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# ROOT SPECTRAL ESTIMATION FOR LOCATION BASED ON TOA

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## ABSTRACT

*As it is well known, Non Line Of Sight (NLOS) and multipath propagation bias Time Of Arrival (TOA) estimate, reducing the accuracy of positioning algorithms. High-resolution first arriving path detector from propagation channel estimates based on the Minimum Variance (MV) and Normalized Minimum Variance (NMV) estimates of the power delay profile yields high accurate TOA estimation even in high multipath scenarios. The aim of this paper is to present root versions of the MV and NMV algorithms and to analyse the improvements in positioning accuracy obtained by the new approaches.*

## 1. INTRODUCTION

An accurate estimation of the first arrival is required in positioning systems. Unfortunately, multipath propagation and NLOS condition imposed by the wireless channel bias the first arrival estimation. In this kind of environments the delayed replicas displace the maximum of the impulse response; therefore, estimation techniques based on a maximum search cannot ensure an accurate estimation. In addition, due to resolution limitations, techniques based on correlation are not able to separate the direct path from the replicas if the LOS signal is highly attenuated or the echoes are close to the direct path. In order to overcome this problem high-resolution methods based on SVD (Singular Value Decomposition) or ML (Maximum Likelihood) techniques have been proposed. Both of them provide a complete channel description at expense of a high computational burden. On the one hand, SVD criteria as MUSIC, root-MUSIC [1] and TLS-ESPRIT [2] require an eigendecomposition to separate the signal subspace from the noise subspace, being not always easy to establish the right dimension of the signal subspace. On the other hand, the ML solution [3], [4] leads to an expensive multidimensional search.

Minimum Variance (MV) [5] and Normalized Minimum Variance (NMV) [6] are widely used as spectral estimators. These techniques were successfully developed and applied to location systems in [7] and [8] achieving an accurate estimation of the first arrival, even in the presence of high multipath and when the LOS signal is highly attenuated. The computational cost of the MV solutions is affordable and any previous knowledge of the number of propagation paths is not required.

New methods, which transform the traditional grid search of the MV and NMV estimation into polynomial rooting procedures, are proposed in this paper. The polynomial approaches provide better statistical results than the maximum search ones, reducing the bias and variance of the TOA estimation. Besides, an efficient implementation of the MV method applied to location, based on a Fast Fourier Transform (FFT), is presented. This approach is inspired in the work of Musicus [9], where the author presented an implementation of the spectral MV algorithm based on a FFT. As well, this work paved the way for deriving a fast implementation of the NMV method. The FFT approach allows to reduce the computational burden of the traditional MV and NMV algorithms exposed in [7] and [8].

The rest of the paper is organized as follows. In the next section the signal model is defined. MV and NMV methods are explained in sections 3 and 4, respectively. Simulation results in a mobile scenario are shown in section 5 and finally, in Section 6 some concluding remarks are presented.

## 2. SIGNAL MODEL

We start considering the next received signal:

$$y(t) = \sum_{i=0}^{L-1} \alpha_i g(t-\tau_i) + n(t) \qquad (1)$$

where $L$ is the number of propagation paths, $g(t)$ is the pulse shape of the modulation, $\tau_i$ and $\alpha_i$ are the time delay and the complex time-varying amplitude of the $i$-th arrival,

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respectively, and $n(t)$ is an additive noise uncorrelated with the data.
In the frequency domain, the expression (1) is transformed into a weighted sum of complex exponentials embedded in noise:

$$Y(w) = \sum_{i=0}^{L-1} \alpha_i G(w) e^{-jw\tau_i} + N(w) \quad (2)$$

Sampling the frequency domain received signal, the expression (2) can be formulated as an M dimensional signal vector:

$$\mathbf{y} = \sum_{i=0}^{L-1} \alpha_i \mathbf{G}\mathbf{e}_{\tau_i} + \mathbf{n} \quad (3)$$

where $\mathbf{G}$ is a diagonal matrix containing the DFT of the pulse shaping filter and $\mathbf{e}_{\tau_i}$ is the steering vector for the arrival at timing $\tau_i$:

$$\mathbf{e}_{\tau_i} = \begin{bmatrix} 1 & e^{-j\omega_i} & \cdots & e^{-j(M-1)\omega_i} \end{bmatrix}^T, \omega_i = \frac{2\pi\tau_i}{M} \quad (4)$$

Equation (3) can be rewritten as the sum of the LOS path (which is the desired signal) and a new noise term $\tilde{\mathbf{n}}$ which includes all the echoes (undesired replicas):

$$\mathbf{y} = \alpha_0 \mathbf{G}\mathbf{e}_{\tau_o} + \tilde{\mathbf{n}} \quad (5)$$

## 3. MINIMUM VARIANCE METHOD

As it is well known, the frequency domain model (2) is equivalent to the models used in spectral and spatial analysis [10]. Thus, spectral estimation algorithms can be applied to the TOA estimation problem.
The MV estimation consists of finding the filter $\mathbf{w}$ which maximizes the output SNR and satisfies the constraint $\mathbf{w}^H\mathbf{G}\mathbf{e}_{\tau_o} = 1$, yielding:

$$\mathbf{w}(\tau) = \frac{\mathbf{R}_y^{-1}\mathbf{G}\mathbf{e}_\tau}{\mathbf{e}_\tau^H \mathbf{G}^H \mathbf{R}_y^{-1} \mathbf{G}\mathbf{e}_\tau} \quad (6)$$

$$P(\tau) = \frac{1}{\mathbf{e}_\tau^H \mathbf{G}^H \mathbf{R}_y^{-1} \mathbf{G}\mathbf{e}_\tau} \quad (7)$$

where $\mathbf{R}_\mathrm{y}$ is the sample correlation matrix estimated from a collection of N signal vectors within the time coherence of the delays.
The expression (6) provides the well known matched-filter bank point of view [11]. In the traditional approach, a grid search is performed to estimate the power delay spectrum (7), and then the delay of the LOS path is estimated as the first one above a threshold, which is related to the noise floor level.

### 3.1 FFT-MV

A FFT based approach of the MV estimator can be derived. Let us consider an alternative expression for (7):

$$P(\tau) = \frac{1}{Tr\{\tilde{\mathbf{R}}\mathbf{E}\}} \quad (8)$$

where $Tr\{\cdot\}$ denotes the trace operator, $\tilde{\mathbf{R}} = \mathbf{G}^H \mathbf{R}_y^{-1} \mathbf{G}$ and matrix $\mathbf{E}$ is defined as follows:

$$\mathbf{E} = \mathbf{e}_\tau \mathbf{e}_\tau^H = \begin{bmatrix} 1 & e^{j\omega} & \cdots & e^{j(M-1)\omega} \\ e^{-j\omega} & 1 & e^{j\omega} & e^{j(M-2)\omega} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-j(M-1)\omega} & e^{-j(M-2)\omega} & \cdots & 1 \end{bmatrix} \quad (9)$$

being $\omega = \dfrac{2\pi\tau}{M}$.

The power delay spectrum (8) can be formulated as a DFT, where time delay information is wrapped in ω.

$$P(\tau) = \frac{1}{\sum_{k=-M+1}^{M-1} D(k) e^{-j\omega k}} \quad (10)$$

defining $D(k) = \sum_{n=1}^{M-k} \tilde{\mathbf{R}}(\mathrm{n}, \mathrm{n}+\mathrm{k})$.

Computing (10) as a FFT of length P, where P is the number of desired grid points, a fast implementation of the grid search of the maxima of the expression (8) can be achieved. Moreover, the matrix inversion of the sample data covariance matrix $\mathbf{R}_\mathrm{y}$ can be avoided as explained in [9] and [12] using the Gohberg-Semencul formula and the Levinson-Durbin recursion. Thus, the MV solution can be computed with a very low computational cost.

### 3.2 Polynomial-MV.

Now we introduce a new polynomial rooting procedure related to the traditional MV. The maximization of $P(\tau)$ to find the channel fingers is equivalent to:

$$\min_\tau Tr\{\tilde{\mathbf{R}}\mathbf{E}\} = \min_\tau \sum_{k=-M+1}^{M-1} D(k) e^{-j\omega k} \quad (11)$$

Equation (11) is derived to obtain the local minimums of the function. Being $\rho = e^{-j\omega}$ we obtain:

$$\frac{\partial Tr\{\tilde{\mathbf{R}}\mathbf{E}\}}{\partial \omega} = -j\sum_{k=1}^{M-1} k \left[ D(k)\rho^k - D^*(k)\rho^{-k} \right] = 0 \quad (12)$$

Multiplying by $\rho^{M-1}$, the equation (12) can be rewritten as the following polynomial function:

$$A(\rho) = (M-1)D(M-1)\rho^{2M-2} + \cdots + D(1)\rho^{M} - D^*(1)\rho^{M-2} - \cdots - (M-1)D^*(M-1) = 0 \quad (13)$$

Let us redefine $A(\rho)$ as follows

$$A(\rho) = a_{2M-2}\rho^{2M-2} + \cdots a_k\rho^k + \cdots + a_o = 0 \quad (14)$$

The expression (14) satisfies $a_k = -a^*_{2M-2-k}$. The polynomials that accomplish this property are called *-antisymmetric or *-antipalindromic, and its roots are not guaranteed to be on the unit circle because also conjugate reciprocal roots could satisfy the conjugate symmetry constraint. That is to say, zeros of the polynomial (14) either lie on the unit circle or occur in conjugate reciprocal pairs. Efficient polynomial rooting algorithms can be used to compute the roots which lie on the unity circle [13] with some tolerance.

Since the first time delay is the only one which bears position information, it is determined as the first maximum of the power delay spectrum above a threshold level similar to that exposed in [7]. Clearly, as shown in Figure 1, the roots of the polynomial (12) correspond to critical values of the expression (10). Thus, to estimate the first maximum of (10) over the threshold two alternatives are possible:

1) To compute the second derivative of (11) in order to pick the maxima of the spectrum and to determine the first one above the threshold level in the expression (8), which represents the power delay spectrum.
2) To evaluate the roots of (12) in (8) and to pick the first meaningful delay, that is, the first positive delay corresponding to a root lying on the unit circle, with some tolerance, above the threshold level. This option is preferred.

Using this new polynomial rooting procedure, the one-dimensional grid search performed to obtain the first time delay is avoided and higher accuracy can be achieved. Moreover, the polynomial-MV algorithm reduces bias and variance in TOA estimation.

## 4. NORMALISED MINIMUM VARIANCE METHOD

The MV power delay spectrum estimation is strongly dependent on the filter bandwidth. As well, the presence of the pulse shaping filter $\mathbf{G}$ causes the appearance of significant side lobes. Normalizing the power delay spectrum (7) by the equivalent noise bandwidth, an estimation of the power delay spectrum density with better resolution properties is obtained:

$$\mathrm{S}(\tau) = \frac{\mathrm{P}(\tau)}{\mathbf{w}^{\mathrm{H}}\mathbf{w}} = \frac{\mathbf{e}_\tau^H\mathbf{G}^H\mathbf{R}_{\mathrm{y}}^{-1}\mathbf{G}\mathbf{e}_\tau}{\mathbf{e}_\tau^H\mathbf{G}^H\mathbf{R}_{\mathrm{y}}^{-2}\mathbf{G}\mathbf{e}_\tau} \quad (15)$$

The NMV presented herein is an application for positioning systems of the NMLM method proposed in [6].

### 4.1 FFT-NMV

In order to derive the FFT based approach of the NMV algorithm we define:

$$\Gamma(k) = \sum_{n=1}^{M-k}\mathbf{\Phi}(\mathrm{n},\mathrm{n}+\mathrm{k}) \text{ where } \mathbf{\Phi} = \mathbf{G}^{\mathrm{H}}\mathbf{R}_{\mathrm{y}}^{-2}\mathbf{G} \quad (16)$$

Then, $S(\tau)$ can be expressed as:

$$S(\tau) = \frac{\sum_{k=-M+1}^{M-1} D(k)e^{-j\omega k}}{\sum_{k=-M+1}^{M-1} \Gamma(k)e^{-j\omega k}} \quad (17)$$

The matrix inversion of the data correlation matrix $\mathbf{R_y}$ in (17) can be avoided as explained in [9] and [12] using the Gohberg-Semencul formula and the Levinson-Durbin recursion. Moreover, the expression (17) can be formulated as the quotient of two DFT. Computing two P-point-FFT, where P is the number of desired grid points, we can define $\mathbf{D}$ and $\mathbf{\Gamma}$ as:

$$\mathbf{D} = 2real[FFT\{D(0)\cdots D(M-1)\}] - D^*(0)$$

$$\mathbf{\Gamma} = 2real[FFT\{\Gamma(0)\cdots\Gamma(M-1)\}] - \Gamma^*(0)$$

Using the previous expressions, the power delay spectrum density (17) can be expressed as $\mathbf{S} = \mathbf{D} \oslash \mathbf{\Gamma}$, where Ø denotes element-wise division:

$$\mathbf{S}(m) = \frac{\mathbf{D}(m)}{\mathbf{\Gamma}(m)} \quad \text{m=0,…, P-1} \quad (18)$$

FFT-NMV provides a fast computation of the power delay spectrum density. Finally, the desired time delay is obtained evaluating (18) and picking the first meaningful maximum.

### 4.2 Polynomial-NMV

In this section, a polynomial interpretation for NMV algorithm, similar to the polynomial-MV derived in section 3.2, is presented. The maximization of the power spectrum density $S(\tau)$ must be implemented to find the channel fingers.

$$\max_\tau\{S(\tau)\} = \max_\tau\left\{\frac{Tr\{\tilde{\mathbf{R}}\mathbf{E}\}}{Tr\{\mathbf{\Phi}\mathbf{E}\}}\right\} = \max_\tau\left\{\frac{\sum_{k=-M+1}^{M-1} D(k)\rho^k}{\sum_{k=-M+1}^{M-1}\Gamma(k)\rho^k}\right\} \quad (19)$$

Deriving (19) to obtain the (local) maxima:

$$\frac{\partial S(\tau)}{\partial\omega} = \frac{\partial Tr\{\tilde{\mathbf{R}}\mathbf{E}\}}{\partial\omega}Tr\{\mathbf{\Phi}\mathbf{E}\} - \frac{\partial Tr\{\mathbf{\Phi}\mathbf{E}\}}{\partial\omega}Tr\{\tilde{\mathbf{R}}\mathbf{E}\} = 0 \quad (20)$$

Where $\frac{\partial Tr\{\tilde{\mathbf{R}}\mathbf{E}\}}{\partial\omega}$ has been formulated in equation (12) and:

$$\frac{\partial Tr\{\mathbf{\Phi E}\}}{\partial \omega} = -j\sum_{k=1}^{M-1} k\left[\Gamma(k)\rho^{k} - \Gamma^{*}(k)\rho^{-k}\right] \quad (21)$$

The resultant polynomial is not *-antisymmetric as the polynomial defined in (13).

The roots of (20) are the critical points of the power delay spectrum density. In order to obtain the first time delay a procedure similar to the one exposed in the section 3.2 must be applied. After the computation of the roots of (20), the roots which are close to the unit norm circle are chosen and the power delay spectrum density (17) is evaluated to determine the first arrival. Note that fast polynomial rooting procedures as those presented in [13] and [14] can be used.

## 5. SIMULATION RESULTS

The methods proposed in this paper can provide an accurate estimation of the first time delay. In order to evaluate the statistical properties of the FFT-based solutions and the polynomial-based techniques a mobile trajectory under two different conditions has been simulated.

In Figure 1, the relationship between the estimated MV power delay spectrum and the polynomial-MV roots has been presented. Without loss of generality, only five incoming paths were considered under a SNR of 15 dB. Obviously, the roots provided by the polynomial algorithm correspond to critical values of the power delay spectrum.

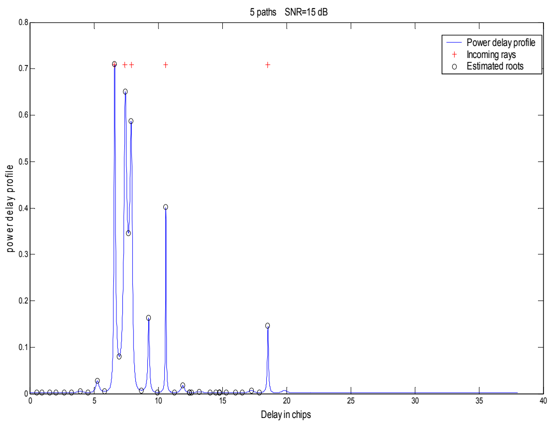


Figure 1. Relationship between the estimated MV power delay spectrum and the polynomial roots for a channel with only five incoming paths under a SNR of 15 dB.

Figure 2 shows the mobile trajectory, consisting of 1404 points, considered in the simulations and the location of the base station that performed the range estimates. In order to obtain the mobile position at each point, several channel estimates were generated. The number of channel estimates depends on the coherence time of the delays and on the rate of provision, which value was set equal to 1500 channels estimates per second.

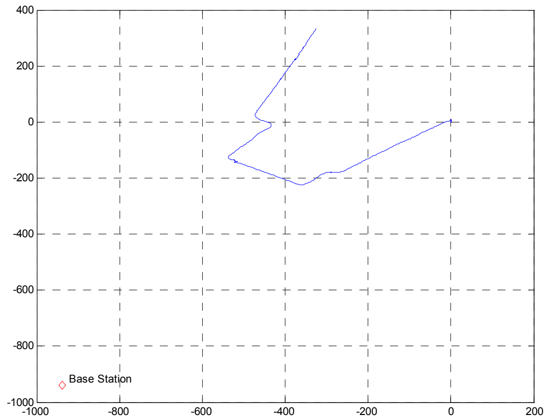


Figure 2 Trajectory considered in the simulations.

Firstly, all the trajectory points are under a LOS situation. Thus, the first arrival corresponds to the direct path and it is the most powerful one. We consider a mean number of the incoming rays of 10 and a delay spread between 3e-7 and 6e-7 seconds, typical values in urban environments. The pulse **G**, considered in this paper, is a raised cosine sampled at two samples per chip. The FFT length in the FFT-MV and FFT-NMV algorithms has been set equal to 4096. Figure 3 shows the Root Mean Square Error (RMSE) and Standard deviation of the range as a function of the SNR. On the one hand, root solutions present less RMSE and less standard deviation than the maximum search ones (computed using the FFT formulation exposed in 3.1 and 4.1). On the other hand, the dependence of the root procedures on SNR is less than the dependence of the maximum search methods. In the same environment, a maximum of the impulse response detector yields a RMSE about 95 metres and a standard deviation about 75 metres.

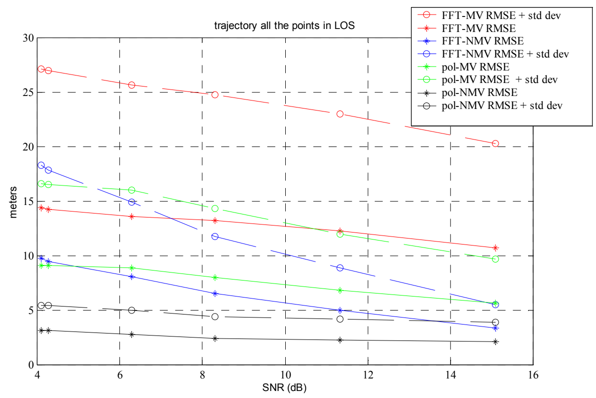


Figure 3. RMSE and standard deviation versus SNR in LOS.

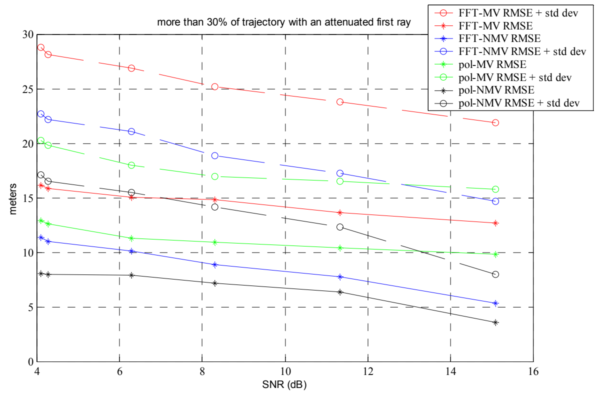


Figure 4. RMSE and standard deviation versus SNR in NLOS.

Next, the same trajectory is considered but in this case, the power of the direct signal is attenuated by a random factor between 0.5 and 1 in more than 30% of the trajectory. In these situations, the mean number of the incoming rays rises to 15. Figure 4 shows the RMSE and standard deviation for different values of SNR. It can be observed that RMSE and standard deviation are slightly increased. In this situation, the maximum of the channel impulse response detector is not considered because if the first path is attenuated, then the obtained estimation will be dramatically biased.

## 6. CONCLUSIONS

In this paper four techniques have been proposed for location purposes. On the one hand, fast implementation approaches of the MV and NMV algorithms based on the FFT have been proposed, which present a dramatically reduction in the computational burden compared to the traditional grid search performed in traditional spectral techniques. On the other hand, two root polynomial procedures derived from the MV and NMV algorithms have been presented. These algorithms present better resolution, less variance and less RMSE.

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