath, and G.B. Giannakis, "Exploiting input cyclostationarity for blind channel identification in OFDM systems", *IEEE Trans. Signal Processing*, vol.47, no.3, pp.848-856, 1999
- [7] J. Chen and G. Gu, *Control-Oriented System Identification - An H<sub>∞</sub> Approach*, John Wiley & Sons, Inc., 2000