

BLIND SEPARATION OF TIME-VARYING SIGNAL MIXTURES USING SPARSENESS AND ZADEH'S TRANSFORM

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ABSTRACT

We consider the general problem of blindly separating time-varying mixtures. Physical phenomena, such as varying attenuation and the doppler effect, can be represented as special cases of a time-varying mixing model. This model can be considered as a linear mixing of time-varying attenuated-and-delayed versions of fixed channel distortions. In this special case, we use Zadeh's transform to project the signals to the time-frequency domain. In this domain, sparse source distribution highlights geometric properties of the mixing coefficients. These coefficients can be used in turn, for inverting the mixing system, and thereby, recover the time-varying filtered versions of the original sources.

1. INTRODUCTION

The problem of blind source separation (BSS) has attracted a great deal of attention of many researchers in recent years. This problem is concerned with estimating some unknown sources from their observed mixtures, without any *a-priori* knowledge about the mixing system. The BSS problem deals with a wide range of setups and scenarios wherein several sensors receive mixtures of some unknown physical sources, such as acoustic, seismic, RF or biological signals.

Approaches and techniques, used for solving the BSS problem of linear mixtures, can be roughly divided into two categories: independent component analysis (ICA) and blind separation using sparse component analysis (SCA). ICA assumes that the sources are statistically independent and, therefore, utilizes separation cost functions based on mutual independency, non-linear correlation, non-gaussianity, or high order statistics (for a review of such methods see [1]). Blind separation using SCA assumes that the sources are sparse or can be projected onto a space of sparse representations by using a proper transform, such as wavelet packets [2]. The sources need not be statistically independent but one should be able to represent them differently in some domain (i.e. there are atoms in the dictionary or moments in time in which only one source is represented or active). This approach adopts a geometric interpretation of the mixing coefficients and the entries of the mixing matrix can be retrieved from the scatter plot of the sparsified mixtures [3].

The majority of the available techniques, have been developed for solving the instantaneous blind source separation (IBSS) problem. In this case the sources are mixed with constant weights and without time delays. This problem is unrealistic in most sensing problems, but applicable for separating mixed images from semi-reflecting lens [4], or in blind separation of MRI tissue signatures [5]. Only in the last decade, researches have begun to address the problem

of blind separation of convolutive mixtures (CBSS), wherein the sources are convolved with some kernel before being mixed. Time delays, multipath and channel distortion can be represented by this kernel. Techniques used for solving the CBSS problem consist of finding a time-domain inverse filter with an ICA-based separation cost function or, alternatively, finding a frequency-domain complex inverse matrix which is an IBSS problem for every frequency (see [6] for a review of relevant methods).

Only few studies have addressed the generalized BSS problem, in which the mixing system is time-varying. In this generalized problem, the channel distortion can change over time (due to temperature changes, for example) and the sources or the sensors can move, thus causing the multipath effect, amplitudes and time delays to vary with time. Attempts to solve this problem have mainly focused on an adaptive online version of the algorithms used for IBSS and CBSS ([7], [8]), or used particle filters for time-varying IBSS problems (see for example [9]).

In this paper we address a special case of time-varying mixtures that can be modelled as a linear mixing of time-varying attenuated-and-delayed versions of fixed channel distortions. This special case accounts for amplitude changes due to signal geometric dissipation, and for delays or frequency change resulting, for example, from the doppler effect. Time varying multipath effects are not represented by this model. In order to solve this blind separation problem of time-varying mixtures, we implement a windowed version of Zadeh's transform, considering the problem in the combined time-frequency domain. The amplitudes and the phases of sparse (or sparsified) sources plotted in this domain lend themselves to a geometric interpretation of the mixing coefficients, used for deriving the unmixing system.

In this paper, results of separating a simulated time-varying mixture are presented and the constraints on the sampling frequency, signal bandwidth, and local stationarity of the mixing system, are discussed

We first present in section 2 the problem of blind separation of time-varying mixtures. The special case of linear mixing of time-varying attenuated-and-delayed version of fixed channel distortions is formulated and the relevances to known physical scenarios is discussed. Section 3 defines Zadeh's transform as well as its modified windowed version. Section 4 illustrates the geometric interpretation of the mixing coefficient as can be retrieved from the mixtures of sparse sources in Zadeh's time-frequency domain. Section 5 presents a method for inverting the mixing system. Finally, simulation results of applying this methods to separate blindly time-varying mixtures are shown in section 6.

2. THE GENERALIZED BSS PROBLEM

The generalized BSS problem is defined as follows:

Let $x_i(t)$ be the observed mixture received by the i^{th} sensor.

$$x_i(t) = \sum_{j=1}^N \int_{-\infty}^{\infty} h_{ij}(t, \tau) \cdot s_j(\tau) d\tau, \quad (1)$$

where $s_j(\tau)$ is the j^{th} source signal and $h_{ij}(t, \tau)$ is the response of the channel from the source j to the sensor i to an impulse given at the time t .

Time domain blind separation of N signals using M mixtures can be defined as follows:

$$\hat{s}_j(t) = \sum_{i=1}^M \int_{-\infty}^{\infty} \tilde{h}_{ji}(t, \tau) \cdot x_i(\tau) d\tau, \quad (2)$$

where $\tilde{h}_{ji}(t, \tau)$ is the response of the unmixing channel from the sensor i , to the source j to an impulse given at time t , and $\hat{s}_j(t)$ is the restored source signal j .

We want to address a special case of time-varying mixtures, in which $h_{ij}(t, \tau)$ can be written as:

$$h(t, \tau) = f(t) \cdot g(k(t) - \tau), \quad (3)$$

where $g(\cdot)$ is a fixed (over time) channel distortion, $f(t)$ is time-varying attenuation and $k(t)$ is a time-varying delay.

Although this special case does not account for time-varying multipath effects, it represents many important physical phenomena and scenarios. For example, $f(t)$ can be interpreted as the attenuation of the source signal, which is a function of the distance between the source and the sensor or, in the context of images, $f(t)$ can be viewed as image acquisition under non-uniform illumination (where t stands for the spatial coordinates). $k(t)$ can be interpreted as the time delay between the emission of the source signal to receiving it at the sensor. This time delay is a function of the distance between the source and the sensor and the speed of the propagating signal and can account for the doppler effect (change in the frequency of the received signal). In images, $k(t)$ can be interpreted as taking a picture through a distorted medium or lens (e.g. a hyperbolic lens).

The time-varying mixing model for this special case is as follows:

$$x_i(t) = \sum_{j=1}^N \int_{-\infty}^{\infty} f_{ij}(t) \cdot g_{ij}(k_{ij}(t) - \tau) \cdot s_j(\tau) d\tau, \quad (4)$$

where $f_{ij}(t)$ is the attenuation function, $k_{ij}(t)$ is the time delay function and $g_{ij}(\cdot)$ is a fixed channel distortion from the source j to the sensor i .

To illustrate this special case, consider the setup depicted in figure 1, illustrating a semi-realistic acoustic scenario. A reporter is speaking to a 2-microphone array, while an emergency vehicle is passing in the nearby road. The functions $f_{ij}(t)$ and $k_{ij}(t)$ are calculated for this scenario as:

$$f_{ij}(t) = \frac{1}{r_{ij}(t)}, \quad k_{ij}(t) = t - \frac{r_{ij}(t)}{c}, \quad (5)$$

where c is the speed of sound.

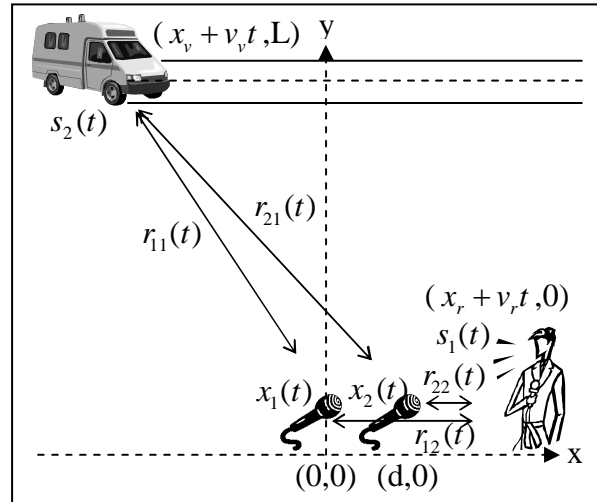


Fig. 1: An acoustic generalized BSS scenario. A reporter, $s_1(t)$, is speaking to a 2-microphone array, $x_1(t)$ and $x_2(t)$, while an emergency vehicle, $s_2(t)$, is passing in the nearby road. The left microphone stands at the origin. The coordinates of the sources and microphones as a function of time are indicated in the figure. v_v is the vehicle speed, v_r is the reporter speed, and r_{ij} are the distances between the i^{th} microphone and the j^{th} source.

We define $p_{ij}(t)$ to be the ratio:

$$p_{ij}(t) = \frac{f_{ij}(t)}{f_{Mj}(t)}, \quad (6)$$

where the index M stands for the last sensor. We define a variable time interval $T_a(t)$ in which $p_{ij}(t)$ is locally stationary:

$$T_a(t) = \epsilon_a \cdot \frac{p_{ij}(t)}{p'_{ij}(t)}, \quad (7)$$

where ϵ_a is some small constant and $p'_{ij}(t)$ is the derivative of p_{ij} . we define $q_{ij}(t)$ to be the difference:

$$q_{ij}(t) = k_{ij}(t) - k_{Mj}(t). \quad (8)$$

we define the variable time interval $T_d(t)$ in which $q_{ij}(t)$ is locally stationary:

$$T_d(t) = \epsilon_d \cdot \frac{q_{ij}(t)}{q'_{ij}(t)}, \quad (9)$$

where ϵ_d is some small constant and $q'_{ij}(t)$ is the derivative of q_{ij} . we assume that for all time instances the following are valid:

$$p_{ij}(t) \ll T_a(t), \quad q_{ij}(t) \ll T_d(t). \quad (10)$$

The motivation for this assumption is given at the end of section 5.

The required solution, using the ICA approach, would be to find $\tilde{h}_{ji}(t, \tau)$ such that the restored signals $\hat{s}_j(t)$ are statistically independent. A possible solution using sparseness would be to identify $f_{ij}(t)$, $k_{ij}(t)$ and $g_{ij}(\cdot)$ through their geometric properties and then invert the system to restore $\hat{s}_j(t)$.

3. ZADEH'S TRANSFORM

Zadeh was the first to present an approach suitable for the analysis of linear time-varying systems in the context of frequency analysis [10]. He proposed to use a function $H(t, \omega)$, obtained by using a non-compatible transform, as if it was the frequency response of a time-invariant system. Zadeh's approach was found to be useful for system synthesis, but had limited value in practice for system analysis. Therefore, Zadeh's transform has rarely been used and was almost forgotten.

A time-invariant system can be defined as follows:

$$x(t) = \int_{-\infty}^{\infty} h(t - \tau) \cdot s(\tau) d\tau. \quad (11)$$

The frequency response of the system to a unit impulse applied at $t = 0$, is $H(\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt$.

A linear time-varying system is defined as follows:

$$x(t) = \int_{-\infty}^{\infty} h(t, \tau) \cdot s(\tau) d\tau. \quad (12)$$

Zadeh defined the frequency response of the time-varying system, to a unit impulse applied at $t = t_0$ as:

$$H(t_0, \omega) = \int_{-\infty}^{\infty} h(t_0, \tau) \cdot e^{-j\omega(t_0 - \tau)} d\tau. \quad (13)$$

Note that in the case of time-invariant system, $h(t, \tau) = h(t - \tau)$, and Zadeh's frequency response is equivalent to the time-invariant frequency response: $H(t, \omega) = H(\omega)$.

Zadeh showed that for linear time-varying systems, $x(t)$ can be found by:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) \cdot H(t, \omega) \cdot e^{j\omega t} d\omega, \quad (14)$$

where $S(\omega)$ is the fourier transform of the source signal $s(t)$.

In the special case of a time-varying system, that its impulse response can be written in the form of (3), Zadeh's time-varying frequency response can be written as:

$$H(t, \omega) = f(t) \cdot e^{-j\omega(t-k(t))} G(\omega), \quad (15)$$

where $G(\omega)$ is the fourier transform of g . Assuming $h(t, \tau)$ has final support, meaning $h(t, \tau < t - T) = 0$ and $h(t, \tau > t + T) = 0$, $x(t)$ can be written as:

$$x(t) = \int_{-\infty}^{\infty} h(t, \tau) \cdot w(t - \tau) \cdot s(\tau) d\tau, \quad (16)$$

where $w(t - \tau)$ is a unit window function centered at $\tau = t$ with support greater or equal $2T$. Or, equivalently, we can write $x(t)$ as:

$$x(t) = \int_{-\infty}^{\infty} h(t, \tau) \cdot s(t, \tau) d\tau, \quad (17)$$

where $s(t, \tau) = w(t - \tau) \cdot s(\tau)$. Then, $x(t)$ can be found from the windowed version of the time frequency representation as:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(t, \omega) e^{j\omega t} d\omega, \quad (18)$$

where $X(t, \omega) = H(t, \omega) \cdot S(t, \omega)$. $X(t, \omega)$ can be interpreted as the frequency content of $x(t)$ in the moment t , as a result of applying a time-invariant filter on $s(t)$. Note that $S(t, \omega)$ can be calculated from:

$$S(t, \omega) = \int_{-\infty}^{\infty} W(\varphi) \cdot S(\omega - \varphi) \cdot e^{-j\varphi t} d\varphi, \quad (19)$$

where $W(\varphi)$ is the fourier transform of $w(t)$. Due to the additivity of Zadeh's transform [11], and in the special case defined by (15), the mixing system can be written in the time-frequency domain as:

$$X_i(t, \omega) = \sum_{j=1}^N f_{ij}(t) \cdot e^{-j\omega(t-k_{ij}(t))} \cdot G_{ij}(\omega) \cdot S_j(t, \omega). \quad (20)$$

4. GEOMETRIC INTERPRETATION OF THE MIXING SYSTEM FOR SPARSE SOURCES

Sparse sources in the time-frequency domain enables tracking of the mixing system, since there are many frequencies and time instances, in which only one source is active. We define the angle $\alpha_i(t, \omega)$ as follows:

$$\alpha_i(t, \omega) = \tan^{-1} \left(\frac{|X_i(t, \omega)|}{|X_M(t, \omega)|} \right), \quad (21)$$

where $X_M(t, \omega)$ is the mixture received at the last sensor.

For naturally sparse (or sparsified) sources in the time-frequency domain, there are instants t_n and frequencies ω_m where only one source is not zero. In those combinations of time and frequency, the mixtures projected onto the time-frequency domain can be written as:

$$X_i(t_n, \omega_m) = f_{ij}(t_n) \cdot e^{-j\omega_m(t_n - k_{ij}(t_n))} \cdot G_{ij}(\omega_m) \cdot S_j(t_n, \omega_m), \quad (22)$$

where j is the index of the non-zero source.

The points $(\alpha_1(t_n, \omega_m), \alpha_2(t_n, \omega_m), \dots, \alpha_{M-1}(t_n, \omega_m), t_n)$ in an M dimensional space, lie on one of the curves $(\beta_{1j}(t, \omega), \beta_{2j}(t, \omega), \dots, \beta_{(M-1)j}(t, \omega), t)$, where $\beta_{ij}(t, \omega)$ is defined as follows:

$$\beta_{ij}(t, \omega) = \tan^{-1} \left(\frac{f_{ij}(t) \cdot |G_{ij}(\omega)|}{f_{Mj}(t) \cdot |G_{Mj}(\omega)|} \right). \quad (23)$$

Note that the above curve is continuous, since $\beta_{ij}(t, \omega)$ does not depend on the value of the source j . Therefore, if we assume that $f(t)$ is a continuous functions of t , $\beta_{ij}(t, \omega)$ is also a continuous function of t , bounded between 0 and $\frac{\pi}{2}$.

We define the phase shift $\Delta\phi_i(t, \omega)$ as follows:

$$\Delta\phi_i(t, \omega) = \angle X_i(t, \omega) - \angle X_M(t, \omega), \quad (24)$$

where \angle stands for the phase of X .

The points $(\Delta\phi_1(t_n, \omega_m), \Delta\phi_2(t_n, \omega_m), \dots, \Delta\phi_{M-1}(t_n, \omega_m), t)$ in an M -dimensional space, lie on one of the curves

$(\Delta\psi_{1j}(t, \omega), \Delta\psi_{2j}(t, \omega), \dots, \Delta\psi_{(M-1)j}(t, \omega), t)$, where $\Delta\psi_{ij}(t, \omega)$ is defined as follows:

$$\Delta\psi_{ij}(t, \omega) = \angle G_{ij}(\omega) - \angle G_{Mj}(\omega) + \omega(k_{ij}(t) - k_{Mj}(t)). \quad (25)$$

In this case too, the above curve is continuous, since $\Delta\psi_{ij}(t, \omega)$ does not depend on the value of the source j . Therefore, if we assume that $k(t)$ is a continuous functions of t , $\Delta\psi_{ij}(t, \omega)$ is also a continuous function of t bounded between 0 and 2π with a 2π fold.

For sparse sources, it is possible to identify the continuous curves $(\beta_{1j}(t, \omega), \beta_{2j}(t, \omega), \dots, \beta_{(M-1)j}(t, \omega), t)$ from the non-continuous curve $(\alpha_1(t, \omega), \alpha_2(t, \omega), \dots, \alpha_{M-1}(t, \omega), t)$, since there are more moments and frequencies, in which only one source is active, than moments and frequencies reflecting simultaneous source activity. Those sparse points $(\alpha_1(t_n, \omega_m), \alpha_2(t_n, \omega_m), \dots, \alpha_{M-1}(t_n, \omega_m), t_n)$ lie densely on the continuous curves.

It is possible to identify the curves $(\Delta\psi_{1j}(t, \omega), \Delta\psi_{2j}(t, \omega), \dots, \Delta\psi_{(M-1)j}(t, \omega), t)$ from the non-continuous curve $(\Delta\phi_1(t, \omega), \Delta\phi_2(t, \omega), \dots, \Delta\phi_{M-1}(t, \omega), t)$ for the same reason. The sparse points $(\Delta\phi_1(t_n, \omega_m), \Delta\phi_2(t_n, \omega_m), \dots, \Delta\phi_{M-1}(t_n, \omega_m), t_n)$ would lie densely on the continuous curves. Methods for such identification of curves in clutter can be borrowed from image processing and computer vision.

5. SEPARATION OF THE SOURCES

It is known that the IBSS problem can be solved up to a scaled version of the original sources. It is also known that the CBSS problem can be solved up to a filtered version of the sources. This leads to the assumption that the blind separation of time-varying mixtures can be solved up to a time-varying filtered version of the sources. Dividing and multiplying the right side of (20) by $f_{Mj}(t) \cdot e^{-j\omega(t-k_{Mj}(t))} \cdot G_{Mj}(\omega)$ introduces $\beta_{ij}(t, \omega)$ and $\Delta\psi_{ij}(t, \omega)$ into the time-frequency mixing model:

$$X_i(t, \omega) = \tan(\beta_{ij}(t, \omega)) \cdot e^{j\Delta\psi_{ij}(t, \omega)} \cdot \hat{S}_j(t, \omega), \quad (26)$$

where

$$\hat{S}_j(t, \omega) = S_j(t, \omega) \cdot f_{Mj}(t) \cdot e^{-j\omega(t-k_{Mj}(t))} \cdot G_{Mj}(\omega), \quad (27)$$

is a time-varying filtered version of $S_j(t, \omega)$.

If $\beta_{ij}(t, \omega) = \frac{\pi}{2}$, a different sensor should be taken as the M^{th} sensor (since $\tan(\beta_{ij}(t, \omega) = \infty)$).

Defining $\vec{X}(t, \omega)$ as the column vector of $X_i(t, \omega)$, $\vec{\hat{S}}(t, \omega)$ as the column vector of $\hat{S}_j(t, \omega)$ and $A(t, \omega)$ as the matrix of the mixing coefficients, $\tan(\beta_{ij}(t, \omega)) \cdot e^{j\Delta\psi_{ij}(t, \omega)}$, the mixing model in the time-frequency domain can be represented by:

$$\vec{X}(t, \omega) = A(t, \omega) \cdot \vec{\hat{S}}(t, \omega). \quad (28)$$

The system can be inverted by:

$$\vec{\hat{S}}(t, \omega) = A^\dagger(t, \omega) \cdot \vec{X}(t, \omega), \quad (29)$$

where $A^\dagger(t, \omega)$ is the pseudo-inverse of $A(t, \omega)$:

$$A^\dagger(t, \omega) = (A(t, \omega)^T \cdot A(t, \omega))^{-1} \cdot A(t, \omega)^T. \quad (30)$$

The separation and reconstruction of $\hat{S}_j(t)$ can be done using the inverse fourier transform:

$$\hat{S}_j(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{S}_j(t, \omega) \cdot e^{j\omega t} d\omega. \quad (31)$$

For practical use, the separation can only be done on a discrete version of the received mixtures. Therefore, going into a discrete version entails some constraints:

- The input and output signals should be band limited, and the sampling rate should obey the nyquist rule (both for $x_i(t)$ and $s_j(t)$).
- The duration of the window should allow enough frequency resolution for the sources to be represented sparsely in the time-frequency domain.

$X_i(t, \omega)$, which is the frequency content of $x(t)$ in the moment t assuming the mixing system was stationary, can only be approximated. The approximation can be calculated the same way $S(t, \omega)$ is calculated using some window w :

$$X_i(t, \omega) \approx \int_{-\infty}^{\infty} W(\varphi) \cdot X_i(\omega - \varphi) \cdot e^{-j\varphi t} d\varphi, \quad (32)$$

The choice of the window is of much importance since $\beta_{ij}(t, \omega)$ and $\Delta\psi_{ij}(t, \omega)$ can only be identified if they are locally stationary in this window. (10) reveals a possible choice for the window duration:

$$\max(q_{ij}) < T < \max(T_a(t), T_d(t)), \quad (33)$$

where T is the window duration and $\max(\cdot)$ stands for the maximum over all elements and time instances. Other choices of windows shape and duration are also possible using for example multiple Gabor windows.

6. SIMULATION RESULTS

An acoustic scenario similar to the one depicted in figure 1 was generated. The records of the reporter and the emergency vehicle where sampled with anti-aliasing filter in the rate of 4K samples per second. L was set to 30 meters, $v_v = 40$ m/s, $v_r = 0.05$ m/s, $c = 340$ m/s, $d = 0.2$ meter, $x_{vehicle} = -162$ meters and $x_{reporter} = 0.2$ meter. The time invariant atmospheric transfer function, $g_{ij}(\cdot)$, was considered to be identical for all ij 's. $X_i(t, \omega)$ was obtained using a window of length 400 samples. Figure 2a shows $|X_1(t, \omega)|$. The doppler effect can clearly be seen. Figure 2b shows $\alpha_i(t, \omega)$, which is equal to every ω , since the atmospheric transfer function, $G_{ij}(\omega)$ is equal for all sensors.

The grey lines indicate $\beta_{ij}(t, \omega)$. Figure 2c shows $\frac{\Delta\phi_i(t, \omega)}{\omega}$, where the grey lines indicate $\frac{\Delta\psi_{ij}(t, \omega)}{\omega}$. Due to the doppler effect, the observations deviate from $\frac{\Delta\psi_{ij}(t, \omega)}{\omega}$ for one of the sources, but it does not interfere with the ability to separate the sources (as mentioned before, the restored sources can be a time-varying filtered version of the true sources). $\beta_{ij}(t, \omega)$ and $\Delta\psi_{ij}(t, \omega)$ were approximated using a polynomial fit. Figure 2d and 2e shows the recovered signals $\hat{S}_j(t, \omega)$ obtained by inverting the mixing system. The mixtures and the recovered sources can be downloaded from <http://visl.technion.ac.il/~kaftory/eusipco2006>.

7. SUMMARY

The proposed new approach to blind separation of time-varying mixtures, projects the signals onto a space of sparse representation. Zadeh's transform enables the identification of the mixing coefficients in the combined time-frequency domain. These coefficients can be inverted, to derive the restored time-varying filtered version of the original sources. The identification of the mixing coefficients can benefit from *a priori* knowledge regarding the time-varying characteristics. Further extension of this study is now in progress using Zadeh's transform in the time-frequency domain and the ICA approach. In this approach the sources need not be sparse, but should be statistically independent. The minimization of a cost function of the independency of $\hat{S}_j(t, \omega)$ in Zadeh's time-frequency domain, restores the sources and the mixing system.

8. ACKNOWLEDGEMENT

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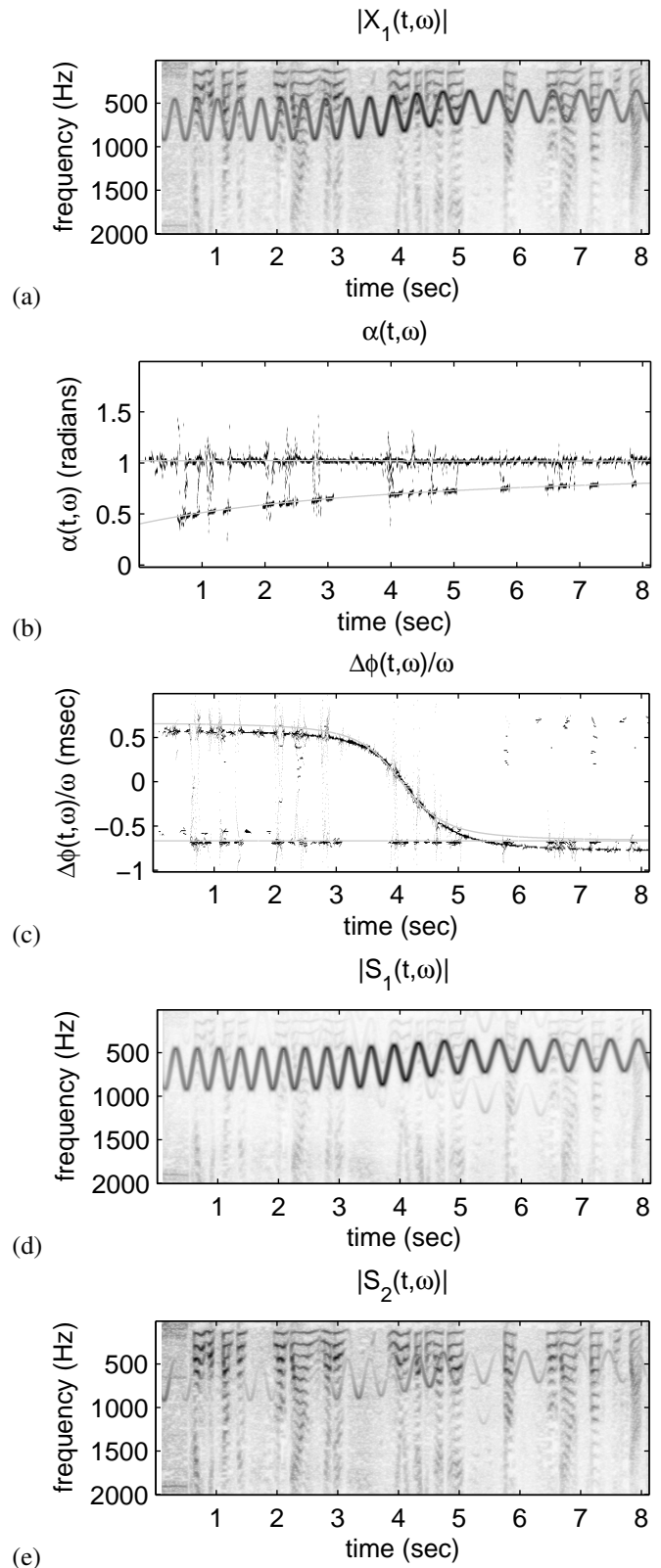


Fig. 2: (a) The amplitude of $X_1(t, \omega)$ as was estimated using (32) with a window of 400 samples. (b) $\alpha(t, \omega)$: $\beta_{ij}(t, \omega)$ can be clearly identified. (c) $\frac{\Delta\phi(t, \omega)}{\omega}$: A doppler effected version of $\frac{\Delta\psi_{ij}(t, \omega)}{\omega}$ can be easily identified. (d),(e) The restored time-frequency sources $\hat{S}_1(t, \omega)$ and $\hat{S}_2(t, \omega)$