SUB-CARRIER SNR ESTIMATION AT THE TRANSMITTER FOR REDUCED FEEDBACK OFDMA

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ABSTRACT

In multiuser OFDMA FDD systems with resource allocation based on the instantaneous channel quality of the users, the feedback overhead can be very large. In this paper, a method to significantly reduce this feedback is proposed. The idea is to let the users feed back the channel quality (the SNR in this paper) of only a sub-set of their strongest sub-carriers. The SNRs on the other sub-carriers are instead estimated from the fed back values. We derive the MMSE estimator of the SNR of a sub-carrier, which uses two fed back SNRs as input. As a comparison, we also study the performance of the LMMSE estimator as well as spline interpolation. Numerical results show that the LMMSE estimator tends to underestimate the SNR compared to the other two estimators, whereas the interpolation tends to overestimate the SNR. System simulations including adaptive modulation and packet losses indicate that the MMSE estimator is the best choice in practice.

1. INTRODUCTION

Opportunistic communication promises increased data rates in multiuser systems [1]. It relies on the principle of multiuser diversity, which exploits the fact that users fade independently. By giving channel access to users with favorable channel conditions and using adaptive modulation, the system throughput can be increased compared with a system that uses fixed user scheduling. Opportunistic scheduling has already entered existing single-carrier standards, like 1xEV-DO [2] and HSDPA [3].

A strong candidate for future wireless standards is orthogonal frequency division multiple access (OFDMA), in particular for the downlink. In OFDMA, several users can simultaneously be scheduled on different frequency sub-carriers. This opens the possibility to exploit the frequency fading that users typically experience. By scheduling users on time instants and frequencies where their channel is strong, transmit power is not wasted on channels with poor conditions.

A problem with channel-aware scheduling for the OFDMA downlink in frequency division duplex (FDD) systems, is the total feedback load which can be very high. To schedule users on different sub-carriers, the base-station requires knowledge of the channel quality on the sub-carriers of all active users. If the number of sub-carriers is high and the users fade fast, the total feedback rate will be overwhelming, even if the channel quality is highly quantized. Different approaches to deal with this problem were treated in [4, 5]. In [4], the sub-carriers are divided into groups of adjacent sub-carriers, called clusters. The users feed back only one channel quality indicator (CQI) per cluster, e.g. the minimum SNR within the cluster. Furthermore, only the CQIs for the strongest clusters are fed back. In [5], the approach was to let each user feed back only one bit per sub-carrier, which signifies if the sub-carrier channel quality is above a threshold. The threshold was adapted as a function of the number of users in order to allow for higher order modulation. In this paper, we use the sub-carrier SNR as the CQI.

In classical pilot-symbol aided OFDM channel estimation, the transmitter sends known pilots on predefined positions on the time-frequency grid. The receiver can then estimate the frequency response of the channel and receive data coherently. In this paper, we reverse this process, by letting the users feed back SNRs from a grid of sub-carriers to the base-station, which then estimates the channel quality on the sub-carriers that were not fed back. In fact, since we consider an opportunistic system, the users only need to feed back SNRs for the parts of the spectrum where the channel gain is high. Consequently, the base-station considers the users for scheduling only in those parts. This effectively reduces the feedback load. We propose and evaluate MMSE and LMMSE estimators as well as simple interpolation.

2. SYSTEM MODEL

2.1 OFDMA Model

The OFDMA downlink of an FDD cellular wireless communication system is considered. Perfect time and frequency synchronization is assumed as well as a wide-sense quasi-stationary (constant during each OFDM symbol) channel with an impulse response that is shorter than the cyclic prefix. Then, the received signal on sub-carrier $n \in \mathcal{N} = \{0,\dots,N-1\}$ at OFDM symbol t for user $k \in \{0,\dots,K-1\}$ can be written as

$$y_{k,n}(t) = c_n(t)H_{k,n}(t) + w_{k,n}(t)$$

where t is the OFDM symbol, $c_n(t)$ is the transmitted symbol, $H_{k,n}(t)$ is the sub-carrier frequency response and $w_{k,n}(t)$ is additive Gaussian noise with user- and sub-carrier-specific variance $\sigma^2_{k,n}$. In this paper, inter-cell interference is neglected. The sub-carrier SNR, $\Gamma_{k,n}(t)$, is defined as

$$\Gamma_{k,n}(t) = \frac{|H_{k,n}(t)|^2}{\sigma^2_{k,n}}.$$

In the following presentation, the time-index t is omitted for brevity.

2.2 Reduced Feedback Scheme

On each sub-carrier, a different user can be scheduled. The scheduling is updated regularly with an interval of several OFDM symbols, here called a block. The scheduling is based on feedback from all users in the form of sub-carrier SNRs. In each block, user k feeds back $\Gamma_{k,n}$ for all $n \in \mathcal{N}_k \subseteq \mathcal{N}_{fb} \subset \mathcal{N}$. The feedback sub-carriers, $\mathcal{N}_{fb}$, are a set of uniformly spaced sub-carriers across the OFDM symbol. The spacing should be dense enough to fulfil the Nyquist sampling theorem for reconstruction. The users select the $S = |\mathcal{N}_k|$ sub-carriers with highest SNR, $\Gamma_{k,n}$, for feedback. The set $\mathcal{N}_k$ is different for different users, since their channels are assumed to be independent.

As an example, consider a system with 128 sub-carriers, i.e. $\mathcal{N} = \{0,1,\dots,127\}$. The sub-carriers eligible for feedback is chosen as every eighth, i.e. $\mathcal{N}_{fb} = \{0,8,\dots 122\}$. From $\mathcal{N}_{fb}$, user k selects the $S = 8$ sub-carriers with highest SNR, for instance $\mathcal{N}_k = \{8,16,32,40,48,96,104,112\}$, as in Figure 1. In order to schedule users on sub-carriers that are not in $\mathcal{N}_{fb}$, the base-station has to estimate the SNR on the sub-carriers in $\mathcal{N} \setminus \mathcal{N}_k$ for each user. This estimation problem is the topic of this paper and is treated

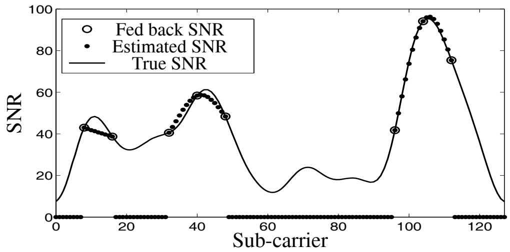


Figure 1: An example feedback and estimation scenario. The user k has chosen sub-carriers $\mathscr{N}_k = \{8,16,32,40,48,96,104,112\}$ for feedback from $\mathscr{N}_{fb} = \{0,8,\dots,120\}$. The base-station estimates the sub-carrier SNRs in between using 2-tap LMMSE, and estimates the others as zero.

in detail in Section 3. Since the users only feed back the SNRs for a subset of the sub-carriers in $\mathscr{N}_{fb}$, many sub-carriers can not be reliably estimated. This can be solved by only estimating sub-carriers close to those fed back in $\mathscr{N}_k$ and estimating the others as zero. This is typically not a problem, since the strongest sub-carriers of a user are located close to those in $\mathscr{N}_k$. To clarify, a sub-carrier SNR of user k is estimated only if the two closest sub-carriers in $\mathscr{N}_{fb}$ were actually fed back in $\mathscr{N}_k$. If not, the sub-carrier SNR is estimated as zero, for the sake of robustness. In the following, the user index k is omitted for brevity.

2.3 Channel Model

Time-dispersive sample-spaced Rayleigh fading channels are considered, with channel impulse response

$$h(m) = \sum_{l=0}^{L-1} \beta_l \delta(m-l)$$

with an exponentially decaying power delay profile,

$$\mathrm{E}\left[|\beta_l|^2\right] = Ae^{-lT/2\tau}$$

where L is the number of taps, β_l are independent zero-mean complex Gaussian, T is the sample period, τ is the root-mean-square delay spread and A is a normalization constant set so that $\sum_{l=0}^{L-1}\mathrm{E}\left[|\beta_l|^2\right] = 1$. This normalization also gives $\mathrm{E}\left[|H_n|^2\right] = 1$, where H_n is the zero-mean complex Gaussian n^{th} point in the DFT of $h(m)$. For this channel model, it is possible to compute the covariance between two sub-carriers as

$$\mathrm{E}\left[H_{n_1}H_{n_2}^*\right] = \frac{\left(1-e^{-T/2\tau}\right)\left(1-e^{-LT/2\tau-j2\pi L(n_1-n_2)/N}\right)}{\left(1-e^{-LT/2\tau}\right)\left(1-e^{-T/2\tau-j2\pi(n_1-n_2)/N}\right)}. \quad (1)$$

The sub-carrier SNR is exponentially distributed with mean

$$\overline{\Gamma}_n = \frac{1}{\sigma_n^2}\mathrm{E}\left[|H_n|^2\right] = \frac{1}{\sigma_n^2} \quad (2)$$

and cross-covariance

$$\mathrm{E}\left[\Gamma_{n_1}\Gamma_{n_2}\right] - \overline{\Gamma}_{n_1}\overline{\Gamma}_{n_2} = \frac{\left|\mathrm{E}\left[H_{n_1}H_{n_2}^*\right]\right|^2}{\sigma_{n_1}^2\sigma_{n_2}^2} \quad (3)$$

which follows from (13) in Appendix A and can be computed from (1). The notation $\overline{x}$ means the expected value of the random x.

The sub-carrier covariance in (1) is needed for the MMSE estimator presented in Section 3.1 and the SNR mean and covariance in (2)-(3) is needed for the LMMSE estimator in Section 3.2.

2.4 Covariance Information at the Transmitter

The MMSE estimators presented in the next section assume that the transmitter knows the cross-covariance between sub-carriers. This information can be obtained in several ways. Since the channel covariance typically is a slowly changing parameter, compared to the instantaneous SNR, the additional overhead of feeding back the covariance from each user would be rather small. Additionally, the covariance can often be parameterized, for instance by L and T/τ in (1), in order to reduce the feedback even more. An alternative, that implies no extra feedback, is that the base-station estimates the sub-carrier covariance from the fed back SNRs of the sub-carriers in $\mathscr{N}_k$. A third option for obtaining downlink covariance information at the base-station without feedback is to estimate it from the uplink [6].

3. SNR ESTIMATORS

In this section, three estimators for estimating sub-carrier SNRs at the *transmitter* based on the few fed back SNRs of the sub-carriers in $\mathscr{N}_k$ are presented. Since the fed back SNR values are concentrated to those frequencies where the channel gain is high, it is difficult to reliably estimate the sub-carriers with low SNR. We solve this by only estimating sub-carrier SNR that can be reliably estimated, i.e. those with high SNR, as in Figure 1. All other estimates are set to zero. This is not a problem since an opportunistic system is considered, where users are to be scheduled on their best sub-carriers.

3.1 MMSE

The MMSE estimator of the SNR takes advantage of the underlying PDF of the SNRs. To derive the estimator, it is assumed that the channel of each sub-carrier is Rayleigh fading, that is, modeled as a zero-mean complex Gaussian random variable.

The MMSE estimator of the SNR of subcarrier n_3, Γ_{n_3} from the known SNRs of two other sub-carriers, Γ_{n_1} and Γ_{n_2}, is given by [7]

$$\widehat{\Gamma}_{n_3}^{\mathrm{MMSE}} = \mathrm{E}\left[\Gamma_{n_3}|\Gamma_{n_1},\Gamma_{n_2}\right]. \quad (4)$$

Even though the joint PDF $f(\Gamma_{n_1},\Gamma_{n_2},\Gamma_{n_3})$ cannot, in general, be expressed in closed form, it is shown in Appendix B that the closed form expression for the MMSE estimator is

$$\mathrm{E}\left[\Gamma_{n_3}|\Gamma_{n_1}\Gamma_{n_2}\right] = \frac{1+\alpha+|\beta|\cos(\angle\beta-\angle\mathbf{R}_{12})\frac{\mathrm{I}_1\left(2\gamma\sqrt{\Gamma_{n_1}\Gamma_{n_2}}\right)}{\mathrm{I}_0\left(2\gamma\sqrt{\Gamma_{n_1}\Gamma_{n_2}}\right)}}{\mathbf{R}_{33}^{-1}}, \quad (5)$$

where $\angle\{\cdot\}$ is the argument (phase) operator, $\mathbf{R}$ is the positive definite covariance matrix of $\mathbf{h}$,

$$\mathbf{h} = \left[\frac{H_{n_1}}{\sigma_{n_1}}\ \frac{H_{n_2}}{\sigma_{n_2}}\ \frac{H_{n_3}}{\sigma_{n_3}}\right]^{\mathrm{T}}, \qquad \mathbf{R} = \mathrm{E}\left[\mathbf{h}\mathbf{h}^{\mathrm{H}}\right] \in \mathbb{C}^{3\times 3}, \quad (6)$$

and the notation $\mathbf{R}_{kl}^{-1}$ is to be interpreted as $[\mathbf{R}^{-1}]_{kl}$. The functions $\mathrm{I}_0(\cdot)$ and $\mathrm{I}_1(\cdot)$ denote the modified Bessel functions of the first kind (zeroth and first order, respectively). The parameters α, β and γ are defined as

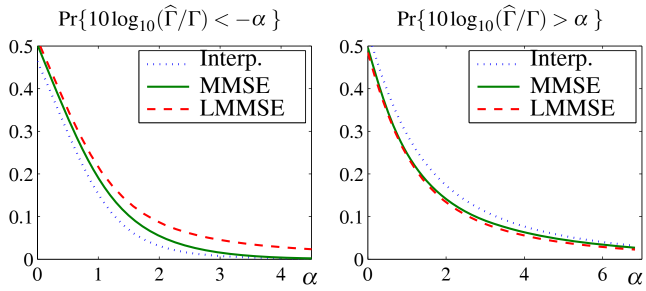
$$\alpha \triangleq \Gamma_{n_1}\frac{\left|\mathbf{R}_{13}^{-1}\right|^2}{\mathbf{R}_{33}^{-1}} + \Gamma_{n_2}\frac{\left|\mathbf{R}_{23}^{-1}\right|^2}{\mathbf{R}_{33}^{-1}}, \quad \beta \triangleq 2\sqrt{\Gamma_{n_1}\Gamma_{n_2}}\frac{\mathbf{R}_{13}^{-1}\left(\mathbf{R}_{23}^{-1}\right)^*}{\mathbf{R}_{33}^{-1}}, \quad (7)$$

and

$$\gamma \triangleq \frac{|\mathbf{R}_{12}|}{\mathbf{R}_{11}\mathbf{R}_{22}-|\mathbf{R}_{12}|^2} = \frac{\sqrt{\mathrm{Cov}[\Gamma_1,\Gamma_2]}}{\overline{\Gamma}_1\overline{\Gamma}_2-\mathrm{Cov}[\Gamma_1,\Gamma_2]}, \quad (8)$$

where (8) follows from (13) in Appendix A. It should be noted that γ is only a function of the mutual statistics of Γ_1 and Γ_2, and independent of their relation to Γ_3.

Note that the computational complexity of evaluating (5) is low, since the fraction of the modified Bessel functions can be efficiently computed from a few terms of the series expansions.



(a) Under-estimation (b) Over-estimation

Figure 2: Shows the probability of under- and over-estimating the SNR, respectively. The filter sizes are 2.

3.2 LMMSE

The linear minimum mean square error estimator of size 2 for the sub-carrier SNR, Γ_{n_3}, based on the fed back observations Γ_{n_1} and Γ_{n_2} is given by [7]

$$\widehat{\Gamma}_{n_3}^{\text{LMMSE}} = \overline{\Gamma}_{n_3} + \mathbf{R}_{n_3\mathbf{x}}\mathbf{R}_{\mathbf{xx}}^{-1}(\mathbf{x} - \bar{\mathbf{x}}) \tag{9}$$

where

$$\begin{aligned} \mathbf{x} &= [\Gamma_{n_1}\ \Gamma_{n_2}]^T \\ \mathbf{R}_{n_3\mathbf{x}} &= \mathrm{E}\left[\Gamma_{n_3}\mathbf{x}^H\right] - \overline{\Gamma}_{n_3}\bar{\mathbf{x}}^H \\ \mathbf{R}_{\mathbf{xx}} &= \mathrm{E}\left[\mathbf{x}\mathbf{x}^H\right] - \bar{\mathbf{x}}\bar{\mathbf{x}}^H. \end{aligned}$$

The covariances are assumed to be known at the transmitter. For the channel model presented in Section 2.3, the covariances are given by (3). The LMMSE estimator of size 4 (four fed back SNRs are used) is of the same form as above.

3.3 Interpolation

The estimation problem in this paper can be seen as an interpolation based on the fed back SNRs. Therefore, the statistical estimators are compared with a piecewise cubic spline interpolation that requires only the fed back SNR values of the sub-carriers in $\mathcal{N}_k$. For the case with two input SNRs, the spline interpolation reduces to linear interpolation.

3.4 Performance Evaluation

The performance of the estimators is evaluated by considering the SNR estimate of sub-carrier n from the SNRs of the sub-carriers $n_2 = n-8$ and $n_3 = n+8$ (i.e. filter-size 2). In total, there are $N = 128$ sub-carriers, and $T/\tau = 2$. In Figure 2 the probability of under- and over-estimating the SNR by α dB is plotted. As can be seen, the LMMSE estimate tends to under-estimate, whereas the interpolation tends to over-estimate the SNR. This behavior is discussed further in Section 4.1.

4. SYSTEM SIMULATION

In order to evaluate the impact of the estimators in Section 3, a multiuser OFDMA single-cell system has been simulated. The cell is populated by $K = 10$ users with i.i.d. channels according to Section 2.3 and temporal block-fading according to Jake's model [8], with a carrier frequency assumption of 2 GHz and user speeds of 20 m/s. The number of sub-carriers $N = 128$, the OFDM symbol duration is 38.33 μs and the block length equals 16 OFDM symbols. The average SNR is 10 dB for all sub-carriers.

In the simulations, the users are assumed to estimate the downlink channel perfectly. Furthermore, the users are assumed to predict the sub-carrier SNRs perfectly (e.g. by [9]), so that the fed back information is not outdated. The feedback is according to Section 2.2, with $S = 5$ and the spacing between the sub-carriers in $\mathcal{N}_{fb}$ varies between 8 and 24. Only the sub-carriers that are located between two sub-carriers in $\mathcal{N}_{fb}$ that were actually fed back are estimated, as in Figure 1. The other sub-carrier SNRs are estimated as zero. On each sub-carrier, the user with highest estimated SNR is scheduled and assigned a modulation order from BPSK, QPSK, 16-QAM, 64-QAM or 256-QAM. The adaptive modulation thresholds are based on a target bit error rate of 10^{-3} and the transmit power is allocated equally over the sub-carriers. The throughput is computed as the number of bits in the successfully received packets, which are 128 bits long. A packet is considered erroneous if at least one bit is erroneous.

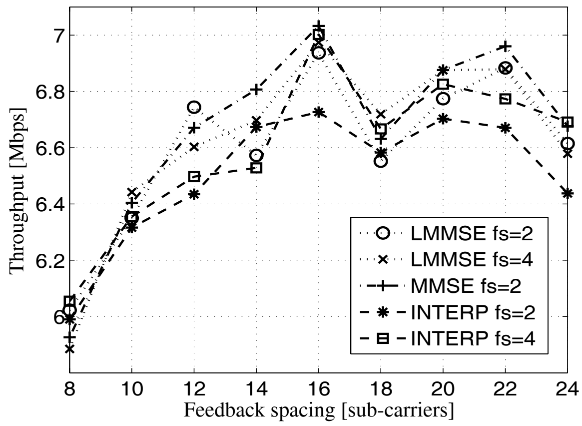


Figure 3: The system throughput for different estimators and filter-sizes (fs) as a function of the spacing in the feedback grid. The total feedback rate is equal in all points.

4.1 Results

For a spacing of 8 sub-carriers, the estimation quality of the estimated sub-carriers is relatively good resulting in few erroneous decisions. A large spacing gives poorer SNR estimates at the base-station. On the other hand, a large feedback spacing enables more sub-carriers per user to be estimated. This can give a higher system throughput, as illustrated in Figure 3, since an overall strong user can be scheduled on more of its sub-carriers.

The two-tap MMSE estimator gives the highest overall throughput when the feedback grid is spaced by 16 sub-carriers. Interestingly, the throughput curves show two distinct peaks, one at a feedback spacing of 16 and one at a feedback spacing of 22. This is the result of two effects of changing the feedback spacing. Up to a feedback spacing of 16, the effect of the reduced estimation quality is more than compensated for by the fact that larger parts of the channels of the strongest users are estimated. This leads to more sub-carriers being allocated to strong users, giving a high throughput. The dip in throughput at feedback spacing 18 is due to the higher packet error rate, induced by worse SNR estimation and higher modulation error rates at the scheduler, which apparently is not compensated for by the ability to give slightly more sub-carriers to the strong users. Increasing the feedback spacing even further gives a similar increase that for lower spacings. Also the fact that some spacings do not divide the number of sub-carriers has an impact on the performance curves. This means that the highest and the lowest numbered sub-carriers in $\mathcal{N}_{fb}$ will be more correlated than the other, due to the circular correlation of the OFDM symbol. This effect gives a slightly decreased throughput for those spacings, but since it affects all algorithms, it does not change the conclusions. It is interesting that even though the packet error rate of the LMMSE is slightly lower than for MMSE (not shown here due to space limitations), MMSE has higher throughput, due to the SNR under-estimation tendency of the LMMSE (see Figure 2). The interpolation loses in performance due to the SNR over-estimation, which gives a higher packet error rate.

Important to note is that the specific feedback spacings that give a high system throughput depend on parameters like the channel delay spread and the packet size, but also on the choice of scheduling metric. We foresee a smaller throughput increase for large feedback spacings if for instance a proportional fair scheduler would have been used, since the strongest user would not be scheduled on as many of its estimated sub-carriers.

5. CONCLUSIONS

In this paper, we have proposed a reduced feedback scheme for multiuser OFDMA systems that employ channel-aware scheduling. The feedback can be greatly reduced if the base-station is equipped with the capability to estimate sub-carrier SNRs from a small set of fed back SNRs. We propose that the users should pick the fed back values from the part of the channel where the SNR is high, where the user should typically be scheduled. We derived the MMSE SNR estimator of size 2, which performed slightly more robustly than the corresponding LMMSE and interpolation estimators. The proposed feedback and estimation scheme was also evaluated in a system simulation. The results showed that a suitable operating point for the spacing of the fed back SNRs can be found, which is a good trade-off between estimator performance and the number of sub-carriers per user that can be estimated.

A. DERIVATION OF THE RELATION BETWEEN SUB-CARRIER AND SNR COVARIANCE

Since the noise power of each carrier n, σ_n^2 is assumed known and static, the SNRs of the carriers are fully determined by the channel coefficients, H_n. The joint PDF of the SNRs, $f(\Gamma_{n_1},\ldots,\Gamma_{n_N})$ can thus be expressed in terms of the channel coefficient statistics, $f(H_{n_1},\ldots,H_{n_N})$. In particular we are interested in the joint PDF, $f(\Gamma_{n_1},\Gamma_{n_2})$, in order to derive the covariance between two SNRs.

The derivations are simplified by stacking H_{n_1} and H_{n_2}, normalized with the standard deviation of the noise in the vector,

$$\mathbf{h}=\left[\frac{H_{n_1}}{\sigma_{n_1}}\ \frac{H_{n_2}}{\sigma_{n_2}}\right]^T,$$

where $\mathbf{h}$ is normalized such that $|\mathbf{h}_i|^2=\Gamma_{n_i}$. For Rayleigh fading channel coefficients, that is, zero-mean complex Gaussian channels, the PDF of $\mathbf{h}$ becomes

$$f(\mathbf{h})=\frac{e^{-\mathbf{h}^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{h}}}{\pi^2\det\mathbf{R}}=\frac{1}{\pi^2\det\mathbf{R}}e^{-\mathbf{R}_{11}^{-1}|\mathbf{h}_1|^2-\mathbf{R}_{22}^{-1}|\mathbf{h}_2|^2-2\Re\left\{\mathbf{R}_{12}^{-1}\mathbf{h}_1\mathbf{h}_2^*\right\}},$$

where $\mathbf{R}$ is the positive definite covariance matrix

$$\mathbf{R}=\mathrm{E}\left[\mathbf{h}\mathbf{h}^{\mathrm{H}}\right]\in\mathbb{C}^{2\times 2} \tag{10}$$

and $\mathbf{R}_{kl}^{-1}$ is to be interpreted as $[\mathbf{R}^{-1}]_{kl}$. The notation $\Re$ and $\Im$ is used to denote real and imaginary parts, respectively. The channel coefficients can be expressed in terms of the SNR as $\mathbf{h}_i=\sqrt{\Gamma_{n_i}}e^{j\phi_i}$, where ϕ_i is distributed uniformly in $[0,2\pi)$. By noting that

$$\mathrm{d}\mathbf{h}_i^{\Re}\,\mathrm{d}\mathbf{h}_i^{\Im}=\left|\frac{\partial(\mathbf{h}_i^{\Re},\mathbf{h}_i^{\Im})}{\partial(\Gamma_{n_i},\phi_i)}\right|\mathrm{d}\Gamma_{n_i}\,\mathrm{d}\phi_i=\frac{\mathrm{d}\Gamma_{n_i}\,\mathrm{d}\phi_i}{2}$$

the joint PDF of Γ_{n_1}, Γ_{n_2}, ϕ_1 and ϕ_2 is obtained as

$$f(\Gamma_{n_1},\Gamma_{n_2},\phi_1,\phi_2)=\frac{e^{-\mathbf{R}_{11}^{-1}\Gamma_{n_1}-\mathbf{R}_{22}^{-1}\Gamma_{n_2}-2\Re\left\{\mathbf{R}_{12}^{-1}\sqrt{\Gamma_{n_1}\Gamma_{n_2}}e^{j(\phi_1-\phi_2)}\right\}}}{4\pi^2\det\mathbf{R}}. \tag{11}$$

The PDF $f(\Gamma_{n_1},\Gamma_{n_2})$ is thus obtained by integrating over ϕ_1 and ϕ_2. This integration can be done by introducing the change of coordinates $(\phi_1,\phi)=(\phi_1,\phi_1-\phi_2)$ which results in

$$\begin{aligned}f(\Gamma_{n_1},\Gamma_{n_2})&=\frac{e^{-\mathbf{R}_{11}^{-1}\Gamma_{n_1}-\mathbf{R}_{22}^{-1}\Gamma_{n_2}}}{2\pi\det\mathbf{R}}\int_0^{2\pi}e^{-2\Re\left\{\mathbf{R}_{12}^{-1}\sqrt{\Gamma_{n_1}\Gamma_{n_2}}e^{j\phi}\right\}}\,\mathrm{d}\phi \qquad (12)\\&=\frac{e^{-\mathbf{R}_{11}^{-1}\Gamma_{n_1}-\mathbf{R}_{22}^{-1}\Gamma_{n_2}}}{\det\mathbf{R}}\,\mathrm{I}_0\left(2\left|\mathbf{R}_{12}^{-1}\right|\sqrt{\Gamma_{n_1}\Gamma_{n_2}}\right)\end{aligned}$$

where the integral is given by (16) in Appendix C.

Using (12) the cross correlation of Γ_{n_1} and Γ_{n_2} is computed as

$$\begin{aligned}\mathrm{E}\left[\Gamma_{n_1}\Gamma_{n_2}\right]&=\int_0^\infty\int_0^\infty\Gamma_{n_1}\Gamma_{n_2}\,f(\Gamma_{n_1},\Gamma_{n_2})\,\mathrm{d}\Gamma_{n_1}\,\mathrm{d}\Gamma_{n_2}\\&\overset{(a)}{=}\frac{1}{\det\mathbf{R}}\int_0^\infty\left[\frac{\Gamma_{n_2}}{\left(\mathbf{R}_{11}^{-1}\right)^2}+\frac{\left|\mathbf{R}_{12}^{-1}\right|^2}{\left(\mathbf{R}_{11}^{-1}\right)^3}\Gamma_{n_2}^2\right]e^{-\frac{\Gamma_{n_2}}{\mathbf{R}_{11}^{-1}\det\mathbf{R}}}\,\mathrm{d}\Gamma_{n_2}\\&=\det\mathbf{R}+2\left|\mathbf{R}_{12}\right|^2=\mathbf{R}_{11}\mathbf{R}_{22}+\left|\mathbf{R}_{12}\right|^2,\end{aligned}$$

where $\mathbf{R}_{22}^{-1}-\left|\mathbf{R}_{12}^{-1}\right|^2/\mathbf{R}_{11}^{-1}=\det\mathbf{R}^{-1}/\mathbf{R}_{11}^{-1}=1/\mathbf{R}_{11}^{-1}\det\mathbf{R}$, and Integral (17) were used in (a). The expression for the covariance between Γ_{n_1} and Γ_{n_2} thus has the surprisingly simple form

$$\mathrm{E}\left[\Gamma_{n_1}\Gamma_{n_2}\right]-\overline{\Gamma}_{n_1}\overline{\Gamma}_{n_2}=\left|\mathbf{R}_{12}\right|^2=\frac{\left|\mathrm{E}\left[H_{n_1}H_{n_2}^*\right]\right|^2}{\sigma_{n_1}^2\sigma_{n_2}^2}. \tag{13}$$

B. DERIVATION OF THE MMSE ESTIMATOR

In order to find the MMSE estimate, $\widehat{\Gamma}_{n_3}^{\mathrm{MMSE}}=\mathrm{E}\left[\Gamma_{n_3}|\Gamma_{n_1},\Gamma_{n_2}\right]$, of the SNR of subcarrier n_3, Γ_{n_3} from known SNRs of two other subcarriers, Γ_{n_1} and Γ_{n_2}, it is necessary to analyze their joint statistics.

Similar to the approach used in Appendix A for deriving $f(\Gamma_{n_1},\Gamma_{n_2})$, the MMSE estimate, $\widehat{\Gamma}_{n_3}^{\mathrm{MMSE}}$, will be derived from the PDF of

$$\mathbf{h}\triangleq\left[\frac{H_{n_1}}{\sigma_{n_1}}\ \frac{H_{n_2}}{\sigma_{n_2}}\ \frac{H_{n_3}}{\sigma_{n_3}}\right]^T.$$

As in Appendix A, $\mathbf{h}$ is re-parameterized as $\mathbf{h}_i=\sqrt{\Gamma_{n_i}}e^{j\phi_i}$. The PDF, $f(\Gamma_{n_1},\Gamma_{n_2},\Gamma_{n_3},\phi_1,\phi_2,\phi_3)$ is given by a straight forward extension of (11) as

$$\begin{aligned}&f(\Gamma_{n_1},\Gamma_{n_2},\Gamma_{n_3},\phi_1,\phi_2,\phi_3)=\\&=\frac{1}{8\pi^3\det\mathbf{R}}e^{-\sum_{m=1}^3\Gamma_{n_m}\mathbf{R}_{mm}^{-1}-2\Re\left\{\sqrt{\Gamma_{n_1}\Gamma_{n_2}}\mathbf{R}_{12}^{-1}e^{j(\phi_1-\phi_2)}\right\}}\\&\quad e^{-2\Re\left\{\sqrt{\Gamma_{n_3}}\left(\sqrt{\Gamma_{n_1}}\mathbf{R}_{13}^{-1}e^{j(\phi_1-\phi_3)}+\sqrt{\Gamma_{n_2}}\mathbf{R}_{23}^{-1}e^{j(\phi_2-\phi_3)}\right)\right\}}\end{aligned}$$

where $\mathbf{R}=\mathrm{E}\left[\mathbf{h}\mathbf{h}^{\mathrm{H}}\right]\in\mathbb{C}^{3\times 3}$ is the covariance matrix of $\mathbf{h}$. By a change of phase variables,

$$(\phi_3,\widetilde{\phi},\Delta\phi)=(\phi_3,\phi_1-\phi_3,\phi_1-\phi_2),$$

it is possible to compute a closed form expression for the joint PDF, $f(\Gamma_{n_1},\Gamma_{n_2},\Gamma_{n_3},\Delta\phi)$, by integrating over ϕ_3 and $\widetilde{\phi}$,

$$\begin{aligned}&f(\Gamma_{n_1},\Gamma_{n_2},\Gamma_{n_3},\Delta\phi)=\int_0^{2\pi}\int_0^{2\pi}f(\Gamma_{n_1},\Gamma_{n_2},\Gamma_{n_3},\phi_3,\widetilde{\phi},\Delta\phi)\,\mathrm{d}\phi_3\,\mathrm{d}\widetilde{\phi}\\&=\frac{\int_0^{2\pi}\mathrm{d}\phi_3}{8\pi^3\det\mathbf{R}}e^{-\sum_{m=1}^3\Gamma_{n_m}\mathbf{R}_{mm}^{-1}-2\Re\left\{\sqrt{\Gamma_{n_1}\Gamma_{n_2}}\mathbf{R}_{12}^{-1}e^{j\Delta\phi}\right\}}\times\\&\quad\times\int_0^{2\pi}e^{-2\Re\left\{\sqrt{\Gamma_{n_3}}\left(\sqrt{\Gamma_{n_1}}\mathbf{R}_{13}^{-1}+\sqrt{\Gamma_{n_2}}\mathbf{R}_{23}^{-1}e^{-j\Delta\phi}\right)e^{j\widetilde{\phi}}\right\}}\mathrm{d}\widetilde{\phi}\\&=\frac{e^{-\Gamma_{n_1}\mathbf{R}_{11}^{-1}-\Gamma_{n_2}\mathbf{R}_{22}^{-1}}}{2\pi\det\mathbf{R}}e^{-\Gamma_{n_3}\mathbf{R}_{33}^{-1}-2\Re\left\{\sqrt{\Gamma_{n_1}\Gamma_{n_2}}\mathbf{R}_{12}^{-1}e^{j\Delta\phi}\right\}}\\&\quad\times\mathrm{I}_0\left(2\left|\sqrt{\Gamma_{n_1}}\mathbf{R}_{13}^{-1}+\sqrt{\Gamma_{n_2}}\mathbf{R}_{23}^{-1}e^{-j\Delta\phi}\right|\sqrt{\Gamma_{n_3}}\right),\end{aligned}$$

where Integral (16) derived in Appendix C was used in the last step. The argument of $I_0(\cdot)$ is computed using the law of cosines as

$$\left|\sqrt{\Gamma_{n_1}}\mathbf{R}_{13}^{-1}+\sqrt{\Gamma_{n_2}}\mathbf{R}_{23}^{-1}e^{-j\Delta\phi}\right|^2=\Gamma_{n_1}\left|\mathbf{R}_{13}^{-1}\right|^2+\Gamma_{n_2}\left|\mathbf{R}_{23}^{-1}\right|^2+ +2\sqrt{\Gamma_{n_1}\Gamma_{n_2}}\left|\mathbf{R}_{13}^{-1}\mathbf{R}_{23}^{-1}\right|\cos(\angle\mathbf{R}_{13}^{-1}-\angle\mathbf{R}_{23}^{-1}+\Delta\phi) \triangleq \mathbf{R}_{33}^{-1}\left[\alpha+\Re\left\{\beta e^{j\Delta\phi}\right\}\right],$$

where α and β are defined in (7). The conditional PDF is given by, $f(\Gamma_{n_3},\Delta\phi|\Gamma_{n_2},\Gamma_{n_3})\triangleq f(\Gamma_{n_1},\Gamma_{n_2},\Gamma_{n_3},\Delta\phi)/f(\Gamma_{n_1},\Gamma_{n_2})$, which if written out becomes

$$f(\Gamma_{n_3},\Delta\phi|\Gamma_{n_1},\Gamma_{n_2})=\frac{e^{-\widetilde{\alpha}}}{2\pi f(\Gamma_{n_1},\Gamma_{n_2})\det\mathbf{R}}e^{-\Re\left\{\widetilde{\beta}e^{j\Delta\phi}\right\}}\times \times e^{-\mathbf{R}_{33}^{-1}\Gamma_{n_3}}\mathrm{I}_0\left(2\sqrt{\Gamma_{n_3}\mathbf{R}_{33}^{-1}\left[\alpha+\Re\left\{\beta e^{j\Delta\phi}\right\}\right]}\right), \quad (14)$$

where

$$\widetilde{\alpha}\triangleq\Gamma_{n_1}\mathbf{R}_{11}^{-1}+\Gamma_{n_2}\mathbf{R}_{22}^{-1}, \qquad \widetilde{\beta}\triangleq 2\sqrt{\Gamma_{n_1}\Gamma_{n_2}}\mathbf{R}_{12}^{-1}. \quad (15)$$

Even though it is intractable to compute $f(\Gamma_{n_3}|\Gamma_{n_1},\Gamma_{n_2})$ due to the complicated dependence on $\Delta\phi$ it is possible to compute closed form expressions for $\mathrm{E}[\Gamma_{n_3}|\Gamma_{n_1},\Gamma_{n_2}]$, that is, the MMSE estimator of Γ_{n_3}. This is a fortunate consequence of the fact that the dependence on $\Delta\phi$ can be simplified by first integrating over Γ_{n_3}. Note that only the last factors in (14) depend on Γ_{n_3}, and the integration with respect to Γ_{n_3} can be performed in closed form.

$$\mathrm{E}[\Gamma_{n_3}|\Gamma_{n_1},\Gamma_{n_2}]=\int_0^{2\pi}\int_0^{\infty}\Gamma_{n_3}f(\Gamma_{n_3},\Delta\phi|\Gamma_{n_1},\Gamma_{n_2})\,\mathrm{d}\Gamma_{n_3}\,\mathrm{d}\Delta\phi =\frac{e^{-\widetilde{\alpha}}}{2\pi f(\Gamma_{n_1},\Gamma_{n_2})\det\mathbf{R}}\int_0^{2\pi}e^{-\Re\left\{\widetilde{\beta}e^{j\Delta\phi}\right\}}g(\Delta\phi)\,\mathrm{d}\Delta\phi,$$

where $g(\Delta\phi)$ is given by Integral (17) as

$$g(\phi)\triangleq\int_0^{\infty}\Gamma_{n_3}e^{-\mathbf{R}_{33}^{-1}\Gamma_{n_3}}\mathrm{I}_0\left(2\sqrt{\Gamma_{n_3}\mathbf{R}_{33}^{-1}\left[\alpha+\Re\left\{\beta e^{j\Delta\phi}\right\}\right]}\right)\mathrm{d}\Gamma_{n_3} =\frac{e^{\alpha}}{\left|\mathbf{R}_{33}^{-1}\right|^2}\left(1+\alpha+\Re\left\{\beta e^{j\phi}\right\}\right)e^{\Re\{\beta e^{j\phi}\}}.$$

The conditional expected value is thus given by

$$\mathrm{E}[\Gamma_{n_3}|\Gamma_{n_1},\Gamma_{n_2}]=\frac{e^{\alpha-\widetilde{\alpha}}}{2\pi\left(\mathbf{R}_{33}^{-1}\right)^2 f(\Gamma_{n_1},\Gamma_{n_2})\det\mathbf{R}}\times \left[(1+\alpha)\int_0^{2\pi}e^{\Re\left\{(\beta-\widetilde{\beta})e^{j\phi}\right\}}\mathrm{d}\phi+\Re\left\{\beta\int_0^{2\pi}e^{j\phi}e^{\Re\left\{(\beta-\widetilde{\beta})e^{j\phi}\right\}}\mathrm{d}\phi\right\}\right]$$

which can be evaluated in closed form as

$$\mathrm{E}[\Gamma_{n_3}|\Gamma_{n_1},\Gamma_{n_2}]=\frac{e^{\alpha-\widetilde{\alpha}}}{\left(\mathbf{R}_{33}^{-1}\right)^2\det\mathbf{R}\,f(\Gamma_{n_1},\Gamma_{n_2})}\times \times\left[(1+\alpha)\mathrm{I}_0\left(\left|\beta-\widetilde{\beta}\right|\right)+\Re\left\{\beta\frac{\beta^*-\widetilde{\beta}^*}{\left|\beta-\widetilde{\beta}\right|}\right\}\mathrm{I}_1\left(\left|\beta-\widetilde{\beta}\right|\right)\right]$$

where the integrals are given by (16) and α,β, $\widetilde{\alpha}$ and $\widetilde{\beta}$ are defined in (7) and (15), respectively. Let ${}_m\mathbf{R}$ denote the m:th principal submatrix of $\mathbf{R}$, i.e. the matrix formed by the first m rows and columns of $\mathbf{R}$. Hence ${}_2\mathbf{R}$ is the correlation matrix of the first two entries in $\mathbf{h}$, as defined in (10), and is the correlation matrix used in (12), where $f(\Gamma_{n_1},\Gamma_{n_2})$ is stated. This allows the expression for the MMSE estimate to be simplified by noting:

1. $\mathbf{R}_{33}^{-1}\det\mathbf{R}=\det{}_2\mathbf{R}$
2. $\alpha-\widetilde{\alpha}=-({}_2\mathbf{R})_{11}^{-1}\Gamma_{n_1}-({}_2\mathbf{R})_{22}^{-1}\Gamma_{n_2}$
3. $\beta-\widetilde{\beta}=-2\sqrt{\Gamma_{n_1}\Gamma_{n_2}}\,({}_2\mathbf{R})_{12}^{-1}$
4. Since $\angle({}_2\mathbf{R})_{12}^{-1}=\angle\mathbf{R}_{12}+\pi$ it follows that $\Re\{\beta e^{\angle(\beta^*-\widetilde{\beta}^*)}\}=|\beta|\cos(\angle\beta-\angle\mathbf{R}_{12})$.

This, combined with (12), results in the MMSE estimate in (5).

C. COMMON INTEGRALS

Two integrals frequently used in this work, found in well sorted tables of integrals, are evaluated as:

$$\int_0^{2\pi}e^{in\phi}e^{\alpha\Re\{h^*e^{i\phi}\}}\mathrm{d}\phi=\int_0^{2\pi}e^{in\phi}e^{\alpha|h|\Re\{e^{i(\phi-\angle h)}\}}\mathrm{d}\phi =e^{in\angle h}\int_0^{2\pi}e^{in(\phi-\angle h)}e^{\alpha|h|\cos(\phi-\angle h)}\mathrm{d}\phi =e^{in\angle h}\int_0^{2\pi}\cos(n\phi)e^{\alpha|h|\cos\phi}\mathrm{d}\phi =2\pi\frac{h^n}{|h|^n}\mathrm{I}_n(\alpha|h|), \quad (16)$$

where $n\in\mathbb{Z}$, $\alpha\in\mathbb{R}$ and $h\in\mathbb{C}$, and

$$\int_0^{\infty}xe^{-ax}\mathrm{I}_0\left(2b\sqrt{x}\right)\mathrm{d}x=\frac{1}{a^2}\left(1+\frac{b^2}{a}\right)e^{\frac{b^2}{a}}, \qquad a>0. \quad (17)$$

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