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# BIDIRECTIONAL MIMO EQUALIZER DESIGN

*Davide Mattera*(*) *Francesco Palmieri*(**) *Gianluca D'Angelo*

* Università degli Studi di Napoli Federico II,
Dipartimento di Ingegneria Elettronica e delle Telecomunicazioni,
Via Claudio, 21 I-80125 Napoli Italy
** Seconda Università degli Studi di Napoli,
Dipartimento di Ingegneria dell'Informazione,
Via Roma, 29 I-81031 Aversa Italy

**ABSTRACT**

*The paper considers an important variation of the MIMO channel Decision Feedback (DF) equalizer, the bidirectional MIMO equalizer, that combines the classical forward DF equalizer (DFE) with the backward DFE, which is based on imposing anticausal properties to the feedback filter. With reference to the minimum mean square error (MMSE) criterion, we extend the bidirectional equalization from the single-input single-output (SISO) scenario to the more general multiple-input multiple-output (MIMO) scenario where different definitions of anticausal systems can be given. An original variation of the bidirectional DF equalizer is also proposed in order to reduce its performance loss in the presence of the error propagation.*

## 1. INTRODUCTION

Recent advances in wireless communications have motivated the study of the equalization over dispersive MIMO channels. Maximum likelihood equalization techniques for a MIMO dispersive channel present computational complexity that increases exponentially with the number of input sequences as well as with channel memory order and, therefore, suboptimum receivers are often considered. An important class of equalizers is the DF equalizer, which is a symbol-by-symbol equalizer and, at least in SISO scenario, performs very closely to the Viterbi equalizer [1] over many channels and exhibits better tolerance to phase jitter [2].

MIMO DF equalizer design is usually performed in the indirect case (i.e., perfect estimates of channel response as well as of data and noise spectra are available) under the adoption of Zero Forcing (ZF) or Minimum Mean Square Error (MMSE) criterion and under the assumption (motivated by analytical tractability) of correct previous decisions. The general design procedure for MIMO scenario has been derived in [3, 4] with reference to the MMSE criterion; such a procedure requires the factorization of a discrete-time spectral matrix; the simplification of such a procedure has been considered in [5].

Several variations of the DF equalizer have been proposed in the literature; among them, one of the most important improvements is represented by its bidirectional version [6], particularly suited to the modern packet-switching transmissions. Such an equalizer, derived only with reference to a SISO scenario, recursively works on the received sequence both in a forward and in a backward fashion, thus obtaining two estimates of each transmitted symbol, to be used for the final decision. Formally, the backward DFE is designed by imposing that the feedback filter is anticausal while the forward DFE is designed by imposing that the feedback filter is causal. Since the definitions of causal and anticausal filter admit several variations in the MIMO scenario, a detailed analysis is required to provide its extension to the MIMO scenario.

In this paper we derive the MIMO bidirectional DF equalizer (Bi-DFE) according to the MMSE criterion and we analyze its performances in the presence of error propagation. Moreover, in order to improve them, an original two-stage variation of such an equalizer is proposed.

## 2. BASIC NOTATIONS

Let us consider the vector $\mathbf{x}(n)$ of discrete-time processes; its autocorrelation matrix is defined as $\mathbf{R}_x(m) \stackrel{\triangle}{=} \langle \mathbf{E}[\mathbf{x}(n)\mathbf{x}^H(n-m)] \rangle = \mathbf{R}_x^H(-m)$ with the superscript $H$ denoting the Hermitian transpose, $\mathbf{E}$ the statistical average and $\langle\cdot\rangle$ the temporal average. The spectral matrix of $\mathbf{x}(n)$ is defined as the (two-sided) z-transform of the autocorrelation matrix $\mathbf{R}_x(m)$, i.e., $\mathbf{S}_x(z) \stackrel{\triangle}{=} \sum_{m=-\infty}^{+\infty} \mathbf{R}_x(m) z^{-m} = \mathbf{S}_x^H(z^{-*})$, where the last equality follows from $\mathbf{R}_x(m)=\mathbf{R}_x^H(-m)$ with the asterisk $*$ denoting the complex conjugate. Therefore $\mathbf{S}_x(e^{j2\pi\nu})$ is the power spectral density of $\mathbf{x}(n)$.

## 3. DF FILTER DESIGN

Let us consider the following MIMO discrete-time channel model:

$$\mathbf{r}(n) = \mathbf{h}(n) \otimes \mathbf{x}(n) + \eta(n) \tag{1}$$

where $\otimes$ denotes MIMO discrete-time convolution, $\mathbf{x}(n)$ is the input information-bearing vector with spectral matrix $\mathbf{S}_x(z)$; such a matrix describes the spatial and temporal correlation of the process $\mathbf{x}(n)$. The channel is described by the MIMO impulse response $\mathbf{h}(n)$ and the zero-mean disturbance $\eta(n)$, uncorrelated with $\mathbf{x}(n)$, has spectral matrix $\mathbf{S}_\eta(z)$. Note that the disturbance can include not only the background noise but also the interferences from signals different from $\mathbf{x}(n)$. The considered discrete-time channel model is central in the problem of receiver design for a variety of modern communication systems.

When DF equalization method is adopted, the equalized sequence $\mathbf{z}(n)$ is given by

$$\mathbf{z}(n) = \mathbf{f}(n) \otimes \mathbf{r}(n) - \mathbf{b}(n) \otimes \hat{\mathbf{x}}(n) \tag{2}$$

where $\mathbf{f}(n)$ is the feedforward MIMO linear and time-invariant (LTI) filter, $\mathbf{b}(n)$ is the feedback MIMO LTI filter, and $\hat{\mathbf{x}}(n)$ represents an estimation of $\mathbf{x}(n)$ obtained by hard-decisions based on the same sequence $\mathbf{z}(n)$. Both the forward and backward DF equalizers share the same structure while the difference consists in the response of the two filters $\mathbf{F}(z)$ and $\mathbf{B}(z)$, where we denote with the upper-case letter the z-transform of the impulse response. In particular only the optimum MMSE filters in the forward DFE are usually derived while the backward DFE is not considered, as in [3, 4], or it is briefly sketched as in [5]. In this section, instead, we provide a simple and detailed derivation of the optimum MMSE backward DFE leaving just sketched the well-known derivation of the forward counterpart.

The adoption of the MMSE criterion means to search for the optimum filters $\mathbf{F}(z)$ and $\mathbf{B}(z)$ (expressed in the z-domain) that minimize the cost function $\int_{-\frac{1}{2}}^{\frac{1}{2}} \mathrm{Tr}\mathbf{S}_e(e^{j2\pi\nu})\mathrm{d}\nu$ where $Tr$ denotes the trace operator and $\mathbf{S}_e(z)$ denotes the spectral matrix of the error sequence $\mathbf{e}(n) \stackrel{\triangle}{=} \mathbf{z}(n) - \mathbf{x}(n)$. Under the typical assumption $\mathbf{x}(n) = \hat{\mathbf{x}}(n)$, it can be shown that

$$\begin{aligned}\mathbf{S}_e(z) &= \left[\mathbf{F}(z)\mathbf{H}(z) - \widetilde{\mathbf{B}}(z)\right]\mathbf{S}_x(z)\left[\mathbf{H}^H(z^{-*})\mathbf{F}^H(z^{-*})\right.\\ &- \left.\widetilde{\mathbf{B}}^H(z^{-*})\right] + \mathbf{F}(z)\mathbf{S}_\eta(z)\mathbf{F}^H(z^{-*})\end{aligned}$$

with $\widetilde{\mathbf{B}}(z) \stackrel{\triangle}{=} \mathbf{B}(z) + \mathbf{I}$. Such a result is easily derived since the output $\mathbf{y}(n)$ of a LTI filter $\mathbf{H}(z)$ with input $\mathbf{x}(n)$ has spectral matrix $\mathbf{S}_y(z) = \mathbf{H}(z)\mathbf{S}_x(z)\mathbf{H}^H(z^{-*})$. The matrix $\mathbf{S}_e(z)$ can be easily re-written as

$$\begin{aligned}\mathbf{S}_e(z) &= \Delta F(z)\mathbf{S}_r(z)\Delta F^H(z^{-*}) + \widetilde{\mathbf{B}}(z)\mathbf{S}_B(z)\widetilde{\mathbf{B}}^H(z^{-*})\\ \Delta F(z) &\stackrel{\triangle}{=} [\mathbf{F}(z) - \mathbf{F}^{(\mathrm{opt})}(z)]\\ \mathbf{S}_r(z) &\stackrel{\triangle}{=} \mathbf{H}(z)\mathbf{S}_x(z)\mathbf{H}^H(z^{-*}) + \mathbf{S}_\eta(z)\\ S_B(z) &\stackrel{\triangle}{=} \mathbf{S}_x(z) - \mathbf{S}_x(z)\mathbf{H}^H(z^{-*})\mathbf{S}_r^{-1}(z)\mathbf{H}(z)\mathbf{S}_x(z)\\ \mathbf{F}^{(\mathrm{opt})}(z) &\stackrel{\triangle}{=} \widetilde{\mathbf{B}}(z)\mathbf{S}_x(z)\mathbf{H}^H(z^{-*})\mathbf{S}_r^{-1}(z). \qquad (3)\end{aligned}$$

This implies that $\mathbf{F}^{(\mathrm{opt})}(z)$ is the optimum choice for $\mathbf{F}(z)$ and it depends on the utilized feedback filter; relation (3) with $\widetilde{\mathbf{B}}(z) = I$ allows one to determine the optimum MMSE linear filter. The optimum feedback filter minimizes $\int_{-\frac{1}{2}}^{\frac{1}{2}} \mathrm{Tr}\left[\widetilde{\mathbf{B}}(e^{j2\pi\nu})\mathbf{S}_B(e^{j2\pi\nu})\widetilde{\mathbf{B}}^H(e^{j2\pi\nu})\right]\mathrm{d}\nu$. Let us note that, according to the matrix inversion lemma, $\mathbf{S}_B(z) = \mathbf{S}_C^{-1}(z)$ where $\mathbf{S}_C(z) \stackrel{\triangle}{=} \mathbf{S}_x^{-1}(z) + \mathbf{H}^H(z^{-*})\mathbf{S}_\eta^{-1}(z)\mathbf{H}(z)$. Since the sequence $\hat{\mathbf{x}}(n)$ is obtained by hard-decisions on the already obtained values of $\mathbf{x}(n)$, not all the values of $\hat{\mathbf{x}}(n)$ are available at time-step $n$ and, therefore, a constraint on the utilized filter $\mathbf{B}(z)$ is needed; in particular, the forward DFE is obtained by imposing the causality of the filter while the backward DFE is obtained by imposing the anticausality of the filter. In order to derive the bidirectional equalizer, we first need to derive the backward DF equalizer. Since in MIMO case there are different anticausality properties, we review a number of relevant definitions to be utilized.

We say that $\mathbf{H}(z)$ is anticausal when it is analytic in $|z| \leq 1$, i.e., it is has all poles outside $C$, and $\mathbf{H}(0)$ is upper triangular. We say that $\mathbf{H}(z)$ is causal when it analytic in $|z| \geq 1$, i.e., all poles are internal to the unit circle $C$, and $\mathbf{H}(\infty)$ is lower triangular. We say that $\mathbf{H}(z)$ is strictly anticausal when all its poles are external to $C$ and $\mathbf{H}(0)$ is strictly upper triangular (i.e., upper triangular with null diagonal entries); we say that $\mathbf{H}(z)$ is strongly anticausal when all its poles are external to $C$ and $\mathbf{H}(0)$ is null. We say that a square matrix is monic when its diagonal entries are equal to one. We say that $\mathbf{H}(z)$ is anticanonical when it is square, anticausal, $\mathbf{H}(0)$ is monic and $\mathbf{H}^{-1}(z)$ is anticausal; note that the inverse of an anticanonical filter is also anticanonical and the product of two anticanonical filters is also an anticanonical filter. We say that $\mathbf{H}(z)$ is canonical when it is square, causal, $\mathbf{H}(\infty)$ is monic and $\mathbf{H}^{-1}(z)$ is causal. Note that if $\mathbf{H}(z)$ is anticanonical, then $\mathbf{H}^H(z^{-*})$ is canonical and *vice versa*.

For backward DF filter design we can impose that $\mathbf{B}(z)$ is strictly anticausal, which implies that $\widetilde{\mathbf{B}}(z)$ is anticausal with $\widetilde{\mathbf{B}}(0)$ monic, but we can also simply impose that $\mathbf{B}(z)$ is strongly anticausal, which implies that $\widetilde{\mathbf{B}}(z)$ is anticausal with $\widetilde{\mathbf{B}}(0) = \mathbf{I}$. In the strictly anticausal case the first component of $\hat{\mathbf{x}}(n)$ (i.e., the decision taken at time $n$ about the first component of $\mathbf{x}(n)$) is utilized for obtaining the second component of $\mathbf{z}(n)$, and so on. Note the dependence on the chosen order of the components of $\mathbf{x}(n)$ that we do not consider for optimization here. In the strongly causal case only decisions relative to successive time instants are utilized to obtain $\mathbf{z}(n)$ and the re-ordering of $\mathbf{x}(n)$ is irrelevant.

In order to determine the optimum $\widetilde{\mathbf{B}}(z)$ in such set of filters, we need to minimize the following cost function $\int_{-\frac{1}{2}}^{\frac{1}{2}} \mathrm{Tr}\left[\widetilde{\mathbf{B}}(e^{j2\pi\nu})\mathbf{S}_B(e^{j2\pi\nu})\widetilde{\mathbf{B}}^H(e^{j2\pi\nu})\right]\mathrm{d}\nu = \int_C \mathrm{Tr}\left[\widetilde{\mathbf{B}}(z)\mathbf{S}_B(z)\widetilde{\mathbf{B}}^H(z^{-*})\right]\frac{\mathrm{d}z}{j2\pi z}$ with $\int_C$ denoting the integral on the contour $|z| = 1$. We first consider the case where the square matrix $\mathbf{S}_B(z) = \mathscr{D}_B$ is a constant diagonal matrix (with positive entries) and the following optimization problem

$$\begin{cases}\widetilde{\mathbf{B}}^{(opt)}(z) = \arg\min\limits_{\widetilde{\mathbf{B}}(z)} \int_C \mathrm{Tr}\left[\widetilde{\mathbf{B}}(z)\mathscr{D}_B\widetilde{\mathbf{B}}^H(z^{-*})\right]\dfrac{\mathrm{d}z}{j2\pi z}\\ \widetilde{\mathbf{b}}(0) = \mathbf{B}_0\end{cases} \qquad (4)$$

where $\mathbf{B}_0$ is an assigned matrix, and $\widetilde{\mathbf{b}}(k)$ (with $(i,\ell)$ entry $\widetilde{b}_{i,\ell}(k)$) denotes the stable time impulse response of $\widetilde{\mathbf{B}}(z)$. At this aim we note that the cost function in (4) can be re-written as

$$\begin{aligned}\int_C \sum_{i=1}^N \sum_{\ell=1}^N D_i|\widetilde{B}_{i,\ell}(z)|^2 \frac{\mathrm{d}z}{j2\pi z} &= \sum_{i=1}^N \sum_{\ell=1}^N \int_{-\frac{1}{2}}^{\frac{1}{2}} D_i|\widetilde{\mathscr{B}}_{i,\ell}(\nu)|^2\mathrm{d}\nu\\ &= \sum_{i=1}^N \sum_{\ell=1}^N \sum_{k=-\infty}^{+\infty} D_i|\widetilde{b}_{i,\ell}(k)|^2 \qquad (5)\end{aligned}$$

where $\widetilde{B}_{i,\ell}(z)$ denotes the $(i,\ell)$ entry of $\widetilde{\mathbf{B}}(z)$ and $\widetilde{\mathscr{B}}_{i,\ell}(\nu)$ denotes $\widetilde{B}_{i,\ell}(z)$ for $z = e^{j2\pi\nu}$. Therefore, problem (4) obviously admits the solution $\widetilde{b}_{i,\ell}^{(opt)}(k) = B_{0i,\ell}\delta(k)$ or, equivalently, $\widetilde{\mathbf{B}}^{(opt)}(z) = \mathbf{B}_0$ and the value of the cost function at the optimum is

$$\mathrm{Tr}\left[\mathbf{B}_0\mathscr{D}_B\mathbf{B}_0^H\right]. \qquad (6)$$

Consider now the same problem with a different constraint

$$\begin{cases} \widetilde{\mathbf{B}}^{(opt)}(z) = \arg\min_{\widetilde{\mathbf{B}}(z)} \int_C Tr\left[\widetilde{\mathbf{B}}(z)\mathscr{D}_B\widetilde{\mathbf{B}}^H(z^{-*})\right]\frac{\mathrm{d}z}{j2\pi z} \\ \widetilde{\mathbf{B}}(z) \quad \text{anticanonical} \\ \widetilde{\mathbf{B}}(0) = \mathbf{B}_0. \end{cases} \tag{7}$$

It admits solution only if $\mathbf{B}_0$ is an anticanonical matrix. In such a case, the solution is obviously $\widetilde{\mathbf{B}}^{(opt)}(z) = \mathbf{B}_0$ with minimum value (6). In fact, optimum solution of unconstrained problem (4) satisfies also the constraint, imposed in (7), of being an anticanonical filter with assigned value for $z = 0$. Consider now the following problem

$$\begin{cases} \widetilde{\mathbf{B}}^{(opt)}(z) = \arg\min_{\widetilde{\mathbf{B}}(z)} \int_C Tr\left[\widetilde{\mathbf{B}}(z)\mathscr{D}_B\widetilde{\mathbf{B}}^H(z^{-*})\right]\frac{\mathrm{d}z}{j2\pi z} \\ \widetilde{\mathbf{B}}(z) \quad \text{anticanonical.} \end{cases} \tag{8}$$

From the solution of problem (7), it easily follows that the problem (8) admits the solution $\widetilde{\mathbf{B}}^{(opt)}(z) = \mathbf{B}_0^{(opt)}$ where

$$\begin{cases} \mathbf{B}_0^{(opt)} = \arg\min_{\mathbf{B}_0} Tr\left[\mathbf{B}_0\mathscr{D}_B\mathbf{B}_0^H\right] \\ \mathbf{B}_0 \quad \text{anticanonical.} \end{cases} \tag{9}$$

Since $D_i > 0$ and

$$Tr\left[\mathbf{B}_0\mathscr{D}_B\mathbf{B}_0^H\right] = \sum_{i=1}^{N}\sum_{\ell=1}^{N} D_i|B_{0i,\ell}|^2, \tag{10}$$

the solution of (9) is obtained when $\mathbf{B}_0^{(opt)}$, constrained at unit diagonal entries, has null nondiagonal entries, i.e., $\mathbf{B}_0^{(opt)} = \mathbf{I}$; therefore, the solution of (8) is $\widetilde{\mathbf{B}}^{(opt)}(z) = \mathbf{I}$.

To solve the optimization problem in the general case of non-constant matrix $\mathbf{S}_B(z)$, we need to consider that, given the spectral matrix $\mathbf{S}_B(z) = \mathbf{S}_B^H(z^{-*}) \geq 0$, there exist two canonical filters $\mathbf{G}_B(z)$ and $\mathscr{G}_B(z)$ (called here the left factor and the right factor, respectively) and two diagonal matrices $\mathbf{D}_B$ and $\mathscr{D}_B$ with positive diagonal entries such that $\mathbf{S}_B(z) = \mathbf{G}_B(z)\mathbf{D}_B\mathbf{G}_B^H(z^{-*}) = \mathscr{G}_B^H(z^{-*})\mathscr{D}_B\mathscr{G}_B(z)$.

Consider therefore the following problem

$$\begin{cases} \widetilde{\mathbf{B}}^{(opt)}(z) = \arg\min_{\widetilde{\mathbf{B}}(z)} \int_C \mathrm{Tr}\left[\mathbf{A}(z)\mathscr{D}_B\mathbf{A}^H(z^{-*})\right]\frac{\mathrm{d}z}{j2\pi z} \\ \mathbf{A}(z) = \widetilde{\mathbf{B}}(z)\mathscr{G}_B^H(z^{-*}) \\ \widetilde{\mathbf{B}}(z) \quad \text{anticausal} \\ \widetilde{\mathbf{b}}(0) = \mathbf{B}_0 \end{cases} \tag{11}$$

where $\widetilde{\mathbf{b}}(k)$ denotes the time transform of $\widetilde{\mathbf{B}}(z)$. From the solution of the problem (4), it follows that the optimum $\widetilde{\mathbf{B}}(z)$ is such that, at the optimum, $\mathbf{A}^{(opt)}(z) \triangleq \widetilde{\mathbf{B}}^{(opt)}(z)\mathscr{G}_B^H(z^{-*}) = \mathbf{A}(0) = \widetilde{\mathbf{B}}(0)\mathscr{G}_B^H(\infty)$. Since $\widetilde{\mathbf{B}}(z)$ is constrained to be anticausal, it follows that

$$\widetilde{\mathbf{B}}^{(opt)}(z) = \mathbf{B}_0\mathscr{G}_B^H(\infty)\mathscr{G}_B^{-H}(z^{-*}). \tag{12}$$

At the optimum, the value of the cost function is

$$\mathrm{Tr}\left[\mathbf{A}(0)\mathscr{D}_B\mathbf{A}^H(0)\right] = \mathrm{Tr}\left[\mathbf{B}_0\mathbf{q}_0\mathbf{B}_0^H\right] \tag{13}$$

where $\mathbf{q}_0 \triangleq \mathscr{G}_B^H(\infty)\mathscr{D}_B\mathscr{G}_B(\infty)$. Let us consider two possible cases regarding the assigned matrix $\mathbf{B}_0$ in the problem (11):

a) the special choice $\mathbf{B}_0 = \mathbf{I}$ (strongly anticausal filtering) implies that $\widetilde{\mathbf{B}}^{(opt)}(z) = \mathscr{G}_B^H(\infty)\mathscr{G}_B^{-H}(z^{-*})$ with optimum value $Tr[\mathbf{q}_0]$;

b) the matrix $\mathbf{B}_0$, which is upper triangular since $\mathbf{B}(z)$ is anticausal, is constrained to be monic (strictly anticausal filtering). Then, $\mathbf{B}_0$ is chosen among all anticanonical matrices in order to minimize the value of the cost function at the optimum. Note that $\widetilde{\mathbf{B}}^{(opt)}(z)$ is a polynomial in $z$ and it is anticanonical provided that $\mathbf{B}_0$ is anticanonical. The property of $\mathscr{G}^H(z^{-*})$ to be anticanonical is crucial to guarantee that the optimum $\widetilde{\mathbf{B}}^{(opt)}(z)$ among all anticausal filters (with anticanonical matrix $\mathbf{B}_0$) can be obtained by searching among the $\widetilde{\mathbf{B}}(z)$ that are anticanonical. This means that the problem (11) with this further choice of $\mathbf{B}_0$ among all anticanonical matrices becomes equivalent to the following problem

$$\begin{cases} \widetilde{\mathbf{B}}^{(opt)}(z) = \arg\min_{\mathbf{A}(z)} \int_C \mathrm{Tr}\left[\mathbf{A}(z)\mathscr{D}_B\mathbf{A}^H(z^{-*})\right]\frac{\mathrm{d}z}{j2\pi z} \\ \mathbf{A}(z) = \widetilde{\mathbf{B}}(z)\mathscr{G}_B^H(z^{-*}) \\ \widetilde{\mathbf{B}}(z) \quad \text{anticanonical.} \end{cases}$$

Note that, as $\widetilde{\mathbf{B}}(z)$ varies over all anticanonical filters, $\mathbf{A}(z)$ describes the set of all anticanonical filters. In fact, for any anticanonical filter $\mathbf{C}(z)$, there exists an anticanonical matrix $\widetilde{\mathbf{B}}(z) = \mathbf{C}(z)\mathscr{G}_B^{-H}(z^{-*})$ such that $\mathbf{A}(z) \triangleq \widetilde{\mathbf{B}}(z)\mathscr{G}_B^H(z^{-*}) = \mathbf{C}(z)$. We have used properties that the product of two anticanonical filter and the inverse of an anticanonical filter are anticanonical. From the solution of problem (8) it follows that $\widetilde{\mathbf{B}}^{(opt)}(z)\mathscr{G}_B^H(z^{-*}) = \mathbf{I}$, or, equivalently,

$$\widetilde{\mathbf{B}}^{(opt)}(z) = \mathscr{G}_B^{-H}(z^{-*}). \tag{14}$$

From the previous results it follows that, for strictly anticausal feedback filter $\mathbf{B}(z)$, the optimum MMSE $\widetilde{\mathbf{B}}(z)$ is $\mathscr{G}_B^{-H}(z^{-*})$ while, for strongly causal feedback filter, the optimum MMSE $\widetilde{\mathbf{B}}(z)$ is $\mathscr{G}_B^H(\infty)\mathscr{G}_B^{-H}(z^{-*})$ where $\mathscr{G}_B(z)$ is the right factor of $\mathbf{S}_B(z)$.

The problem has been solved for the anticausal case. Analogously, it is easy to show that, for strictly causal feedback filter, the optimum MMSE $\widetilde{\mathbf{B}}(z)$ is $\mathbf{G}_B^{-1}(z)$ while, for strongly anticausal feedback filter, the optimum MMSE $\widetilde{\mathbf{B}}(z)$ is $\mathbf{G}_B(\infty)\mathbf{G}_B^{-1}(z)$. Such anticausal filters start the backward recursion from the successive training sequence. The results of forward and backward recursion on the same data may be

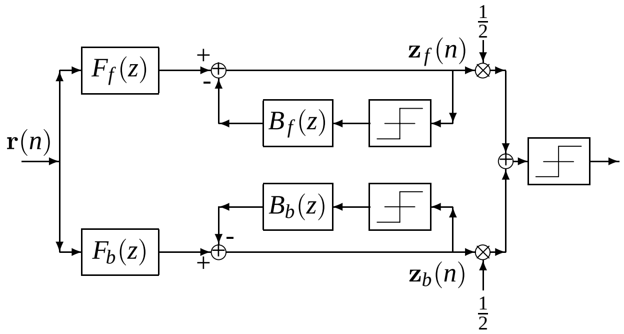


Figure 1: The structure of the bidirectional DF equalizer.

used to conceive a special MIMO multistage equalizer which focuses on the part of the sequence where one of the two DF equalizers has failed.

Note that $\mathbf{S}_C(z) = \mathbf{S}_B^{-1}(z) = \mathscr{G}_B^{-1}(z)\mathscr{D}_B^{-1}\mathscr{G}_B^{-H}(z^{-*})$ and, therefore, the filter $\mathscr{G}_B^{-1}(z)$ is the left factor of $\mathbf{S}_C(z)$. It is certainly easier to manipulate $\mathbf{S}_C(z)$ when the inversions of the matrices $\mathbf{S}_x(z)$ and $\mathbf{S}_\eta(z)$ are particularly simple (e.g., they are constant diagonal matrices [i.e., spatially and temporally uncorrelated signal and noise processes]).

The result reported in the present section extends those reported in [4] for the following reasons: (a) the backward DFE is explicitly derived; (b) a discrete-time model is adopted; (c) non-stationarity in the transmitted sequence and in the disturbance sequence is admitted; (d) optimization is not only performed over strictly causal feedback filters but also over strongly causal filters; (e) optimum feedforward filter is determined by standard algebra rather than by calculus of variations; (f) in the derivation of the result not only factorization of matrix $\mathbf{S}_B(z)$ but also of matrix $\mathbf{S}_C(z)$ is used. Note, however, that in [4] the equivalence of the MMSE and geometric-MSE criteria with respect to the design of the feedforward filter is also shown and the optimization over a possible linear transmitting filter under geometric MSE criterion is also performed.

## 4. BIDIRECTIONAL DF EQUALIZER

The bidirectional DF equalizer (Bi-DFE), showed in Fig. 1, utilizes both the forward and backward DFE; its decisions are taken on the basis of sequence $\mathbf{z}(n)$

$$\mathbf{z}(n) \triangleq \frac{1}{2}[\mathbf{z}_f(n) + \mathbf{z}_b(n)] \tag{15}$$

where $\mathbf{z}_f(n)$ and $\mathbf{z}_b(n)$ represent the outputs of the forward and backward equalizers, respectively. More sophisticated ways to combine $\mathbf{z}_f(n)$ and $\mathbf{z}_b(n)$ are not considered here. An extension could be based on the joint optimization of the forward and backward filters assuming that they are combined as in Fig. 1.

## 5. VARIABLE THRESHOLD BIDIRECTIONAL DECISION-FEEDBACK EQUALIZER

In order to limit the effects of the error propagation on the Bi-DFE considered in the previous section, we propose here an original variation of such an equalizer. The structure of the variable threshold bidirectional DF equalizer (VT-Bi-DFE) is based on the unidirectional DFEs and on the Bi-DFE. It operates in two stages: in the first stage it operates as a Bi-DFE; in the second stage, the equalized sequence obtained in the previous stage is used as input of the feedback filters in the forward and backward DFEs; moreover, the final decision is taken by using a non-standard threshold.

Such a threshold modification refers to the case of binary shift keying (BPSK) input symbols and it is aimed at maximizing the correlation between the equalized sequence and the outputs of the forward and backward DFE after the first stage. More specifically, the threshold is set at the end of the first stage according to the following procedure:

a) determine the $p$th component of the threshold vector $\mathbf{\Lambda}_f$ such that the decisions $\mathbf{D}_f(n)$ taken according to the following rule

$$\mathbf{z}_{f,1}(n) \gtrless \mathbf{\Lambda}_f$$

and the decisions $\mathbf{D}_{bi}(n)$ taken according to the following rule

$$\frac{\mathbf{z}_{f,1}(n) + \mathbf{z}_{b,1}(n)}{2} \gtrless 0$$

provide the smallest value of $\sum_{i=1}^{\ell} |D_{f,p}(n) - D_{bi,p}(n)|$ where $\ell$ is number of the symbols considered for the decision by the equalizer, $D_{f,p}(n)$ and $D_{bi,p}(n)$ are the $p$th components of $\mathbf{D}_f(n)$ and $\mathbf{D}_{bi}(n)$, respectively, $\mathbf{z}_{f,1}(n)$ is the output sequence obtained by the forward DFE after the first stage, and $\mathbf{z}_{b,1}(n)$ is the output sequence obtained by the backward DFE after the first stage. The thresholds $\mathbf{\Lambda}_f$ are determined separately for each component by calculating $\sum_{i=1}^{\ell} |D_{f,p}(n) - D_{bi,p}(n)|$ for different values of the variable and choosing the value minimizing it.

b) determine the $p$th component of the threshold vector $\mathbf{\Lambda}_b$ such that the decisions $\mathbf{D}_b(n)$ taken according to the following rule

$$\mathbf{z}_{b,1}(n) \gtrless \mathbf{\Lambda}_b$$

and the decisions $\mathbf{D}_{bi}(n)$ provide the smallest value of $\sum_{i=1}^{\ell} |D_{b,p}(n) - D_{bi,p}(n)|$, where $D_{b,p}(n)$ is the $p$th component of $\mathbf{D}_b(n)$. The thresholds $\mathbf{\Lambda}_b$ are determined as in the previous step.

c) the final threshold vector $\mathbf{\Lambda}$ is

$$\mathbf{\Lambda} \triangleq \frac{\mathbf{\Lambda}_f + \mathbf{\Lambda}_b}{2}$$

In the second stage, the two feedback filters $\mathbf{B}_f(z)$ and $\mathbf{B}_b(z)$ are fed with the decisions $\mathbf{D}_{bi}(n)$ while the final decisions are taken according to the following rule

$$\frac{\mathbf{z}_{f,2}(n) + \mathbf{z}_{b,2}(n)}{2} \gtrless \mathbf{\Lambda}$$

where $\mathbf{z}_{f,2}(n)$ is the output sequence obtained by the forward DFE after the second stage, and $\mathbf{z}_{b,2}(n)$ is the output sequence obtained by the backward DFE after the second stage.

## 6. PERFORMANCE ANALYSIS

The performance analysis is carried out by computer simulation in a dispersive MIMO scenario with two inputs and two outputs and a channel memory equal to 6. The average bit error rate (BER) reported here is obtained by averaging the BERs of the two BPSK transmissions; the two BERs are obtained by averaging the results over 100 independent trials; in each trial each tap of the FIR MIMO channel is generated

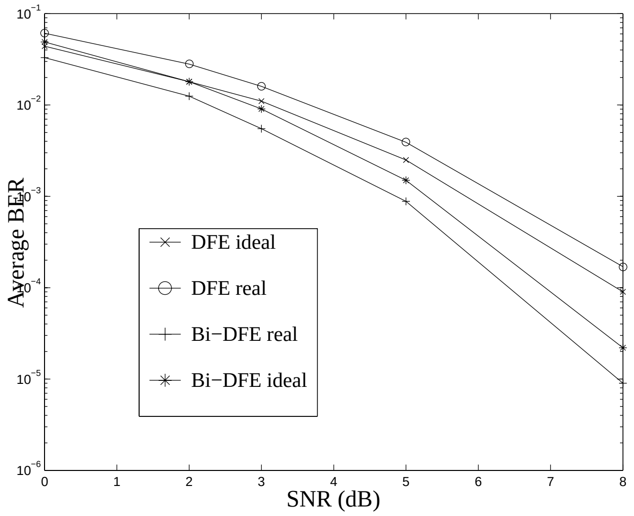


Figure 2: The effect of the error propagation on the performances of the DFE and the Bi-DFE.

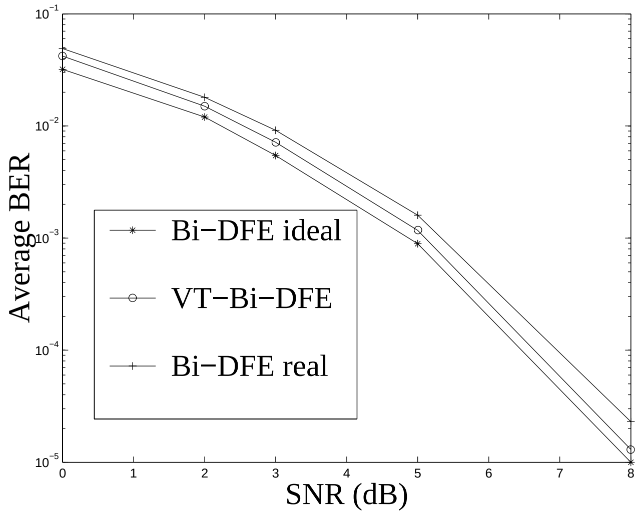


Figure 3: The comparison between the Bi-DFE and the VT-Bi-DFE.

randomly according to a circular complex-valued Gaussian distribution and with channel taps independent of each other, $10^6$ symbols are observed for each trial. The energy of the resulting MIMO channel impulse response is also normalized. The signal-to-noise ratio (SNR) is defined as $\frac{1}{\sigma^2}$ where the noise spectral matrix is $S_\eta(z) = \sigma^2\mathbf{I}$ and the signal spectral matrix is set to $S_x(z) = \mathbf{I}$.

The filter implementation is based on a FIR structure with coefficients obtained by truncation of the infinite-length impulse response of each matrix entry. Such a response is calculated by using a symbolic matrix toolbox operating in the ideal condition where $\mathbf{H}(z)$, $\mathbf{S}_x(z)$ and $\mathbf{S}_\eta(z)$ are perfectly known; the feedback filter in the forward DFE is constrained to be strongly causal while the feedback filter in the backward DFE is constrained to be strongly anticausal. The order of each FIR filter is chosen sufficiently long to span a significant portion of its impulse response, whose poles and zeros are determined by symbolic computation. Note that the anticausal portion of the impulse response of the feedforward filter in the forward DFE accounts for the implementation delay and it is chosen to the same value for all the matrix entries.

Fig. 2 shows the effects of the error propagation on the forward DFE; since the order of the FIR filters is sufficiently long, the backward DFE achieves the same performances of the forward DFE (both in presence and in absence of the error propagation) and, therefore, its performances are not reported here. Note that, in terms of the average BER, the effects of the error propagation is limited.

Fig. 2 also allows one to compare the performances of the unidirectional and bidirectional DFEs in the ideal scenario where the error propagation is not present and in the presence of the error propagation. The obtained results show that the advantage of the Bi-DFE over the unidirectional DFE is obtained in both cases. Note that also the performances of the Bi-DFE are affected by the presence of the error propagation. The results reported in Fig. 3 show that the proposed VT-Bi-DFE achieves enhanced performances with respect to the bidirectional DFE in presence of the error propagation.

## 7. CONCLUSIONS

The bidirectional DF equalizer, already proposed in SISO scenario [6], is proposed for the MIMO scenario in order to improve the performance of the classical DF equalizer. The performance analysis carried out by computer simulation shows that the proposed equalizer improves the performances of the classical DFE. An original variation of the bidirectional DF equalizer, based on a two-stage DF equalization, is able to reduce the performance loss of the Bi-DFE due to the effects of the error propagation.

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