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BLIND MMSE-BASED RECEIVERS FOR RATE AND DATA DETECTION IN VARIABLE-RATE CDMA SYSTEMS

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ABSTRACT

In this paper the problem of rate detection for a variable data rate code division multiple access (CDMA) system is addressed. A multiuser scenario is considered wherein each user may transmit at one out of a set of possible data rates in each data-frame. In particular a data rate and information symbol detection strategy based on the use of a bank of linear minimum mean square error (MMSE) filters is here proposed; as many filters as the number of available data rates are considered and a decision on the estimated data rate is taken based on rate matched to the filter with the minimum mean output energy (MOE). Analytical expressions for the MOE are derived in order to give theoretical grounds to our detection strategy; moreover, the effectiveness of the proposed detection strategy is also validated through simulation results that show satisfactory performance levels.

1. INTRODUCTION

In this paper the problem of detecting the data-rate and the information symbols transmitted by each user in a variable-rate direct-sequence code division multiple access system (DS/CDMA) is considered. Variable-rate CDMA systems wherein each user transmits at one out of a set of available data-rates are nowadays of widespread interest, since they are able to support transmission of heterogeneous kinds of data , such as streaming video, voice, bulk data, etc., with different data-rates. These systems are part of current 3G wireless networks, and, also, they can support adaptive rate strategies that allow to achieve an higher system throughput [1, 2] than a fixed data-rate CDMA systems. Obviously, in a variable-rate CDMA system the receiver generally needs knowledge of the actual data-rate of each user in order to perform data detection, and this information is usually conveyed on a separate multiplexed channel. Alternatively, it may be extracted in some way from the incoming signal.

In this paper we focus on this latter approach, namely we are interested to the problem of detecting, possibly in a blind fashion, the data-rate and the data information bits of each user. Previous contributions in this area have mainly addressed the problem of frame rate detection with regard to a specific second-generation (2G) or 3G cellular standard [3, 4]. A key remark, however, is that none of these papers explicitly takes into account the presence of the multiple access interference (MAI). More recently, the paper [5] has addressed the problem of blind rate-detection only (i.e. with no joint data detection) for variable data-rate CDMA systems, while the problem of *joint* detection of the data-rate and of the information symbols in such systems has been recently addressed in [6]. In particular, in [6] optimal (i.e. based on the maximum likelihood strategy) non-blind detectors have been proposed; in this paper, instead, we looks at simpler receiver structures, that are amenable to blind implementations and with a computational complexity that is much lower than that of the receivers in [6].

Next Section contains the considered system model, while the detector design is illustrated in Section 3. Section 4 contains an analysis of the proposed detection strategies; in particular, it is shown that the MMSE filter matched to the true data rate has minimal MOE, thus giving a theoretical ground to the receivers. Moreover, asymptotic expressions for the high SNR regime are also given. Finally, numerical results are illustrated in Section 5.

2. SYSTEM MODEL

We consider a synchronous DS/CDMA network with K active users employing BPSK modulation (extension to larger cardinality signaling constellations is trivial); we also assume that the propagation channel introduces frequency-flat fading, that each user may transmit, in each data-frame, at one out of S available data rates $R_1 < R_2 < \ldots < R_S$, and that the data rates $R_2, \ldots, R_S$ are all integer multiples of the lowest data-rate R_1, i.e. we have

$$R_i = m_i R_1, \qquad \forall i = 2, \ldots, S \tag{1}$$

where m_i are positive integers. In each frame slot, each user, based on the kind of data to be transmitted, and, possibly, on the propagation channel state, selects one out of the S available data-rates for its transmission. The multirate access scheme that is here considered is the variable spreading length (VSL) [7], i.e. all the signature waveforms have the same chip interval T_c and different spreading lengths. In particular, the signatures corresponding to the rate R_i have a spreading length equal to $1/(R_i T_c) = T_{b,i}/T_c$, with $T_{b,i}$ denoting the symbol interval for the users transmitting at rate R_i. Based on the above assumptions, the complex envelope of the received signal in a given frame slot is written as

$$r(t) = \sum_{k=0}^{K-1} \sum_{p_k=0}^{\frac{Pr_k}{R_1}-1} A_k \alpha_k(p_k) b_k(p_k) s_k^{r_k}(t - p_k/r_k) + n(t) \ . \tag{2}$$

In this equation, r_k is a random variate taking on values in the set $\{R_1, \ldots, R_S\}$, and denoting the data-rate of the k-th active user, P is the number of data-bits transmitted in each frame by the users at rate R_1 (thus implying that Pr_k/R_1 is the number of bits transmitted by the users at rate r_k), $b_k(\cdot) \in \{+1, -1\}$ denotes the bit stream transmitted by the $k-$th user, $\alpha_k(p_k)$ is a complex gain accounting for the channel propagation effects, while the waveform $n(t)$ represents the ambient noise, that we assume to be a complex zero-mean Gaussian white random process with power spectral density $2\mathcal{N}_0$. Moreover, $s_k^{r_k}(t)$ and A_k are respectively the rate-r_k signature waveform and the amplitude of the k-th user transmitted signal. Denoting by $N_{r_k} = 1/(r_k T_c)$ the processing gain for the k-th user (recall that in the VSL access strategy the processing gain for each user is tied to its data-rate), by $\{\beta_k^{r_k}(n)\}_{n=0}^{N_{r_k}-1}$ the spreading code (at rate r_k) of the $k-$th user, and by $\phi_{T_c}(\cdot)$ a square-root raised cosine bandlimited waveform, we have $s_k^{r_k}(t) = \sum_{n=0}^{N_{r_k}-1} \beta_k^{r_k}(n)\phi_{T_c}(t - nT_c)$, assuming that $s_k^{r_k}(t)$ are unit-energy waveform. At the receiver, the signal $r(t)$ is passed through a filter matched to the waveform $\phi_{T_c}(t)$ and sampled at rate $1/T_c$. The resulting samples can be stacked in a PN_{R_1}-dimensional vector, which we denote by $\boldsymbol{r}$, and which is expressed as

$$\boldsymbol{r} = \sum_{k=0}^{K-1} \sum_{p_k=0}^{Pr_k/R_1-1} A_k \alpha_k(p_k) b_k(p_k) \boldsymbol{s}_k^{r_k}(p_k) + \boldsymbol{n} \; . \tag{3}$$

In the above equation, the vector $\boldsymbol{s}_k^{r_k}(p_k)$ is the discrete-time version of the waveform $s_k^{r_k}(t - p_k/r_k)$ (notice that this vector, although being PN_{R_1}-dimensional, has only N_{r_k} non-zero entries), while the vector $\boldsymbol{n}$ is the discrete-time version of the ambient noise, and is a white complex zero-mean Gaussian random vector with covariance matrix $E[\boldsymbol{n}\boldsymbol{n}^H] = 2\mathcal{N}_0 \boldsymbol{I}_{PN_{R_1}}$ (with $(\cdot)^H$ denoting conjugate transpose and $\boldsymbol{I}_n$ the identity matrix of order n). Now the PN_{R_1}-dimensional vector $\boldsymbol{r}$ can be split in P distinct N_{R_1}-dimensional vectors, $\boldsymbol{r}(0), \ldots, \boldsymbol{r}(P-1)$, i.e. $\boldsymbol{r} = [\boldsymbol{r}^T(0), \ldots, \boldsymbol{r}^T(P-1)]^T$ with

$$\boldsymbol{r}(p) = \sum_{k=0}^{K-1} A_k {\boldsymbol{S}_k}^{r_k} {\boldsymbol{b}_k}^{r_k}(p) + \boldsymbol{n}(p) \qquad p = 0, \ldots, P-1, \tag{4}$$

wherein the $N_{R_1} \times (r_k/R_1)$-dimensional matrix ${\boldsymbol{S}_k}^{r_k}$ is expressed as

$$\boldsymbol{S}_k^{r_k} = \boldsymbol{I}_{r_k/R_1} \otimes [\beta_k^{r_k}(0), \ldots, \beta_k^{r_k}(N_{r_k}-1)]^T \tag{5}$$

with $\otimes$ denoting the Kronecker product and

$$\begin{aligned} \boldsymbol{b}_k^{r_k}(p) = \Big[& \alpha_k \left(p\tfrac{r_k}{R_1}\right) b_k \left(p\tfrac{r_k}{R_1}\right), \ldots \\ & \ldots, \alpha_k \left((p+1)\tfrac{r_k}{R_1} - 1\right) b_k \left((p+1)\tfrac{r_k}{R_1} - 1\right)\Big]^T . \end{aligned} \tag{6}$$

3. MINIMUM MEAN OUTPUT ENERGY RECEIVERS

The detection strategy that we propose is based on the use of a bank of blind linear MMSE filters, each one matched to a given data-rate for each user (see Fig. 1). In what follows, we assume that the channel coefficients $\alpha_k(\cdot)$ are kept constant over a temporal interval of length $T_{b,1}$, so that the vector $\boldsymbol{b}_k^{r_k}(p)$ can be written as:

$$\boldsymbol{b}_k^{r_k}(p) = \alpha_k(p)\boldsymbol{b}_{k,d}^{r_k}(p) \tag{7}$$

wherein $\boldsymbol{b}_{k,d}^{r_k}(p)$ is an r_k/R_1-dimensional vector with binary entries. Now assuming that the user 0 is the one of interest, the received vector in (4) can be written as

$$\boldsymbol{r}(p) = A_0 \boldsymbol{S}_0^{r_0} \boldsymbol{b}_0^{r_0}(p) + \boldsymbol{h}(p) \qquad p = 0, \ldots, P-1 \; , \tag{8}$$

wherein the vector $\boldsymbol{h}(p) = \sum_{k=1}^{K-1} A_k \boldsymbol{S}_k^{r_k} \boldsymbol{b}_k^{r_k}(p) + \boldsymbol{n}(p)$ represents the multiple access interference (MAI) plus the noise contribute. We suppose that the amplitude A_k, the SK spreading sequences and the complex gain $\alpha_k(\cdot)$ of the k-th user, for $k = 0, \ldots, K-1$, are known at the receiver side. Moreover, let $\boldsymbol{M}_0(p) = \mathrm{E}\left[\boldsymbol{h}(p)\boldsymbol{h}(p)^H\right]$ be the covariance matrix of the vector $\boldsymbol{h}(p)$, which is given by

$$\boldsymbol{M}_0(p) = \sum_{k=1}^{K-1} A_k^2 \left|\alpha_k(p)\right|^2 \boldsymbol{S}_k^{r_k} \boldsymbol{S}_k^{r_k^H} + 2N_0 \boldsymbol{I}_{N_{R_1}}, \tag{9}$$

wherein $|\cdot|$ denotes the Euclidean norm of a complex scalar.

In order to detect the data-rate of the 0th user, we consider a bank of blind MMSE filter, one for each of the data-rates available

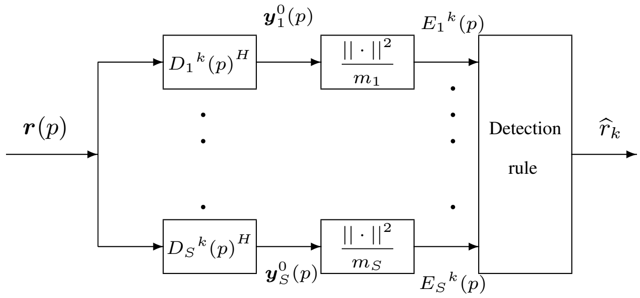

Figure 1: Block scheme of the proposed receiver.

to user 0. Accordingly, the data vector $\boldsymbol{r}(p)$ undergoes the linear transformations:

$$\boldsymbol{y}_i^0(p) = \boldsymbol{D}_i^0(p)^H \boldsymbol{r}(p) \quad i = 1, \ldots, S \tag{10}$$

wherein

$$\boldsymbol{D}_i^0(p) = \boldsymbol{R}_0(p)^{-1} \boldsymbol{S}_0^{R_i} A_0 \alpha_0(p) \quad i = 1, \ldots, S \tag{11}$$

is the MMSE filter matched to the i-th rate, and

$$\boldsymbol{R}_0(p) = A_0^2 \left|\alpha_0(p)\right|^2 \boldsymbol{S}_0^{r_0} \boldsymbol{S}_0^{r_0\,H} + \boldsymbol{M}_0(p) \; . \tag{12}$$

Note that, in practical implementations, this matrix may be replaced by the sample covariance matrix $\widehat{\boldsymbol{R}} = \dfrac{1}{P}\displaystyle\sum_{p=0}^{P-1} \boldsymbol{r}(p)\boldsymbol{r}(p)^H$, which, for increasing P, converges to the true covariance matrix (conditioned upon the actual data rates). Note also that the mean output energy per bit at the output of the i-th filter in the p-th symbol interval is expressed as

$$E_i^0(p) = \mathrm{E}\left[\frac{\left\|\boldsymbol{y}_i^0(p)\right\|^2}{m_i}\right] \quad \text{for} \quad i = 1, \ldots, S \; . \tag{13}$$

The proposed rate detection strategies are described in what follows.

Minimum Mean Output Energy Receiver (MOER). This receiver processes jointly the P mean output energies $\{{E_i}^k(p)\}_{p=0}^{P-1}$, $i = 1, \ldots, S$, for one user at a time, i.e. for $k = 0, \ldots, K-1$, in order to take a data rate decision according to the following detection rule

$$\widehat{r}_k = R_l \quad : \quad l = \arg \min_{i \in \{1, \ldots, S\}} \sum_{p=0}^{P-1} E_i^k(p) \; . \tag{14}$$

The information bit decision for each symbol interval is given by:

$$\boldsymbol{b}_{k,d}{}^{R_l}(p) = \mathrm{sign}\left[\mathcal{R}(\boldsymbol{D}_l^k(p)^H \boldsymbol{r}(p))\right] \tag{15}$$

for $k = 0, \ldots, K-1$ and $p = 0, \ldots, P-1$.

Windowed Minimum Mean Output Energy Receiver (WMOER). The WMOER processes separately the P mean output energies $E_i^k(p)$ for $i = 1, \ldots, S$ and for each user at a time, taking for each symbol interval independent decisions on the data rates and the information bits. In this receiver the rate detection rule is given for $p = 0, \ldots, P-1$ by:

$$\widehat{r}_k(p) = R_l \quad : \quad l = \arg \min_{i \in \{1, \ldots, S\}} E_i^k(p) \; . \tag{16}$$

The result of such operation is a $P \times K$-dimensional matrix of estimated rates $\boldsymbol{R}_e$ and, since each user transmits with the same rate during the whole data-frame, a majority rule is applied on each column of $\boldsymbol{R}_e$ to estimate the vector of the K rates $r_s = (\widehat{r}_0, \widehat{r}_1, \ldots, \widehat{r}_{K-1})$. Finally, the information symbols are to be detected and the test in (15) is applied given the detected rates $\widehat{r}_i$ for $i = 0, \ldots, K-1$.

4. ANALYSIS OF THE MEAN OUTPUT ENERGIES.

In this section we give analytical formulas for the mean output energy, and show that the MMSE filter matched to the true data-rate achieves minimal mean output energy.

The single user case. Consider, for the sake of simplicity, a two rates system with $S = 2$ and $K = 1$, and assume that the user data rate is $r_0 = R_1$, i.e. $m_1 = 1$. According to the signal model in (4) we can write the MMSE filter output as

$$\boldsymbol{y}_1^0(p) = \boldsymbol{S}_0^{R_1\,H}\left(A_0^2 |\alpha_0(p)|^2 \boldsymbol{S}_0^{R_1}\boldsymbol{S}_0^{R_1\,H} + 2\mathcal{N}_0 \boldsymbol{I}_{N_{R_1}}\right)^{-1}\boldsymbol{S}_0^{R_1}\boldsymbol{b}_{0,d}^{R_1}(p) \times A_0^2 |\alpha_0(p)|^2 + \boldsymbol{S}_0^{R_1\,H}\left(A_0^2 |\alpha_0(p)|^2 \boldsymbol{S}_0^{R_1}\boldsymbol{S}_0^{R_1\,H} + 2\mathcal{N}_0 \boldsymbol{I}_{N_{R_1}}\right)^{-1}\boldsymbol{n}(p) \times A_0 |\alpha_0(p)| \ . \tag{17}$$

Applying the matrix inversion lemma given in [8, p. 196] we observe that $\boldsymbol{S}_0^{R_1\,H}\boldsymbol{S}_0^{R_1} = 1$ but in order to simplify the generalization of the obtained expressions to the case $m_1 \neq 1$, we consider $\boldsymbol{S}_0^{R_1\,H}\boldsymbol{S}_0^{R_1} = \boldsymbol{I}_{m_1}$. Thus we can write

$$A_0^2 |\alpha_0(p)|^2 \boldsymbol{S}_0^{R_1\,H}\boldsymbol{R}_0(p)^{-1}\boldsymbol{S}_0^{R_1} = \frac{A_0^2 |\alpha_0(p)|^2}{2\mathcal{N}_0}\boldsymbol{S}_0^{R_1\,H} \times \left[\boldsymbol{I}_{N_{R_1}} - \boldsymbol{S}_0^{R_1}\left(\boldsymbol{I}_{m_1} + \frac{A_0^2 |\alpha_0(p)|^2}{2\mathcal{N}_0}\boldsymbol{S}_0^{R_1\,H}\boldsymbol{S}_0^{R_1}\right)^{-1}\frac{\boldsymbol{S}_0^{R_1\,H}}{2\mathcal{N}_0} \times A_0^2 |\alpha_0(p)|^2\right]\boldsymbol{S}_0^{R_1} = \frac{A_0^2 |\alpha_0(p)|^2}{A_0^2 |\alpha_0(p)|^2 + 2\mathcal{N}_0} \tag{18}$$

and defining $\mathrm{SNR} = \dfrac{A_0^2}{2\mathcal{N}_0}$ the equation (17) can be rewritten as

$$\boldsymbol{y}_1^0(p) = \frac{\mathrm{SNR}\,|\alpha_0(p)|^2}{\mathrm{SNR}\,|\alpha_0(p)|^2 + 1}\boldsymbol{b}_{0,d}^{R_1}(p) + \frac{\mathrm{SNR}\,|\alpha_0(p)|\,\boldsymbol{S}_0^{R_1\,H}\boldsymbol{n}(p)}{A_0(\mathrm{SNR}\,|\alpha_0(p)|^2 + 1)} \ . \tag{19}$$

According to this equation we can easily calculate the output energy $E_1^0(p)$ as

$$E_1^0(p) = \mathrm{E}\left[\frac{\left\|\boldsymbol{y}_1^0(p)\right\|^2}{m_1}\right] = \frac{SNR\,|\alpha_0(p)|^2}{SNR\,|\alpha_0(p)|^2 + 1} \ . \tag{20}$$

Now in this case we can consider the case of vanishingly small noise floor, i.e. $SNR \to \infty$ obtaining

$$\lim_{2\mathcal{N}_0 \to 0} E_1^0(p) = 1 \ . \tag{21}$$

Thus it has been shown that in the hypothesis of correct data rate the mean output energy in the region of high SNR approaches one. Then we can also calculate the following limit:

$$\lim_{2\mathcal{N}_0 \to 0}\boldsymbol{R}_0(p)^{-1}\boldsymbol{S}_0^{R_1} = \lim_{2\mathcal{N}_0 \to 0}\left[\boldsymbol{I}_{N_{R_1}} - A_0^2 |\alpha_0(p)|^2 \times \boldsymbol{S}_0^{R_1}\left(\boldsymbol{I}_{m_1} + \frac{A_0^2 |\alpha_0(p)|^2}{2\mathcal{N}_0}\boldsymbol{S}_0^{R_1\,H}\boldsymbol{S}_0^{R_1}\right)^{-1}\frac{\boldsymbol{S}_0^{R_1\,H}}{2\mathcal{N}_0}\right]\frac{\boldsymbol{S}_0^{R_1}}{2\mathcal{N}_0} = \lim_{2\mathcal{N}_0 \to 0}\left(\boldsymbol{S}_0^{R_1} - \frac{SNR\,|\alpha_0(p)|^2}{SNR\,|\alpha_0(p)|^2 + 1}\boldsymbol{S}_0^{R_1}\right)/2\mathcal{N}_0 = \frac{\boldsymbol{S}_0^{R_1}}{|\alpha_0(p)|^2 A_0{}^2} \ , \tag{22}$$

and it can be noted, as expected in the single user case, that the MMSE filter performs the matched filter. Considering now the output of the second MMSE filter we can write:

$$\boldsymbol{y}_2^0(p) = A_0^2 |\alpha_0(p)|^2 \boldsymbol{S}_0^{R_2\,H}\boldsymbol{R}_0(p)^{-1}\boldsymbol{S}_0^{R_1}\boldsymbol{b}_{0,d}^{R_1}(p) + A_0 |\alpha_0(p)| \times \boldsymbol{S}_0^{R_2\,H}\boldsymbol{R}_0(p)^{-1}\boldsymbol{n}(p) = \boldsymbol{S}_0^{R_2\,H}(A_0^2 |\alpha_0(p)|^2 \boldsymbol{S}_0^{R_1}\boldsymbol{S}_0^{R_1\,H} + 2\mathcal{N}_0\boldsymbol{I}_{N_{R_1}})^{-1} \times \boldsymbol{S}_0^{R_1}\boldsymbol{b}_{0,d}^{R_1}(p)A_0^2 |\alpha_0(p)|^2 + A_0 |\alpha_0(p)|\,\boldsymbol{S}_0^{R_2\,H}(A_0^2 |\alpha_0(p)|^2 \boldsymbol{S}_0^{R_1}\boldsymbol{S}_0^{R_1\,H} + 2\mathcal{N}_0\boldsymbol{I}_{N_{R_1}})^{-1}\boldsymbol{n}(p) \tag{23}$$

and applying the inversion lemma to the matrix $\boldsymbol{R}_0(p)^{-1}$ we obtain the following expression

$$\boldsymbol{y}_2^0(p) = \frac{A_0^2 |\alpha_0(p)|^2}{2\mathcal{N}_0}\boldsymbol{S}_0^{R_2\,H}(\boldsymbol{I}_{N_{R_1}} - \frac{A_0^2 |\alpha_0(p)|^2}{A_0^2 |\alpha_0(p)|^2 + 2\mathcal{N}_0}\boldsymbol{S}_0^{R_1} \times \boldsymbol{S}_0^{R_1\,H})\boldsymbol{S}_0^{R_1}\boldsymbol{b}_{0,d}^{R_1}(p) + \frac{A_0 |\alpha_0(p)|}{2\mathcal{N}_0}\boldsymbol{S}_0^{R_2\,H}(\boldsymbol{I}_{N_{R_1}} - \frac{A_0^2 |\alpha_0(p)|^2}{A_0^2 |\alpha_0(p)|^2 + 2\mathcal{N}_0} \times \boldsymbol{S}_0^{R_1}\boldsymbol{S}_0^{R_1\,H})\boldsymbol{n}(p) = \frac{A_0^2 |\alpha_0(p)|^2}{A_0^2 |\alpha_0(p)|^2 + 2\mathcal{N}_0}\boldsymbol{S}_0^{R_2\,H}\boldsymbol{S}_0^{R_1}\boldsymbol{b}_{0,d}^{R_1}(p) + \frac{A_0 |\alpha_0(p)|}{2\mathcal{N}_0}\boldsymbol{S}_0^{R_2\,H}(\boldsymbol{I}_{N_{R_1}} - \frac{A_0^2 |\alpha_0(p)|^2}{A_0^2 |\alpha_0(p)|^2 + 2\mathcal{N}_0}\boldsymbol{S}_0^{R_1}\boldsymbol{S}_0^{R_1\,H})\boldsymbol{n}(p) \ . \tag{24}$$

According to this last expression we can easily calculate the mean output energy

$$E_2^0(p) = \frac{A_0^4 |\alpha_0(p)|^4}{(A_0^2 |\alpha_0(p)|^2 + 2\mathcal{N}_0)^2}\,\mathrm{trace}(\boldsymbol{S}_0^{R_2\,H}\boldsymbol{S}_0^{R_1}\boldsymbol{S}_0{}^{R_1\,H}\boldsymbol{S}_0^{R_2}) + \frac{A_0^2 |\alpha_0(p)|^2}{2\mathcal{N}_0}\,\mathrm{trace}\left[\boldsymbol{S}_0^{R_2\,H}\left(\boldsymbol{I}_{N_{R_1}} - \frac{A_0^2 |\alpha_0(p)|^2 \boldsymbol{S}_0^{R_1}\boldsymbol{S}_0^{R_1\,H}}{A_0^2 |\alpha_0(p)|^2 + 2\mathcal{N}_0}\right)^2\boldsymbol{S}_0^{R_2}\right] = \frac{A_0^2 |\alpha_0(p)|^2}{2\mathcal{N}_0}\,\mathrm{trace}\left[\boldsymbol{S}_0^{R_2\,H}\boldsymbol{S}_0^{R_2}\right] - \frac{A_0^4 |\alpha_0(p)|^4}{(A_0^2 |\alpha_0(p)|^2 + 2\mathcal{N}_0)2\mathcal{N}_0}\,\mathrm{trace}[\boldsymbol{S}_0^{R_2\,H}\boldsymbol{S}_0^{R_1}\boldsymbol{S}_0{}^{R_1\,H}\boldsymbol{S}_0^{R_2}] = \mathrm{SNR}\,|\alpha_0(p)|^2 m_2 - \frac{\mathrm{SNR}^2 |\alpha_0(p)|^4}{\mathrm{SNR}\,|\alpha_0(p)|^2 + 1}\,\mathrm{trace}[\boldsymbol{S}_0^{R_2\,H}\boldsymbol{S}_0^{R_1}\boldsymbol{S}_0{}^{R_1\,H}\boldsymbol{S}_0^{R_2}] \ . \tag{25}$$

In order to calculate the second term of this last equation we assume that the spreading codes $\{\beta_k^{r_k}(n)\}_{n=0}^{N_{r_k}-1}$, $k = 0, \ldots, K-1$ are i.i.d. random variates thus we obtain:

$$\mathrm{E}\left[\mathrm{trace}(\boldsymbol{S}_0^{R_2\,H}\boldsymbol{S}_0^{R_1}\boldsymbol{S}_0{}^{R_1\,H}\boldsymbol{S}_0^{R_2})\right] = \mathrm{trace}\left(\boldsymbol{I}_{N_{R_1}}\frac{m_2 m_1}{N_{R_1}^2}\right) = \frac{m_2 m_1}{N_{R_1}} \ . \tag{26}$$

With this assumption the mean output energy $E_2^0(p)$ can be written as

$$E_2^0(p) = \mathrm{SNR}\,|\alpha_0(p)|^2 m_2\left(1 - \frac{m_1 \mathrm{SNR}\,|\alpha_0(p)|^2}{N_{R_1}(\mathrm{SNR}\,|\alpha_0(p)|^2 + 1)}\right) \ . \tag{27}$$

It can be noted that

$$\lim_{2\mathcal{N}_0 \to 0} E_2^0(p) = \lim_{SNR \to \infty} E_2^0(p) = \infty \tag{28}$$

showing that the mean output energy of the MMSE filter with a wrong data rate hypothesis is higher than that of the filter which assumes a correct data rate. In particular it can be easily found that

$$\lim_{2\mathcal{N}_0 \to 0}\boldsymbol{R}_0(p)^{-1}\boldsymbol{S}_0^{R_2} = \lim_{2\mathcal{N}_0 \to 0}(\boldsymbol{I}_{N_{R_1}} - \frac{A_0^2 |\alpha_0(p)|^2}{A_0^2 |\alpha_0(p)|^2 + 2\mathcal{N}_0}\boldsymbol{S}_0^{R_1}\boldsymbol{S}_0^{R_1\,H})\frac{\boldsymbol{S}_0^{R_2}}{2\mathcal{N}_0} = \boldsymbol{N}_\infty \tag{29}$$

denoting with $\boldsymbol{N}_\infty$ a $N_{R_1} \times m_2$-dimensional matrix with entries ∞.

The multi-user case. Assuming a multi-user scenario the covariance matrix of the observables $\boldsymbol{R}_0(p)$ can be written as:

$$\boldsymbol{R}_0(p) = A_0^2 |\alpha_0(p)|^2 \boldsymbol{S}_0^{r_0} \boldsymbol{S}_0^{r_0\,H} + \sum_{k=1}^{K-1} A_k^2 |\alpha_k(p)|^2 \boldsymbol{S}_k^{r_k} \boldsymbol{S}_k^{r_k^H} + 2N_0 \boldsymbol{I}_{N_{R_1}} \tag{30}$$

and grouping in $\boldsymbol{R}_I(p) = \sum_{k=1}^{K-1} A_k^2 |\alpha_k(p)|^2 \boldsymbol{S}_k^{r_k} \boldsymbol{S}_k^{r_k^H}$ the MAI contribute, we can consider the following full-rank matrix

$$\boldsymbol{R}_m(p) = \boldsymbol{R}_I(p) + 2N_0 \boldsymbol{I}_{N_{R_1}} \ . \tag{31}$$

Using a singular value decomposition (SVD) [9] $\boldsymbol{R}_m(p)$ can be written as

$$\boldsymbol{R}_m(p) = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^H \tag{32}$$

wherein $\boldsymbol{U}$ is an unitary matrix whose first r columns represents the unit vectors spanning the linear space defined by $\boldsymbol{R}_I(p)$ while its remaining $N_{R_1} - r$ columns are the unit vectors spanning the complex vector space $\mathcal{C}^{N_{R_1}-r}$. Finally $\boldsymbol{\Lambda}$ is defined as

$$\boldsymbol{\Lambda} = \mathrm{diag}\left(\underbrace{\lambda_1 + 2\mathcal{N}_0, \ldots, \lambda_r + 2\mathcal{N}_0}_{r}, \underbrace{2\mathcal{N}_0, \ldots, 2\mathcal{N}_0}_{N_{R_1}-r}\right) \tag{33}$$

wherein $\mathrm{diag}(\cdot)$ represents the diagonal matrix, while λ_i for $i = 1, \ldots, r$ are the eigenvalues corresponding to the first r columns of $\boldsymbol{U}$. According to this position equation (30) if $r_0 = R_1$ can be written as

$$\boldsymbol{R}_0(p) = A_0^2 |\alpha_0(p)|^2 \boldsymbol{S}_0^{R_1} \boldsymbol{S}_0^{R_1\,H} + \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^H \ . \tag{34}$$

Then applying the matrix inversion lemma, the matrix $\boldsymbol{R}_0(p)^{-1}$ is expressed as

$$\boldsymbol{R}_0(p)^{-1} = \boldsymbol{U}\boldsymbol{\Lambda}^{-1}\boldsymbol{U}^H - \boldsymbol{U}\boldsymbol{\Lambda}^{-1}\boldsymbol{U}^H A_0^2 |\alpha_0(p)|^2 \boldsymbol{S}_0^{R_1} \times \left(\boldsymbol{I}_{m_1} + A_0^2 |\alpha_0(p)|^2 \boldsymbol{S}_0^{R_1\,H} \boldsymbol{U}\boldsymbol{\Lambda}^{-1}\boldsymbol{U}^H \boldsymbol{S}_0^{R_1}\right)^{-1} \boldsymbol{S}_0^{R_1\,H} \boldsymbol{U}\boldsymbol{\Lambda}^{-1}\boldsymbol{U}^H \ . \tag{35}$$

Now we can calculate the following expression

$$\boldsymbol{R}_0(p)^{-1}\boldsymbol{S}_0^{R_1} = \boldsymbol{U}\boldsymbol{\Lambda}^{-1}\boldsymbol{U}^H \boldsymbol{S}_0^{R_1} (\boldsymbol{I}_{m_1} - (\boldsymbol{I}_{m_1} + A_0^2 |\alpha_0(p)|^2 \times \boldsymbol{S}_0^{R_1\,H} \boldsymbol{U}\boldsymbol{\Lambda}^{-1}\boldsymbol{U}^H \boldsymbol{S}_0^{R_1})^{-1} \boldsymbol{S}_0^{R_1\,H} \boldsymbol{U}\boldsymbol{\Lambda}^{-1}\boldsymbol{U}^H \boldsymbol{S}_0^{R_1} A_0^2 |\alpha_0(p)|^2) = \boldsymbol{U}\boldsymbol{\Lambda}^{-1}\boldsymbol{U}^H \boldsymbol{S}_0^{R_1} (\boldsymbol{S}_0^{R_1\,H} \boldsymbol{U}(\boldsymbol{I}_{N_{R_1}} + A_0^2 |\alpha_0(p)|^2 \boldsymbol{\Lambda}^{-1}) \boldsymbol{U}^H \boldsymbol{S}_0^{R_1})^{-1} \tag{36}$$

and after some algebraic manipulation we obtain

$$\boldsymbol{R}_0(p)^{-1}\boldsymbol{S}_0^{R_1} = \boldsymbol{U}\mathrm{diag}\left(\frac{2\mathcal{N}_0}{\lambda_1 + 2\mathcal{N}_0}, \ldots, \frac{2\mathcal{N}_0}{\lambda_r + 2\mathcal{N}_0}, 1, \ldots, 1\right) \times \boldsymbol{U}^H \boldsymbol{S}_0^{R_1} \left(\boldsymbol{S}_0^{R_1\,H} \boldsymbol{U}\mathrm{diag}\left(2\mathcal{N}_0 + \frac{2\mathcal{N}_0 A_0^2 |\alpha_0(p)|^2}{\lambda_1 + 2\mathcal{N}_0}, \ldots, 2\mathcal{N}_0 + \frac{2\mathcal{N}_0 A_0^2 |\alpha_0(p)|^2}{\lambda_r + 2\mathcal{N}_0}, 2\mathcal{N}_0 + A_0^2 |\alpha_0(p)|^2, \ldots, 2\mathcal{N}_0 + A_0^2 |\alpha_0(p)|^2\right) \times \boldsymbol{U}^H \boldsymbol{S}_0^{R_1}\right)^{-1} \ . \tag{37}$$

Indeed this last expression allows to consider the system behavior in the high SNR region and in particular we have

$$\lim_{2\mathcal{N}_0 \to 0} \boldsymbol{R}_0(p)^{-1}\boldsymbol{S}_0^{R_1} = \boldsymbol{U}\mathrm{diag}(0, \ldots, 0, 1, \ldots, 1)\boldsymbol{U}^H \boldsymbol{S}_0^{R_1} \times \left(\boldsymbol{S}_0^{R_1\,H} \boldsymbol{U}\mathrm{diag}(0, \ldots, 0, 1, \ldots, 1) A_0^2 |\alpha_0(p)|^2 \boldsymbol{U}^H \boldsymbol{S}_0^{R_1}\right)^{-1} = \boldsymbol{S}_0^{R_1 \perp} \left(\boldsymbol{S}_0^{R_1 \perp H} \boldsymbol{S}_0^{R_1 \perp} A_0^2 |\alpha_0(p)|^2\right)^{-1} \tag{38}$$

wherein $\boldsymbol{S}_0^{R_1 \perp}$ represents the projection of $\boldsymbol{S}_0^{R_1}$ on the orthogonal complement of the subspace spanned by the matrix $\boldsymbol{R}_I(p)$. Thus it has been shown that the linear MMSE filter reduces to the decorrelating receiver in the high SNR region. We can apply this last result in order to derive the expression of the output energy in the case of correct data rate hypothesis, i.e.

$$\mathrm{E}\left[\left\|\boldsymbol{y}_1^0(p)\right\|^2\right] = \mathrm{E}\left[\mathrm{trace}(\boldsymbol{D}_1^0(p)^H \boldsymbol{r}(p)\boldsymbol{r}^H(p)\boldsymbol{D}_1^0(p))\right] = \mathrm{trace}(\boldsymbol{D}_1^0(p)^H \boldsymbol{R}_0(p)\boldsymbol{D}_1^0(p)) = \mathrm{trace}(A_0^2 |\alpha_0(p)|^2 \boldsymbol{S}_0^{R_1\,H} \boldsymbol{R}_0(p)^{-1} \times \boldsymbol{S}_0^{R_1}) = \mathrm{trace}(A_0^2 |\alpha_0(p)|^2 \boldsymbol{S}_0^{R_1\,H} (A_0^2 |\alpha_0(p)|^2 \boldsymbol{S}_0^{r_0} \boldsymbol{S}_0^{r_0\,H} + \boldsymbol{R}_m(p))^{-1} \boldsymbol{S}_0^{R_1}) \ . \tag{39}$$

Thus using the result given in (38) we obtain:

$$\lim_{2\mathcal{N}_0 \to 0} E_1^0(p) = \mathrm{trace}\left(\boldsymbol{S}_0^{R_1\,H} \boldsymbol{S}_0^{R_1 \perp} \left(\boldsymbol{S}_0^{R_1 \perp H} \boldsymbol{S}_0^{R_1 \perp}\right)^{-1}\right) \frac{1}{m_1} = \mathrm{trace}\left(\frac{\boldsymbol{S}_0^{R_1 \perp H} \boldsymbol{S}_0^{R_1 \perp}}{m_1} \left(\boldsymbol{S}_0^{R_1 \perp H} \boldsymbol{S}_0^{R_1 \perp}\right)^{-1}\right) = \frac{\mathrm{trace}\left(\boldsymbol{I}_{m_1}\right)}{m_1} = 1 \tag{40}$$

showing that also in the multiuser case, when the correct data rate is assumed, the mean output energy reduces to one as $2\mathcal{N}_0$ vanishes. Finally, it is derived the expression of the mean output energy for the second MMSE filter obtaining in this case:

$$E_2^0(p) = \mathrm{trace}[A_0^2 |\alpha_0(p)|^2 \boldsymbol{S}_0^{R_2\,H} (A_0^2 |\alpha_0(p)|^2 \boldsymbol{S}_0^{r_0} \boldsymbol{S}_0^{r_0\,H} + \boldsymbol{R}_m(p))^{-1} \boldsymbol{S}_0^{R_2}]/m_2 \ . \tag{41}$$

Now we can repeat the same steps follow to derive equation (37) in order to calculate the expression

$$\boldsymbol{R}_0(p)^{-1}\boldsymbol{S}_0^{R_2} = \boldsymbol{U}\boldsymbol{\Lambda}^{-1}\boldsymbol{U}^H [\boldsymbol{I}_{N_{R_1}} - A_0^2 |\alpha_0(p)|^2 \boldsymbol{S}_0^{R_1} (\boldsymbol{I}_{m_1} + A_0^2 |\alpha_0(p)|^2 \boldsymbol{S}_0^{R_1\,H} \boldsymbol{U}\boldsymbol{\Lambda}^{-1}\boldsymbol{U}^H \boldsymbol{S}_0^{R_1})^{-1} \boldsymbol{S}_0^{R_1\,H} \boldsymbol{U}\boldsymbol{\Lambda}^{-1}\boldsymbol{U}^H] \boldsymbol{S}_0^{R_2} \ , \tag{42}$$

and defining $\boldsymbol{\Lambda}_1 = \boldsymbol{I}_{N_{R_1}} + A_0^2 |\alpha_0(p)|^2 \boldsymbol{\Lambda}^{-1}$ we obtain

$$\boldsymbol{R}_0(p)^{-1}\boldsymbol{S}_0^{R_2} = \boldsymbol{U}\boldsymbol{\Lambda}^{-1}\boldsymbol{U}^H [\boldsymbol{I}_{N_{R_1}} - A_0^2 |\alpha_0(p)|^2 \boldsymbol{S}_0^{R_1} \times (\boldsymbol{S}_0^{R_1\,H} \boldsymbol{U}\boldsymbol{\Lambda}_1^{-1}\boldsymbol{U}^H \boldsymbol{S}_0^{R_1})^{-1} \boldsymbol{S}_0^{R_1\,H} \boldsymbol{U}\boldsymbol{\Lambda}^{-1}\boldsymbol{U}^H] \boldsymbol{S}_0^{R_2} \ . \tag{43}$$

Then as $2\mathcal{N}_0$ vanishes we obtain the following results

$$\lim_{2\mathcal{N}_0 \to 0} \boldsymbol{R}_0(p)^{-1}\boldsymbol{S}_0^{R_2} = \boldsymbol{U}\mathrm{diag}(\frac{1}{\lambda_1}, \ldots, \frac{1}{\lambda_r}, \infty, \ldots, \infty) \times \boldsymbol{U}^H (\boldsymbol{I}_{N_{R_1}} - \boldsymbol{S}_0^{R_1} (\boldsymbol{S}_0^{R_1 \perp H} \boldsymbol{S}_0^{R_1 \perp})^{-1} \boldsymbol{S}_0^{R_1 \perp H}) \boldsymbol{S}_0^{R_2} = \boldsymbol{N}_\infty \tag{44}$$

and

$$\lim_{2\mathcal{N}_0 \to 0} E_2^0(p) = \infty \ . \tag{45}$$

It has been thus shown that the proposed receivers work correctly if the minimum output energy is picked out and the corresponding data rate is the estimated one.

5. NUMERICAL RESULTS

In this section we report and discuss some simulation results for the probability of erroneous rate detection (PERD) and for the bit error rate in the case of correct rate detection (BER|CRD) for the proposed receivers. In particular we have considered a synchronous variable rate DS/CDMA system assuming known the set of the possible spreading sequences with largest processing gain $N_{R_1} = 16$ and we have defined the rate ratio vector $\boldsymbol{m} = [m_1, \ldots, m_S]$. In all the simulation results it has been assumed frequency-flat Rayleigh fading. Figure 2 reports the probability of erroneous rate detection (PERD) versus the energy contrast $E_b/\mathcal{N}_0$[dB] for the two proposed receivers assuming $S = 2$ and $m = [1,2]$ and with the sample covariance matrix $\widehat{\boldsymbol{R}}$. It can be noted that for two active users, at a PERD of about 10^{-2}, doubling the P value there is a performance improvement of about 2[dB] for the WMOER and of about 1 [dB] for the MOER receiver. Indeed, in this situation, the WMOER receiver performs better than the MOER so that, for a frame length of $P = 200$ and for 10000 data frame, zero probability of erroneous rate detection has been observed. In Figure 3 we have reported the (BER|CRD) versus the energy contrast $E_b/\mathcal{N}_0$[dB] and it can be noted that although this probability decreases for higher P values, the two receivers present the same performance levels. Then, in Figure 4, we have reported the PERD for both the cases of the true covariance matrix $\boldsymbol{R}_0(p)$ and the sample covariance matrix $\widehat{\boldsymbol{R}}$, for $K = 4$ and $K = 2$ active users. It can be noted that, in the ideal case wherein the covariance matrix is perfectly estimated, the MOER receiver performs better that the WMOER one although it seems to be more sensible to the covariance matrix estimation errors. The impact of the covariance matrix estimation error on the system performance is thus an issue currently under investigation.

6. CONCLUSIONS

In this paper the problem of blind data rate and information symbol detection for a variable-rate DS/CDMA system has been considered. In particular a detection scheme employing a bank of linear MMSE filters has been proposed taking a data rate decision based on the minimum filter output energy. Analytical expressions for the filter output energies have been derived and the simulation results confirm that all the proposed receivers achieve good performance levels both in terms of probability of error rate detection and of bit error rate.

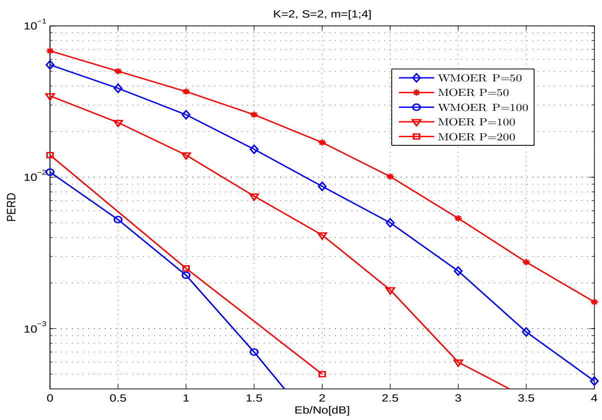

Figure 2: Probability of erroneous rate detection versus the energy contrast with the sample covariance matrix $\widehat{\boldsymbol{R}}$.

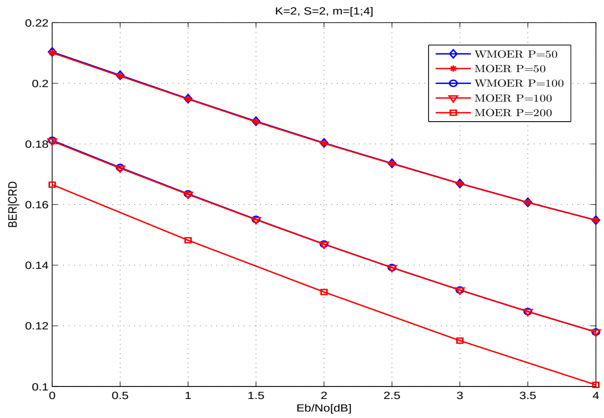

Figure 3: Bit error probability in the case of correct rate detection versus the energy contrast with the sample covariance matrix $\widehat{\boldsymbol{R}}$.

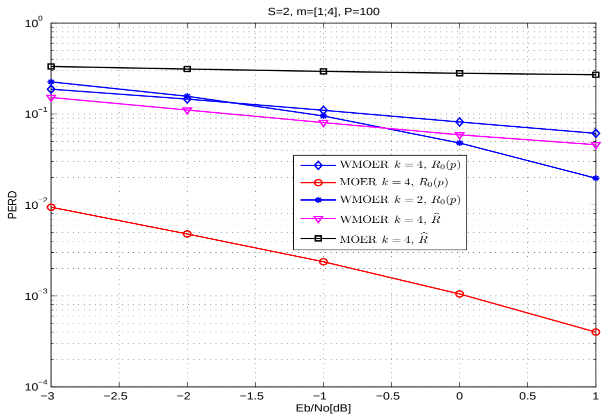

Figure 4: Probability of erroneous rate detection versus the energy contrast.

REFERENCES

[1] A. Goldsmith and S. Chua, "Adaptive coded modulation for fading channels," *IEEE Trans. Commun.*, Vol. 46, pp. 595-602, May 1998.

[2] S. A. Jafar and A. Goldsmith, "Adaptive multirate CDMA for uplink throughput maximization," *IEEE Trans. Wireless Commun.*, Vol. 2, pp. 218-228, March 2003.

[3] G. Yang and S. Kellel, "Optimal frame rate detection for CDMA mobile stations with variable-rate data transmission," *Proc. of the 1999 Fifth Asia-Pacific Conference on Communications and Fourth Optoelectronics and Communications Conference*, Bejing, China, October 1999.

[4] I. Held and A. Chen, "Reduced complexity blind rate detection for second and third generation cellular CDMA systems," *Proc. of the 1999 IEEE Global Communications Conference (GLOBECOM '99)*, Rio de Janeiro (Brasil), December 1999.

[5] S. Buzzi and A. De Maio, "Code-aided blind detection of the transmission rate for multirate direct-sequence CDMA systems," *IEEE Trans. Signal Processing*, Vol. 53, pp. 2182-2192, June 2005.

[6] S. Buzzi and S. Sardellitti, "Joint rate and data detection in variable-rate CDMA systems," *Proc. of IEEE Radio and Wireless Symposium (RWS '06)*, San Diego (CA), USA, January 2006.

[7] S. Buzzi, M. Lops and A.M. Tulino, "Blind adaptive multiuser detection for asynchronous dual-rate DS/CDMA systems," *IEEE Journal Selected Areas Commun.*, Vol. 19, pp. 233-244, Feb. 2001.

[8] S. Verdú, *Multiuser detection*. New York: Cambridge Univ. Press, 1998.

[9] S. Buzzi, M. Lops and A.M. Tulino, "Partially blind adaptive MMSE interference rejection in asynchronous DS/CDMA networks over frequency-selective fading channels," *IEEE Trans. Commun.*, Vol. 49, pp. 94-108, January 2001.