DESIGN OF MMSE FILTERBANK PRECODER AND EQUALIZER FOR MIMO FREQUENCY SELECTIVE CHANNELS

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ABSTRACT

In this paper, we consider the problem of designing minimum mean squared error (MMSE) filterbank precoder and equalizer for multiple input multiple output (MIMO) frequency selective channels. We derive the conditions to be satisfied by the optimal precoder-equalizer pair, and provide an iterative algorithm for solving them. The optimal design is very general, in that it is not constrained by channel dimensions, channel order, channel rank, or the input constellation. We also discuss some pertinent differences between the filterbank approach and the space-time approach to the design of optimal precoder and equalizer. Simulation results demonstrate that the proposed design performs better than the space-time systems while supporting a higher data rate.

1. INTRODUCTION

MIMO communication systems have been the focus of attention in the recent past due to the ever increasing demand for high data rates [1]. The design of a MIMO communication system throws up many signal processing challenges, such as achieving a high data rate and reliable equalization in the presence of multipath effects. One effective approach to addressing these issues is to linearly process the input signal to the MIMO channel using a precoder. A corresponding decoder/equalizer operates at the receiver. Precoders can be designed even in the absence of channel knowledge at the transmitter: a popular example being the MIMO-OFDM systems. However, there are many practical scenarios, like WLAN, in which the assumption of channel knowledge (either instantaneous or long-term) at the transmitter is reasonable [2][3]. In such cases, precoders can be designed to provide substantial performance gains.

Precoder designs for MIMO frequency-selective channels can be broadly classified into two categories: space-time precoding (STP) and filterbank precoding (FBP). In STP, a block of channel output vectors is processed at a time. Sufficient zero-padding is introduced between the blocks to avoid inter-block interference. In this framework, precoders designed to meet different criteria such as minimizing MSE or maximizing information rate etc., can be be shown to possess a common structure that involves eigenmode transmission and converts the frequency selective channel into a set of parallel flat fading channels [2][4].

In FBP, the MIMO frequency selective channel is viewed as a polynomial matrix. The precoder design task involves the construction of precoder and equalizer polynomial matrices to meet the performance criterion. The main advantage of FBP is that precoders that require nominal redundancy can be designed, as opposed to the ST precoders that require redundancy of at least the channel length. FBP is also less complex computationally than STP.

When the MIMO channel has more transmitters than receivers, the FB precoder can be designed to pre-equalize the channel completely, i.e, no equalizer is required at the receiver [5][3]. Similarly, when the channel is tall, FIR equalization can be achieved without precoding [6][7]. In [8], we derived the the expression for minimum redundancy required for the FB precoder to render an arbitrary FIR channel of any dimensions FIR invertible. It was also shown that the data rate can be increased by utilizing the block pseudocirculant representation of the channel matrix. But obtaining jointly optimal FB precoder and decoder for a general channel is a difficult task. In [10], an iterative procedure for the joint design of precoder and equalizer is developed, for binary input vectors, with the assumption that the channel is *communicable*.

In this paper, we derive the equations to be satisfied by the MMSE-optimal precoder-equalizer pair for a given channel. We develop an iterative algorithm to solve the equations. The result is general, in the sense that it holds for any channel dimension, channel rank, channel order, and input constellation.

While comparing our scheme with the STP approach, we point out the reason for the difficulty in obtaining closed form solution for the optimal precoder-equalizer in FBP. We also discuss some other salient differences between the FBP and STP approaches. The simulation results demonstrate the significant performance gains provided by our design over the STP approach.

2. PRECODER DESIGN PROBLEM

Consider the MIMO signal model given by

$$y(n) = \sum_{k=0}^{L_H-1} H(k)x(n-k) + v(n) \qquad (1)$$

where $x(n)$ is the N-length input vector, $y(n)$ is the M-length received signal vector, $H(k)$,$k = 0, 1, ..., L_H - 1$ is the time domain representation of the $M \times N$ frequency selective MIMO channel, and $v(n)$ is the M-length noise vector. In the absence of noise, we can write

$$\mathbf{y}(z) = \mathbf{H}(z)\mathbf{x}(z) \tag{2}$$

In [8][9], we showed that an $N \times K$ precoder $\mathbf{E}(z)$ that makes the *composite channel* $\mathbf{H}(z)\mathbf{E}(z)$ FIR invertible exists iff $K \leq r_m$, where r_m is the *minimum rank* of $\mathbf{H}(z)$. ($r_m = \min_z\{rank[\mathbf{H}(z)]\}$). It was seen that no redundancy is required in case of random rectangular channels (i.e, $K = N$), since random rectangular polynomial matrices are polynomially invertible with probability 1. For random square channels, it was seen that $r_m = M - 1$, and a redundancy of 1 symbol per channel use is required. The data rate (number of symbols per channel use) can be further increased by using the *pseudocirculant* channel representation [8][11]. In [11], the question of optimality of the precoder was not considered, and a zero-padding precoder was used. In the next section, we consider the problem of jointly optimizing (in the MMSE sense) the precoder/decoder pair.

3. OPTIMAL PRECODER DESIGN

Let $\mathbf{F}(z)$ denote the $K \times M$ equalizer for the composite channel $\mathbf{H}(z)\mathbf{E}(z)$. Then, for perfect reconstruction in the absence of noise, we require

$$\mathbf{F}(z)\mathbf{H}(z)\mathbf{E}(z) = z^{-d}I \tag{3}$$

[1]where d is the decoding delay. Alternatively, we can write

$$\mathsf{F}\mathcal{H}\mathcal{E} = \mathsf{J}_d \tag{4}$$

where

$$\mathsf{F} = [\ F(0) \quad F(1) \quad \cdots \quad F(L_F-1)\ ] \tag{5}$$

$\mathcal{H}$ is a block Sylvester matrix of the form

$$\mathcal{H} = \begin{bmatrix} H(0) & \ldots & H(L_H-1) & & \\ & \ddots & & \ddots & \\ & & H(0) & \ldots & H(L_H-1) \end{bmatrix} \tag{6}$$

and $\mathcal{J}_d = [\ o_{K\times Kd} \quad I_{K\times K} \quad 0_{K\times K(q+L_E-d-2)}\ ]$. $\mathcal{E}$ too has the same block Sylvester form as $\mathcal{H}$. F is $K \times ML_F$, $\mathcal{H}$ is $ML_F \times K(L_H + L_F - 1)$, $\mathcal{E}$ is $N(L_H + L_F - 1) \times K(L_H + L_F + L_E - 2)$, and J_d is $K \times K(L_H + L_F + L_E - 2)$. L_F is chosen to make the matrix $\mathcal{H}\mathcal{E}$ tall, i.e, $L_F \geq \frac{K(L_H+L_E-2)}{M-K}$. When the noise term is included, the system model becomes

$$\hat{x}(n) = \mathsf{F}\mathcal{H}\mathcal{E}\mathcal{X}(n) + \mathsf{F}\mathcal{V}(n) \tag{7}$$

where $\mathcal{X}(n)$ and $\mathcal{V}(n)$ are the blocked versions of $x(n)$ and $v(n)$, of dimensions $K(L_H + L_F + L_E - 2) \times 1$ and $ML_F \times 1$ respectively. We assume that the input symbols are white with unit variance. The covariance matrix of the error vector $e(n) = \widehat{x}(n) - x(n)$ can be written as

$$R_{ee} = (\mathsf{F}\mathcal{H}\mathcal{E} - \mathsf{J}_d)(\mathsf{F}\mathcal{H}\mathcal{E} - \mathsf{J}_d)^* + \mathsf{F}\mathcal{R}_{\mathcal{V}\mathcal{V}}\mathsf{F}^* \tag{8}$$

[1]The perfect reconstruction condition can be expressed more generally as $\mathbf{F}(z)\mathbf{H}(z)\mathbf{E}(z) = diag(z^{-d_0}, \ldots, z^{-d_{K-1}})$. In this paper, we use $\mathbf{F}(z)\mathbf{H}(z)\mathbf{E}(z) = z^{-d}I$ for the sake of simplified notation.

where $\mathcal{R}_{\mathcal{V}\mathcal{V}}$ is the covariance matrix of the zero mean circularly symmetric Gaussian noise.

Let p_0 indicate the power constraint on the precoder. Defining $\mathsf{E} = [\ E(0) \quad E(1) \quad \cdots \quad E(L_E-1)\ ]$, the power constraint can be written as $tr[\mathsf{E}\mathsf{E}^*] \leq p_0$. The MMSE cost function can now be written as

$$\begin{aligned}\Psi(\mathsf{F}, \mathcal{E}, \mu) &= tr[(\mathsf{F}\mathcal{H}\mathcal{E} - \mathsf{J}_d)(\mathsf{F}\mathcal{H}\mathcal{E} - \mathsf{J}_d)^* + \mathsf{F}\mathcal{R}_{\mathcal{V}\mathcal{V}}\mathsf{F}^*] \\ &\quad + \mu(tr[\mathsf{E}\mathsf{E}^*] - p_0)\end{aligned} \tag{9}$$

where μ is the Lagrange multiplier. Partially differentiating (9) w.r.t F and equating to zero, we get

$$\mathcal{H}\mathcal{E}\mathsf{J}_d^* = \mathcal{H}\mathcal{E}\mathcal{E}^*\mathcal{H}^*\mathsf{F}^* + \mathcal{R}_{\mathcal{V}\mathcal{V}}\mathsf{F}^* \tag{10}$$

Since $\mathcal{E}$ has the block-Sylvester structure, it is difficult to differentiate (9) w.r.t $\mathcal{E}$. We require $\mathbf{E}(z)$ to expressed in the row expanded form, like F. Towards this end, observe that the zero-forcing condition (3) implies

$$\mathbf{E}^*(z)\mathbf{H}^*(z)\mathbf{F}^*(z) = z^{-d}I \tag{11}$$

This can be rewritten as

$$\bar{\mathsf{E}}\bar{\mathcal{H}}\bar{\mathcal{F}} = \mathsf{J}_d \tag{12}$$

where $\bar{\mathsf{E}} = [\ E^*(0) \quad E^*(1) \quad \cdots \quad E^*(L_E-1)\ ]$ is the $K \times NL_E$ row expanded matrix. $\bar{\mathcal{H}}$ and $\bar{\mathcal{F}}$ are the block-Sylvester matrices corresponding to $\mathbf{H}^*(z)$ and $\mathbf{F}^*(z)$, of sizes $NL_E \times M(L_H + L_E - 1)$ and $M(L_H + L_E - 1) \times K(L_H + L_E + L_F - 2)$ respectively. The cost function (9) can now be written as

$$\begin{aligned}\Psi(\mathsf{E}, \mathcal{F}, \mu) &= tr[(\bar{\mathsf{E}}\bar{\mathcal{H}}\bar{\mathcal{F}} - \mathsf{J}_d)(\bar{\mathsf{E}}\bar{\mathcal{H}}\bar{\mathcal{F}} - \mathsf{J}_d)^* + \mathsf{F}\mathcal{R}_{\mathcal{V}\mathcal{V}}\mathsf{F}^*] \\ &\quad + \mu(tr[\bar{\mathsf{E}}\bar{\mathsf{E}}^*] - p_0)\end{aligned} \tag{13}$$

The partial differentiation of (13) w.r.t $\bar{\mathsf{E}}$ gives

$$\bar{\mathcal{H}}\bar{\mathcal{F}}\mathsf{J}_d^* = \bar{\mathcal{H}}\bar{\mathcal{F}}\bar{\mathcal{F}}^*\bar{\mathcal{H}}^*\bar{\mathsf{E}}^* + \mu\bar{\mathsf{E}}^* \tag{14}$$

From (10) and (14), we obtain

$$\mathsf{F} = \mathsf{J}_d\mathcal{E}^*\mathcal{H}^*(\mathcal{H}\mathcal{E}\mathcal{E}^*\mathcal{H}^* + \mathcal{R}_{\mathcal{V}\mathcal{V}})^{-1} \tag{15}$$

$$\bar{\mathsf{E}} = \mathsf{J}_d\bar{\mathcal{F}}^*\bar{\mathcal{H}}^*(\bar{\mathcal{H}}\bar{\mathcal{F}}\bar{\mathcal{F}}^*\bar{\mathcal{H}}^* + \mu I)^{-1} \tag{16}$$

An iterative solution to (15) and (16) can be obtained if a μ can be found that satisfies both (16) and the power constraint.

Finding μ

From (16), we can write

$$\bar{\mathsf{E}} = B^*(A + \mu I)^{-1} \tag{17}$$

where $A = \bar{\mathcal{H}}\bar{\mathcal{F}}\bar{\mathcal{F}}^*\bar{\mathcal{H}}^*$ and $B = \bar{\mathcal{H}}\bar{\mathcal{F}}\mathsf{J}_d^*$. Since A is positive semidefinite, we can denote its eigen decomposition by

$$A = \Xi\Lambda\Xi^* \tag{18}$$

where Ξ is unitary and the eigenvalues λ_i are greater than or equal to zero. Now,

$$\begin{aligned}\bar{\mathsf{E}}\bar{\mathsf{E}}^* &= B^*[(A + \mu I)^{-1}]^*(A + \mu I)^{-1}B \\ &= B^*MB\end{aligned} \tag{19}$$

where $M = [(A+\mu I)^{-1}]^*(A+\mu I)^{-1} = (A+\mu I)^{-2}$. Therefore

$$\begin{aligned} tr[\bar{\mathsf{E}}\bar{\mathsf{E}}^*] &= tr[B^*MB] \\ &= \sum_{i=0}^{K-1} b_i^* M b_i \end{aligned} \quad (20)$$

where b_i is the i^{th} column of B.

From (18), we have

$$(A+\mu I)^{-1} = \Xi(\Lambda+\mu I)^{-1}\Xi^* \quad (21)$$

and

$$M = \Xi(\Lambda+\mu I)^{-2}\Xi^* \quad (22)$$

Let $c_i = \Xi^* b_i$. Therefore, we have $b_i = \Xi c_i$. (20) can now be written as

$$\begin{aligned} tr[\bar{\mathsf{E}}\bar{\mathsf{E}}^*] &= \sum_{i=0}^{K-1} c_i^* \Xi^* M \Xi c_i \\ &= \sum_{i=0}^{K-1} c_i^* (\Lambda+\mu I)^{-2} c_i \end{aligned} \quad (23)$$

The input power constraint $tr[\bar{\mathsf{E}}\bar{\mathsf{E}}^*] - p_0 \leq 0$ can now be written as

$$\sum_{i=0}^{K-1}\sum_{j=0}^{K-1} \frac{c_{ij}^* c_{ij}}{(\lambda_j+\mu)^2} - p_0 \leq 0 \quad (24)$$

where c_{ij} is the j^{th}element of c_i. Therefore, to obtain μ, we need to solve

$$\sum_{i=0}^{K-1}\sum_{i=0}^{K-1} c_{ij}^* c_{ij} (\prod_{l\neq j}(\lambda_l+\mu)^2) - p_0(\prod_{i=0}^{K-1}(\lambda_i+\mu)^2) = 0 \quad (25)$$

Any real, positive root of (25) will give us the required μ. If no such root exists, then $\mu = 0$.

From (11) and (16), we can see that $\bar{\mathsf{E}}$ is the row expanded form of the MMSE inverse of $\mathbf{H}^*(z)\mathbf{F}^*(z)$. The squared norm $tr[\bar{\mathsf{E}}\bar{\mathsf{E}}^*]$ of the MMSE inverse depends on the noise variance. Comparing (15) and (16), we see that μI plays the role of the noise correlation matrix. Recall that the norm of the MMSE inverse is upper bounded by the norm of the ZF inverse. Putting $\mu = 0$ in (16) gives the ZF equalizer. As μ is increased, $tr[\bar{\mathsf{E}}\bar{\mathsf{E}}^*]$ decreases monotonically, as is evident from (24). Thus, when $\mathbf{E}^*(z)$ is viewed as the MMSE equalizer of $\mathbf{H}^*(z)\mathbf{F}^*(z)$, μ becomes the noise variance required to make $tr[\bar{\mathsf{E}}\bar{\mathsf{E}}^*] = p_0$. Also, since the MMSE cost function (13) decreases monotonically with increase in p_0, we are assured of finding a $\mu \geq 0$.

In practice, it is not necessary to perform the computationally expensive root finding routine to find μ. As already mentioned, $tr[\bar{\mathsf{E}}\bar{\mathsf{E}}^*]$ decreases monotonically with increasing μ. Thus, at every iteration, a simple gradient search algorithm can be used to find μ up to a desired accuracy.

4. OBSERVATIONS

1. The design of the optimal precoder and equalizer in STP also uses the block-Sylvester representation of the channel matrix. Even the MMSE cost function in STP is similar in form to the FBP cost function in (9). In STP, a closed form solution can be obtained for the optimal precoder-equalizer pair: the optimal precoder is made up of the significant eigenvectors of $\mathcal{H}^*\mathcal{R}_{\mathcal{V}\mathcal{V}}{}^{-1}\mathcal{H}$ [4][2]. But obtaining such a closed form solution in FBP is difficult, because
   1. $\mathcal{E}$ has the block-Sylvester structure in FBP, whereas it is a completely free variable in STP.
   2. From (8) and (15), the error covariance matrix for FBP can be written as $R_{ee} = \mathsf{J}_d(I + \mathcal{E}^*\mathcal{H}^*\mathcal{R}_{\mathcal{V}\mathcal{V}}{}^{-1}\mathcal{H}\mathcal{E})^{-1}\mathsf{J}_d^*$, i.e, R_{ee} is a $K \times K$ principal submatrix of $(I + \mathcal{E}^*\mathcal{H}^*\mathcal{R}_{\mathcal{V}\mathcal{V}}{}^{-1}\mathcal{H}\mathcal{E})^{-1}$. Thus, minimizing the trace of the error covariance matrix in FBP entails minimizing the sum of only K diagonal elements of $(I + \mathcal{E}^*\mathcal{H}^*\mathcal{R}_{\mathcal{V}\mathcal{V}}{}^{-1}\mathcal{H}\mathcal{E})^{-1}$, whereas the same criterion in STP leads to the minimization of the sum of *all* elements. Thus, in STP, the sum of diagonal elements can be replaced by the sum of eigenvalues, the minimization of which can be accomplished by eigenmode precoding. In FBP, the sum of K diagonal elements that needs to be minimized can only be lower bounded by the sum of the K least eigenvalues, using the majorization inequality.
2. From (15) and (16), we see that each iteration involves finding the optimal F given $\mathcal{E}$ and vice-versa. Therefore, the iterations can be shown to converge (to at least a local minima) in a manner similar to the proof of convergence of the Lloyd-Max algorithm for optimal quantizer design [12]. Fig. 1 shows the decrease in the mean squared error (MSE) as a function of the number of iterations for the case of 4×4 channels of length 5, with uniform power delay profile. The precoder was of size 4×3 with $L_E = 1$. The SNR was 10 dB. The result was averaged over 1000 randomly generated channels. As can be observed from the plot, substantial drop in MSE occurs within the first few iterations.
3. Even though both FBP and STP use the block-Sylvester representation of the channel matrix, to achieve the same data rate, the dimension of the block-Sylvester matrix is much higher in STP than in FBP. This is because, for an $M\times N$ channel of length L_H, STP requires a zero padding of NL_H symbols between successive blocks. For example, consider the case of 4×4 channels with 16 taps (as in the case of HIPERLAN-B channels with a sampling period of 50 ns). In the case of FBP, for a rate of 0.75, we need $L_F = 46$, so that $\mathcal{H}\mathcal{E}$ is 184×183. To achieve the same rate in STP, $\mathcal{H}$ becomes a 400×460 matrix, necessitating the eigen decomposition of a 460×460 matrix. Thus, in the STP approach, the cost of computation puts a limit to the achievable data rate.
4. In the FBP framework, both $\mathcal{H}\mathcal{E}$ and $\bar{\mathcal{H}}\bar{\mathcal{F}}$ have the block-Toeplitz structure. Therefore, in each iteration, (15) and (16) can be solved using Gohberg-

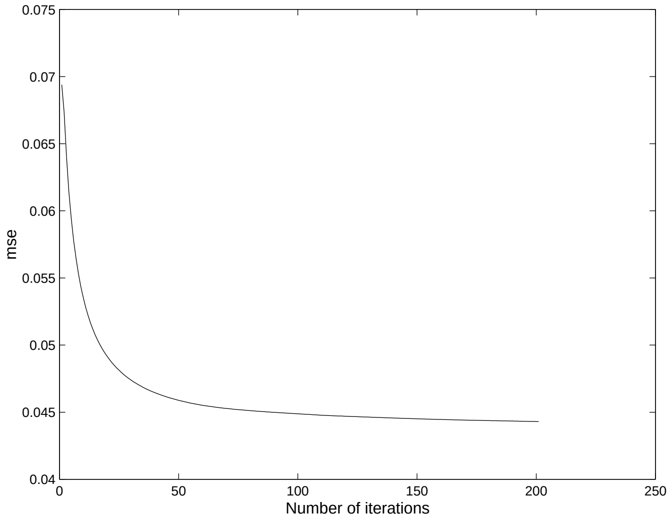

Figure 1: MSE as a function of the number of iterations for designing 4×3 precoders with $L_E = 1$, for 4×4 channels of length 5 with uniform power delay profile.

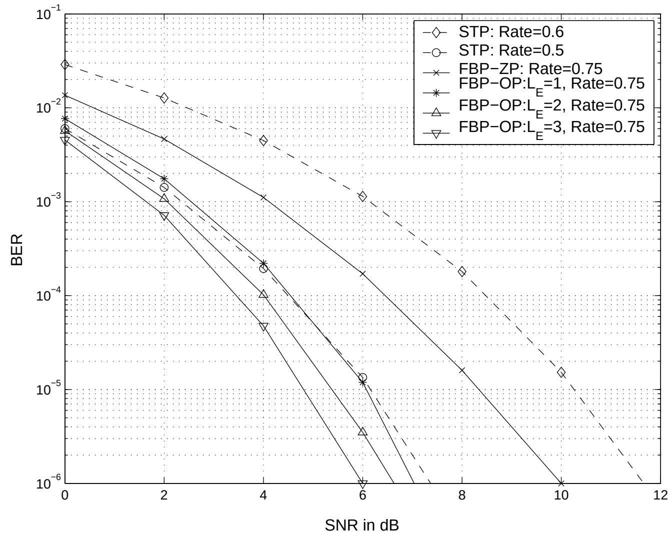

Figure 2: Comparative BER performance of STP and FBP.

Semencul type formulas, that require $\mathcal{O}(n \log^2 n)$ computations [13].

5. SIMULATION RESULTS

Simulations were carried out with 4×4 channel matrices with the IEEE HiperLAN-B power delay profile. The sampling period was 50 ns, resulting in channels of length 16. Each individual channel in the 4×4 channel matrix was generated independently. The results were averaged over 1000 random channels. The noise generated was spatially and temporally white. The input constellation used in the simulations was BPSK. The SNR was calculated as total transmit power divided by total noise variance, summed across all antennas.

As observed in [8], for channels with random coefficients, it is possible to achieve FIR equalization without using channel knowledge at the transmitter, by using a zero-padding precoder of the form $\begin{bmatrix} I_K \\ 0_{(N-K)\times K} \end{bmatrix}$,for example. In the following, we denote the zero-padding precoder by ZP, and the optimal precoder obtained by the iterative solution of (18) and (19) by OP.

Fig. 2 shows the BER performance of STP and FBP with precoders of different lengths. The ST precoders were based on [2, Lemma 1] and involved transmission along the eigenmodes of $\mathcal{H}^* \mathcal{R}_{\mathcal{V}\mathcal{V}}{}^{-1} \mathcal{H}$. All the FB precoders were of size 4×3, i.e, the date rate was 0.75. Each optimal precoder was obtained with 100 iterations. From the figure, we observe that FBP-ZP with a rate of 0.75 outperforms the STP scheme with rate=0.6. Thus, even in the absence of channel knowledge at transmitter, FBP provides better data rate and performance than STP. In addition, since FBP-ZP does not involve any iterative optimization, and requires just an one time evaluation of (15), it is much cheaper computationally than STP. To achieve high data rates, STP utilizes more and more weaker eigenmodes, resulting in a higher error rate. The optimal constant precoder, again with rate=0.75, performs almost on par with the STP scheme with rate=0.5. Precoders with lengths 2 and 3 provide further improvements in performance.

6. CONCLUSION

In this paper, we have provided an iterative algorithm for obtaining the MMSE precoder-equalizer pair for MIMO frequency selective channels. A comparison of our approach with the STP approach has been provided. Simulation results show that our approach provides considerable performance gains over the STP approach.

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