CONTROLLING PARTICLE FILTER REGULARIZATION FOR GPS/INS HYBRIDIZATION

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ABSTRACT

Coupling GPS with Inertial Navigation Systems (INS) is an interesting way of improving navigation performance in terms of accuracy and continuity of service. This coupling is generally performed by using GPS pseudorange measurements to estimate INS estimation errors and sensor biases. Particle filtering techniques are good candidates to solve the corresponding estimation problem due to the nonlinear measurement equation. However, classical particle filter algorithms tends to degenerate for this application because of the small state noise. Regularized particle filters allow to overcome this limitation at the expense of noisy state estimates. A recent regularized particle filter was proposed to control the regularization process by a Metropolis-Hasting step. The method was shown to increase particle filter robustness while decreasing the variance of the estimates. This paper goes further by introducing an appropriate criterion which measures the degeneracy of the particle cloud. This criterion is used to control the regularization which is not applied systematically reducing the algorithm computational cost. The main idea of the proposed strategy is to monitor on line the mean jumps of the predicted measurement likelihood by means of a CUSUM algorithm. Simulation results are proposed to validate the relevance of the criterion and the performance of the overall algorithm.

1. INTRODUCTION

Inertial Navigation Systems (INS) have become standard equipment on planes, ships or submarines due to their reliability and short-term accuracy. INS are based on accelerometers that directly measure the motion of the vehicle in a frame whose orientation is defined by a set of gyrometers. These measurements are integrated to yield the navigation solution, i.e. the position and velocity of the vehicle. This principle makes INS inherently robust to external perturbations but very sensitive to sensor inaccuracies. Indeed, small measurement errors result in unbounded position and velocity errors. External aiding is an efficient way of correcting INS drifts while taking advantage of inertial navigation accuracy. INS are classically coupled with GPS because of GPS availability and world-wide coverage. This paper considers a tight integration whereby GPS measurements are used to calibrate INS sensors and compensate for INS estimation errors.

Particle filters (PFs) are good candidates to solve the estimation problem associated to INS/GPS hybridization, because of the nonlinear measurement equation. PFs belong to the class of sequential Monte Carlo methods, which provide a set of powerful algorithms allowing to handle nonlinear and non Gaussian state space models. These algorithms approximate the posterior distribution of the unknown parameters by a swarm of weighted samples called particles. However, the instability of INS states combined with the small dynamic noise tend to make classical PFs diverge. Regularized Particle filters (RPFs), first introduced in [1], have proved efficient to prevent sample depletion. However, they are known to artificially increase the variance of the estimates. This study proposes some extensions to RPFs which overcome this limitation. First, an efficient degeneracy measurement is introduced to prevent systematic regularization. Second, a Metropolis-Hastings step allows to select relevant particles from the regularization process.

The paper is organized as follows. Section 2 briefly recalls the principles of GPS and INS navigation as well as the associated hybridization state space model. Section 3 introduces RPFs which allow to solve the navigation problem due to the INS/GPS state space model characteristics. The main limitations of RPFs are also outlined. Section 4 presents an improved RPF decomposed in two steps controlling regularization and preserving the particle distribution. Simulation results illustrating the interest of the proposed strategy are shown in Section 5. Conclusions are reported in Section 6.

2. GPS/INS INTEGRATION

GPS/INS integration is motivated by the complementary characteristics of the two systems: INS slow drifts are compensated by GPS long term accuracy whereas INS can coast during GPS outages. The coupling between GPS and INS is classically performed by means of an hybridization filter that fuses information from both navigation systems to compute the mobile dynamics. The preferred embodiment consists of processing GPS measurements to estimate slowly varying INS errors. The state model then describes INS error dynamic behavior while the observation equation relates GPS measurements to the components of the state vector.

2.1. State model

The idea of INS is to integrate acceleration signals to determine velocity and position in a desired frame of reference. A set of onboard sensors are used to achieve this goal:

- 3 accelerometers measure the non gravitational inertial acceleration along their axes,
- 3 gyrometers provide the angular velocity of the vehicle, hence the orientation of the sensor frame.

Denote $\boldsymbol{x}_t$ and $\boldsymbol{u}_t$ the vectors of the unknown motion parameters and the sensor outputs, respectively. These vectors are related through the following differential equation (1):

$$\dot{\boldsymbol{x}}_t = f(\boldsymbol{u}_t, \boldsymbol{x}_t), \tag{1}$$

which is solved online by the INS computer. The navigation solution is expressed in a convenient frame of reference on the basis of the gyrometer outputs. Due to the successive integrations, estimation errors due to sensor biases or misalignments grow unbounded. The equation describing the error dynamics is obtained by linearizing the differential equation (1) around the INS states as follows:

$$\begin{aligned}
\delta\dot{\boldsymbol{x}}_t &= \frac{\partial f}{\partial \boldsymbol{x}_t}(\boldsymbol{x}_{t,\text{ins}}, \boldsymbol{u}_{t,\text{ins}})\,\delta\boldsymbol{x}_t + \frac{\partial f}{\partial \boldsymbol{u}_t}(\boldsymbol{x}_{t,\text{ins}}, \boldsymbol{u}_{\text{ins}})\,\delta\boldsymbol{u}_t, \quad (2)\\
\delta\boldsymbol{x}_t &= \boldsymbol{x}_t - \boldsymbol{x}_{t,\text{ins}}, \quad (3)\\
\delta\boldsymbol{u}_t &= \boldsymbol{u}_t - \boldsymbol{u}_{t,\text{ins}}, \quad (4)
\end{aligned}$$

where the subscript "ins" refers to the quantities sensed or measured by the INS. The hybridization state vector is composed of the estimation and instrumentation errors, denoted as $\delta\boldsymbol{x}_t$ and $\delta\boldsymbol{u}_t$, respectively. A convenient model for INS sensor biases is required to make the state model complete. They are typically represented as first-order Gauss Markov models defined as:

$$\delta\dot{\boldsymbol{u}}_t = A\delta\boldsymbol{u}_t + \boldsymbol{v}_{u,t},$$

where $\boldsymbol{v}_u$ is a white Gaussian noise sequence and $\boldsymbol{A}$ is a diagonal matrix whose elements depends on the correlation time of the sensor biases [2, p. 81].

2.2. Measurement model

GPS navigation is based on distance measurements directly related to the unknown mobile position, i.e., to the INS positioning error. These measurements are obtained from radio-frequency satellite signals processed by an onboard receiver. They are called pseudoranges to account for various degradations ranging from atmospheric delays to non-synchronization of satellite and GPS receiver clocks. The following mathematical model holds for GPS measurements:

$$\boldsymbol{Y}_{t,i} = \boldsymbol{h}_{t,i}(\boldsymbol{x}) + b_t + \boldsymbol{w}_{t,i}, \quad i = 1, \ldots, n_s, \quad (5)$$

where n_s is the number of in-view satellites at time t, $\boldsymbol{w}_{t,i}$ is a white Gaussian noise sequence, b_t is the GPS clock offset, and

$$\boldsymbol{h}_{t,i}(\boldsymbol{x}) = \sqrt{(x_{t,i} - x_t)^2 + (y_{t,i} - y_t)^2 + (z_{t,i} - z_t)^2},$$

where the vectors $(x_{t,i}, y_{t,i}, z_{t,i})$ and (x_t, y_t, z_t) denote the positions of the vehicle and of the ith satellite at time t (expressed in rectangular coordinates). The dependance of the measurements on the state parameters can be made explicit by rewriting the mobile position as:

$$\begin{aligned}
x_t &= x_{t,\text{ins}} + \delta x_t,\\
y_t &= y_{t,\text{ins}} + \delta y_t,\\
z_t &= z_{t,\text{ins}} + \delta z_t,
\end{aligned}$$

where $(\delta x_t, \delta y_t, \delta z_t)$ is the vector of INS positioning errors. The GPS clock offset is considered as an additional unknown and is consequently appended to the hybridization state vector.

2.3. State model analysis

The overall state space model takes the form:

$$\begin{aligned}
\begin{pmatrix} \delta\dot{\boldsymbol{x}}_t \\ \delta\boldsymbol{u}_t \\ \boldsymbol{b}_t \end{pmatrix} &= \begin{pmatrix} \frac{\partial f}{\partial \boldsymbol{x}}_{|\boldsymbol{x}_{\text{ins}}} & \frac{\partial f}{\partial \boldsymbol{x}}_{|\boldsymbol{u}_{\text{ins}}} & [0] \\ [0] & A & [0] \\ [0] & [0] & B \end{pmatrix} \begin{pmatrix} \delta\boldsymbol{x} \\ \boldsymbol{u} \\ \boldsymbol{b}_t \end{pmatrix} + \begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{v}_u \\ \boldsymbol{v}_b \end{pmatrix},\\
\boldsymbol{Y}_t &= \boldsymbol{h}_t(\boldsymbol{x}_t) + b_t + \boldsymbol{w}_t,
\end{aligned}$$

where $\boldsymbol{b}_t = \left(b_t, \dot{b}_t\right)$. The elements of the state matrix are not detailed herein, for simplicity. However, they can be found in many textbooks dealing with inertial navigation such as [2, p. 152] for the receiver clock bias dynamic model and [2, p. 204] for the nominal INS error block. We denote the state vector $\boldsymbol{X}_t = (\delta\boldsymbol{x}_t, \delta\boldsymbol{u}_t, \boldsymbol{b}_t)$ afterwards. The characteristics of the discrete-time equivalent state space model [3] arise many difficulties, causing classical PF algorithms to be inefficient. First, INS positioning errors are in first approximation exponentially unstable. Second, the kinematic components of the dynamical system are nearly noise-free.

3. PARTICLE FILTERING TECHNIQUES

The Bayesian approach to solving estimation problems consists of computing the posterior distribution $p(\boldsymbol{X}_t|\boldsymbol{Y}_{1:t})$ of the unknown state vector $\boldsymbol{X}_t$ given the measurements $\boldsymbol{Y}_{1:t}$. As an alternative to the classical Kalman filter, sequential Monte Carlo methods (SMC) have gained increasing interest for nonlinear systems. The main idea of SMCs is to approximate the posterior distribution of interest by a set of weighted samples called particles as follows:

$$\widehat{p}(\boldsymbol{X}_t|\boldsymbol{Y}_{1:t}) = \sum_{i=1}^{N} w_t^{(i)} \delta\left(\boldsymbol{X}_t - \boldsymbol{X}_t^{(i)}\right).$$

The particle system evolves according to an importance-sampling rule and is made to interact through repeated resampling steps. More precisely, the particles are simulated sequentially according to a proposal distribution:

$$\boldsymbol{X}_t^{(i)} \sim q\left(\boldsymbol{X}_t|\boldsymbol{X}_{1:t-1}^{(i)}, \boldsymbol{Y}_{1:t}\right), \text{ for } i = 1, \ldots, N.$$

They are then assigned importance weights to correct for the discrepancy between the proposal and the target distribution:

$$\begin{aligned}
w_t &= \tilde{w}_t^{(i)} / \sum_{k=1}^{N} \tilde{w}_t^{(k)} \text{ where}, \quad (6)\\
\tilde{w}_t^{(i)} &= \frac{w_{t-1}^{(i)} p\left(\boldsymbol{Y}_t|\boldsymbol{X}_{1:t}^{(i)}, \boldsymbol{Y}_{1:t-1}\right) p\left(\boldsymbol{X}_t^{(i)}|\boldsymbol{X}_{1:t-1}^{(i)}\right)}{q\left(\boldsymbol{X}_t|\boldsymbol{X}_{1:t-1}, \boldsymbol{Y}_{1:t}\right)}. \quad (7)
\end{aligned}$$

According to (7), the most likely particles yield high importance weights. A selection step is finally introduced to prevent degeneracy. This selection is performed by resampling the set of particles according to the obtained approximation of the posterior distribution. Thus, low-weighted particles are discarded whereas the surviving particles are ensured to contribute efficiently to the estimation. However, systematic resampling is known to result in a loss of sample diversity. A measure of degeneracy, called effective sample size and denoted N_{eff}, has been introduced in **Liu** to decide whether resampling is useful or not:

$$N_{\text{eff}} = \frac{1}{\sum_{k=1}^{N}\left(w_t^{(k)}\right)^2}.$$

The selection procedure is carried out whenever N_{eff} is below a given threshold.

The earliest contribution in the field of particle filtering was the seminal paper of [4] which introduced "the bootstrap filter". This algorithm has appealing properties and is straightforward to implement due to a simple proposal distribution defined as the *a priori* dynamic model of the unknown parameters. However, the algorithm has shown deficiencies in some applications, including for instance the cases where the dynamic noise is small. This problem has received much attention in the literature. Improvements to the classical PF have been proposed including efficient sampling strategies and techniques introducing sample diversity.

A special care should be taken to design efficient PFs to cope with INS error estimation. Indeed, the system instability tend to make samples move away from each other until few particles are likely regarding the current measurement. Consequently, the resampling step results in an impoverishment of the particle system. Moreover, diversity cannot be reintroduced at the simulation step due to the small noise affecting the INS error states. The regions of the state space corresponding to high values of the *posterior* distribution $p\left(\boldsymbol{X}_t|\boldsymbol{Y}_{1:t}\right)$ are gradually depleted of particles and the PF fails to track properly the unknown parameters. Although more appropriate proposal distributions allow to slow down the degeneracy, they turn out to be inefficient to prevent it. The most promising solutions allowing to reduce degeneracy are probably the regularized particle filters (RPFs). This section ends with a brief presentation of these filters and a discussion of their shortcomings.

Regularization consists of resampling the particles according to a continuous approximation of the target distribution so that all the particles obtained have different locations. The smoothing is performed by convoluting the discrete PF approximation with a kernel whose properties are provided by the density estimation theory [5]. The resulting continuous approximation takes the following form:

$$\widehat{p}_c(\boldsymbol{X}_t|\boldsymbol{Y}_{1:t}) = K_h * \sum_{i=1}^{N} w_t^{(i)}\delta\left(\boldsymbol{X}_t - {\boldsymbol{X}_t}^{(i)}\right), \qquad (8)$$

$$= \sum_{i=1}^{N} w_t^{(i)} K_h(\boldsymbol{X}_t - {\boldsymbol{X}_t}^{(i)}). \qquad (9)$$

The rescaled kernel K_h appearing in the estimated posterior distribution (8) is defined as:

$$K_h = \frac{(\det S)^{-1/2\,n_x}}{h} K\left(\frac{1}{h}A^{-1}\boldsymbol{x}\right),$$

where h is the kernel bandwidth, n_x is the dimension of the state vector and $S = AA^T$ is the covariance matrix of the particle system. It is important to note that the convergence of $\widehat{p}_c(\boldsymbol{X}_t|\boldsymbol{Y}_{1:t})$ to $p(\boldsymbol{X}_t|\boldsymbol{Y}_{1:t})$ as the number of particles tends to infinity is ensured when the kernel density K satisfies the following conditions:

- K is a symmetric probability density function,
- $\int K(\boldsymbol{X})\mathrm{d}\boldsymbol{X} = 1$,
- $\int \|\boldsymbol{X}\|^2 K(\boldsymbol{X})\mathrm{d}\boldsymbol{X} < \infty$.

The parameter h and the kernel K are usually chosen to minimize the mean integrated square error between the target posterior pdf and the regularized approximation [5]. In the case of equally weighted samples, the optimal kernel is known to be the Epanechnikov Kernel. However, a Gaussian kernel can be used instead, by simplicity. This approach can be seen as introducing additional noise to compensate for the small dynamic noise that the model really exhibits. In this way, sample depletion is avoided at the cost of an increased variance of the estimates.

4. IMPROVED RPF

This study proposes two improvements to the classical RPF which yield tighter state estimates without impairing the algorithm stability. First, a detection criterion is introduced indicating whether regularization should be applied. Second, we propose to introduce a Metropolis-Hastings (MH) step to accept/reject the particles resulting from the regularization. This second step ensures that the simulated samples are distributed according to the target posterior distribution $p(\boldsymbol{X}_t|\boldsymbol{Y}_{1:t})$. The MH step was detailed in [3]. Consequently, this paper focuses on the detection criterion used before regularization.

4.1. Controlling Degeneracy

A way of mitigating regularization shortcomings consists of applying regularization only in cases where an abnormal behavior of the PF has been detected. A measure of degeneracy similar to the effective sample size may not be appropriate for that purpose. Indeed, N_{eff} is related to the variance of the particle set but does not provide any relevance regarding the current measurement. Instead, we propose to control regularization by means of a similarity measure between the predictive distribution $\widehat{p}\left(\boldsymbol{X}_t|\boldsymbol{Y}_{1:t-1}\right)$ and the likelihood function $p\left(\boldsymbol{Y}_t|\boldsymbol{X}_t, \boldsymbol{Y}_{1:t-1}\right)$. The choice of this criterion is motivated by the analysis of PF degeneracy conducted in section 3. The similarity measure used in this paper is:

$$L_t = \int p\left(\boldsymbol{Y}_t|\boldsymbol{X}_t, \boldsymbol{Y}_{1:t-1}\right)\widehat{p}\left(\boldsymbol{X}_t|\boldsymbol{Y}_{1:t-1}\right) d\boldsymbol{X}_t, \qquad (10)$$

where:

$$\widehat{p}\left(\boldsymbol{X}_t|\boldsymbol{Y}_{1:t-1}\right) = \sum_{i=1}^{N} w_{t|t-1}^{(i)}\delta\left(\boldsymbol{X}_t - \boldsymbol{X}_t^{(i)}\right), \qquad (11)$$

$$w_{t|t-1}^{(i)} \propto w_t^{(i)} \frac{p\left(\boldsymbol{X}_t|\boldsymbol{X}_{0:t-1}^{(i)}, \boldsymbol{Y}_{1:t-1}\right)}{q\left(\boldsymbol{X}_t|\boldsymbol{X}_{0:t-1}^{(i)}, \boldsymbol{Y}_{1:t-1}\right)}. \qquad (12)$$

Eqs. (10) and (11) yield the following expression for L_t:

$$L_t = \sum_{i=1}^{N} p\left(\boldsymbol{Y}_t|\boldsymbol{X}_t^{(i)}, \boldsymbol{Y}_{1:t-1}\right) w_{t|t-1}^{(i)}.$$

Note that in the case where the particles are propagated directly according to the state model, L_t reduces to the sum of the importance weights previous to normalization:

$$L_t = \sum_{i=1}^{N} \tilde{w}_t^{(i)},$$

which makes sense. The parameter L_t represents the overall relevance of the set of particles.

This paper argues that the process L_t undergoes a gradual mean value change when the predictive distribution and the likelihood function significantly differ. As a consequence, we propose an on-line detection of this drift to control regularization. The proposed test statistics, inspired by the CUSUM algorithm [6], is the difference between the current value of L_t and its estimated mean $\langle L_t \rangle$. This test statistic behaves as a random walk in the absence of mean value change and starts to grow significantly otherwise. An alarm is set when the test statistics exceeds an appropriate threshold. More precisely, a two-sided test is performed to deal with both increase and decrease of the mean value of L_t. The algorithm is summarized in Table (1). It requires to tune the parameters h and ν which have a strong impact on the probability of false alarm and the probability of non-detection. From a practical point of view, non detection is prejudicial to the PF performance and should be avoided, resulting in small values of h and ν.

Initialization: $t_0, T_0^1, T_0^2 = 0$.

tth iteration:

$$\begin{aligned}
\langle L_t \rangle &= \frac{1}{t-t_0} \sum_{k=t_0+1}^{t} L_k, \\
r_t &= L_t - \langle L_t \rangle, \\
T_t^1 &= \max\left(T_{t-1}^1 + r_t - \nu, 0\right), \\
T_t^2 &= \max\left(T_{t-1}^2 - r_t - \nu, 0\right),
\end{aligned}$$

If $(T_t^1 > h)$ ot $(T_t^2 > h)$,
- set the alarm,
- $T_t^1 = 0$ and $T_t^2 = 0$,
- $t_0 = t$.

Table 1. Two-sided CUSUM.

4.2. Metropolis-Hastings step

Regularization prevents sample impoverishment but results in noisy estimates. Contrary to classical PF, RPF particles are indeed no longer distributed according to the target distribution. A possible remedy consists of propagating only the relevant particles according to a Metropolis-Hastings scheme. As a consequence, the regularization kernel is considered as the proposal distribution of a MH algorithm whose invariant distribution is the posterior distribution $p\left(\boldsymbol{X}_{0:t}|\boldsymbol{Y}_{1:t}\right)$. Particle candidates are generated according to the regularization kernel

$$\boldsymbol{X}_t^{'(i)} \sim K_h\left(\boldsymbol{X}_t^{'(i)} - \boldsymbol{X}_t^{(i)}\right).$$

These candidates are accepted with the usual acceptance probability $\alpha_t = \min\left(1, r_t\right)$, where

$$r_t = \frac{p\left(\boldsymbol{X}_{0:t-1}^{(i)}, \boldsymbol{X}_t^{'(i)}|\boldsymbol{Y}_{1:t}\right) K_h\left(\boldsymbol{X}_t^{(i)} - \boldsymbol{X}_t^{'(i)}\right)}{p\left(\boldsymbol{X}_{0:t}^{(i)}|\boldsymbol{Y}_{1:t}\right) K_h\left(\boldsymbol{X}_t^{'(i)} - \boldsymbol{X}_t^{(i)}\right)}, \qquad (13)$$

otherwise $\boldsymbol{X}_t^{(i)}$ is left unchanged. The acceptance procedure ie repeated several times to improve convergence. However, few iterations are required since the particles already form a point-mass approximation of the posterior distribution. A closer analysis of the acceptance ratio yields:

$$r_t = \frac{p\left(\boldsymbol{X}_t^{'(i)}|\boldsymbol{X}_{t-1}^{(i)}\right) p\left(\boldsymbol{Y}_t|\boldsymbol{X}_t^{'(i)}\right) K_h\left(\boldsymbol{X}_t^{(i)} - \boldsymbol{X}_t^{'(i)}\right)}{p\left(\boldsymbol{X}_t^{(i)}|\boldsymbol{X}_{t-1}^{(i)}\right) p\left(\boldsymbol{Y}_t|\boldsymbol{X}_t^{(i)}\right) K_h\left(\boldsymbol{X}_t^{'(i)} - \boldsymbol{X}_t^{(i)}\right)}.$$

The probability for the particle $\boldsymbol{X}_t^{'(i)}$ to be an offspring of the parent particle $\boldsymbol{X}_{t-1}^{(i)}$ is negligible due to the small process noise. Consequently, $p\left(\boldsymbol{X}_t^{'(i)}|\boldsymbol{X}_{t-1}^{(i)}\right)$ is very low, leading to a low acceptance probability α_t. The renewal of the set of particles is therefore expected to be insufficient. A solution was proposed in a previous paper [3] that consists of moving a block of consecutive particles $\boldsymbol{X}_{t-L:t-1}^{'(i)}$ (with $L > 0$) originated from the candidate $\boldsymbol{X}_t^{'(i)}$. The algorithm then decide between $\boldsymbol{X}_{t-L:t-1}^{'(i)}$ and $\boldsymbol{X}_{t-L:t-1}^{(i)}$ by applying the MH rule.

5. SIMULATION RESULTS

The performance of the algorithm has been tested from several simulated data. To make the simulations realistic, the following INS platform has been implemented:

- computation of the IRS sensor outputs for a given vehicle trajectory depending on the class of sensors,
- on-line solution to the IRS navigation problem on the basis of these measurements.

A slowly maneuvring vehicle is studied, which makes the estimation problem more difficult. Indeed, such trajectories only allow a partial correction of INS errors. In particular, the angles defining the orientation of the mobile cannot be recovered properly. The associated GPS pseudoranges have been generated according to (5) from real GPS satellite orbital parameters. Low-cost IRS sensors have been considered, yielding important drift of IRS positioning estimates.

The good behavior of the proposed RPF is emphasized by comparing different PF strategies. The vertical INS channel is well known to be the most critical due to gravitational effects. Therefore, the analysis focuses on vertical INS errors. All tested PFs operate with 2000 particles. The following set of parameters has been used for the controlled RPF : $h = 10^{-10}, \nu = 10^{-11}$ and $L = 6$. Figures (1), (2), (3) show typical INS drifts and the corresponding estimates obtained respectively from a classical PF, the RPF and the improved RPF. Note that the standard PF has been implemented by using an approximation of the optimal proposal distribution in the sense that it minimizes the variance of the importance weights [7]. However, this algorithm loses track of the vertical IRS error whereas RPFs recover the correct trajectory. Although both RPFs track successfully the vehicle dynamics, the proposed approach clearly yield less noisy estimates.

To better evaluate the performance of the algorithms, the follow-

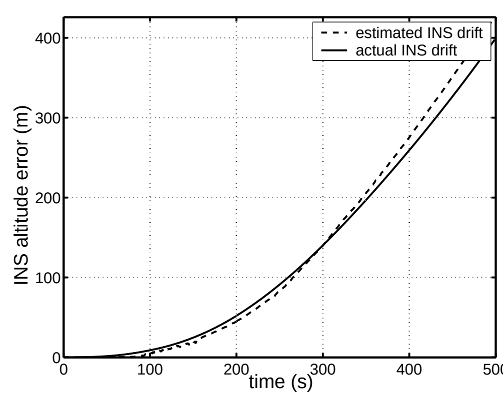

Fig. 1. INS vertical drift estimation-standard PF.

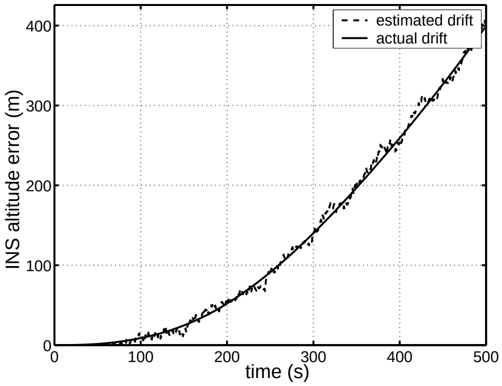

Fig. 2. INS vertical drift estimation-standard RPF.

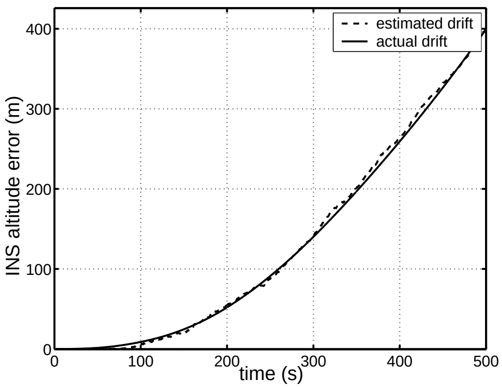

Fig. 3. INS vertical drift estimation-improved RPF.

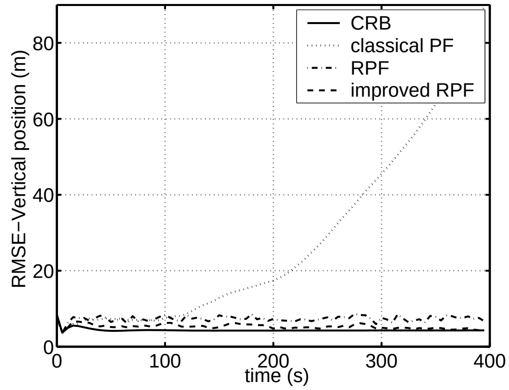

Fig. 4. RMSE-vertical position.

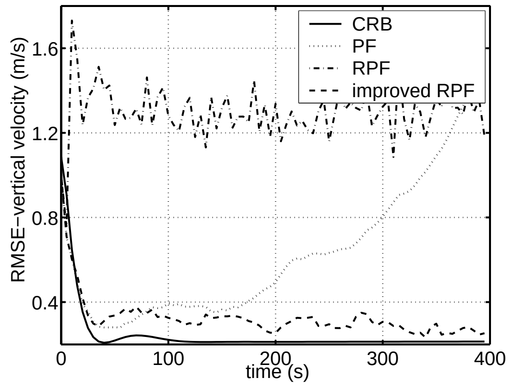

Fig. 5. RMSE-vertical velocity.

ing results have been averaged from 100 Monte Carlo runs. The root mean square estimation error (RMSE) for the jth component of the state vector X_t can then be computed as follows:

$$\sqrt{\mathrm{E}\left(\left(\boldsymbol{X}_t\left[j\right]-\widehat{\boldsymbol{X}}_t\left[j\right]\right)^2\right)} \simeq \sqrt{\frac{1}{M}\sum_{k=1}^{M}\left(\boldsymbol{X}_t\left[j\right]-\widehat{\boldsymbol{X}}_t^k\left[j\right]\right)^2},$$

where $\widehat{\boldsymbol{X}}_t^k$ stands for the kth run estimate. The obtained RMSE are compared to the lower limit provided by the Posterior Cramer Rao Bound, which is the Bayesian version of the classical Cramer Rao Bound. Theoretical background as well as recursive formula to compute the PCRB can be found for instance in [8]. The results are presented on figures (4) and (5). The proposed approach enhances estimation accuracy, especially for the velocity parameters. The estimation error of the standard PF grows unbounded due to degeneracy, contrary to RPFs demonstrating good convergence properties. However, the improved RPF achieves smaller RMSEs than all other filters by decreasing the variance of the estimates.

6. CONCLUSION

INS cannot be used as sole-means of navigation due to their inherent instability. They are therefore integrated with ancillary sensors such as GPS by means of an hybridization filter. Among different existing PF strategies studied, the RPF was selected as the most robust algorithm regarding a small variance of the state noise. However, this approach is known to artificially increase the variance of the estimates. This paper proposed two extensions to benefit from RPF stability while improving the estimation accuracy. Thus, regularization was controlled by means of a MH step which guaranteed that the set of particles was distributed according to the target distribution. In addition, an efficient criterion was introduced to decide whether to regularize. The performance of the resulting algorithm was illustrated through simulation results.

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