14th European Signal Processing Conference (EUSIPCO 2006), Florence, Italy, September 4-8, 2006, copyright by EURASIP

TIME-OF-ARRIVAL ESTIMATION UNDER IMPULSIVE NOISE FOR WIRELESS POSITIONING SYSTEMS

İsa Hacıoğlu, F. Kerem Harmancı, Emin Anarım, Hakan Deliç

Department of Electrical and Electronics Engineering
Boğaziçi University, Bebek 34342 Istanbul, Turkey
{isa.hacioglu,harmanci,anarim,delic@boun.edu.tr}

ABSTRACT

Forward-Backward modified Fractional Lower Order Moment-MUSIC (FB-FLOM-MUSIC), a high resolution spectral estimation algorithm, is proposed for time of arrival (TOA) estimation under a non-Gaussian noise model that accurately represents the impulsive outliers in indoor wireless channels. FB-FLOM-MUSIC is designed for the α-stable noise model, and the first peak of its pseudospectrum is assumed to give the TOA estimate. Simulation results indicate that FLOM-MUSIC clearly outperforms MUSIC in impulsive noise, and that FB-FLOM-MUSIC provides reduced estimation variance at the expense of a slight loss of peakfinding success in moderately dispersed and highly impulsive noise.

1. INTRODUCTION

The popularity of positioning services has been boosted by the recent developments in wireless communication technology. Location awareness has a wide range of application areas both in indoor and outdoor environments. Cellular phone localization in 911 calls, event-localization such as forest-fire detection, earthquake detection, target tracking, etc., patient/child-positioning are some examples of location-based applications. Although the Global Positioning System (GPS) has been quite popular, it does not accurately estimate the target position in environments whose propagation characteristics are not taken into account, e.g., indoor, urban or undersea.

Time-of-arrival (TOA) estimation is used in range-based localization by obtaining the distance between the transmitter and receiver from the flying time of the signal. Once the distances of an object to at least three reference points are measured, its position is calculated by trilateration. TOA estimation has attracted interest for years and it is employed in positioning systems in various wireless telecommunications settings such as GSM, GPS, etc.

One TOA estimation approach which is still in use consists of applying the inverse Fourier transform (IFT) on frequency-domain measurement data [1]. Some researchers have focused on the maximum likelihood (ML) optimization to estimate the propagation delay, which they achieve by maximizing the correlation function of the received signal in [2, 3]. In [4], superresolution methods such as MUltiple SIgnal Classification (MUSIC), which are based on the covariance matrix estimates, are used to estimate the TOA. Although ML estimators have the edge in performance, superresolution techniques seem more advantageous when it comes to computational complexity. In [4], it is shown that MUSIC can be used to estimate the TOA of signals that suffer from multipath in the indoor radio channel and it performs better than traditional IFT and direct-sequence spread spectrum signal-based cross-correlation techniques.

The common assumption of the aforementioned studies for TOA, as well as the signal processing applications in general is that noise is Gaussian. This assumption is justified by the central limit theorem and is very attractive because the Gaussian probability density function is mathematically tractable and the resulting algorithms are computationally less complex. On the other hand, underwater acoustic signals, low-frequency atmospheric noise and the interference emanating from many man-made devices in an office environment are of impulsive nature, and they need to be modeled with distributions that possess heavier tails than the Gaussian [5]. Impulsive noise can be modeled through the α-stable family of distributions, whose appropriateness is theoretically justified by the generalized central limit theorem [5, 6]. Severe performance degradation is inevitable for Gaussian-optimal systems when the acting noise is impulsive.

Tsakalides and Nikias [7] introduce a “covariation matrix” based on the fractional-lower order moments (FLOMs) to replace the covariance matrix of the conventional MUSIC, which is based on the second-order-statistics (SOS-MUSIC). In robust covariation-based (ROC)-MUSIC, both noise and signal components are modeled as complex symmetric alpha-stable (SαS) processes. Liu and Mendel [8] introduce the FLOM-based matrices that can be used with MUSIC applied to angle-of-arrival estimation for finite-variance circular signals under SαS noise (the FLOM-MUSIC method). While FLOM-MUSIC outperforms ROC-MUSIC for signal constellations such as phase modulation (PM) and quadriphase shift-keying (QPSK), ROC-MUSIC is still preferable in cases such as binary phase shift-keying (BPSK) [8]. Altınkaya et al. [9], on the other hand, employ the generalized covariation coefficient, a fractional lower order statistic to replace the correlation matrix.

The objective of this paper is to apply the forward-backward (FB) modified FLOM-MUSIC technique to the frequency response of an indoor communication channel to estimate the TOA in the presence of complex symmetric α-stable noise, and to compare its performance with FLOM-MUSIC and the more conventional SOS-MUSIC and FB-SOS-MUSIC.

2. SIGNAL AND NOISE MODEL

Multipath fading results in the reception of a superposition of multiple delayed and scaled versions of the transmitted signal, leading to phase and amplitude distortion.

Although many objects in the physical environment may not be stationary, the channel variation due to their motion is relatively slow compared to the signal rate in the wireless communication system. In this paper, the channel is considered as locally time-invariant [10]. The low-pass equivalent of such a frequency-selective indoor radio channel's impulse response is given by

$$h(t)=\sum_{k=0}^{P-1} b_k\delta(t-\tau_k) \tag{1}$$

where P is the number of multipath components, $b_k = |b_k|e^{j\theta_k}$ and τ_k are the complex attenuation and propagation delay of the kth path, respectively. In an indoor channel with or without the presence of line-of-sight (LOS), it is assumed that τ_0 represents the TOA. The channel gain b_k is complex Gaussian; that is, $|b_k|$ has the Rayleigh distribution and θ_k is uniformly distributed on $[0,2\pi)$. The kth multipath signal component undergoes a time delay of τ_k.

The received signal $y(t)$ includes the convolution of the channel $h(t)$ and the transmitted signal $s(t)$, together with the additive noise $n(t)$, i.e.,

$$y(t)=\int_{-\infty}^{\infty} s(\tau)h(t-\tau)d\tau+n(t). \tag{2}$$

The additive noise term $n(t)$ in (2) is assumed to be white, complex isotropic SαS.

A real zero-mean SαS random variable X is characterized by two parameters: α, the characteristic exponent and γ, the scale/dispersion parameter. The characteristic exponent, for which $0<\alpha\le 2$, parameterizes the thickness of the tail of the distribution. If $\alpha=2$, the distribution is Gaussian. As α decreases, the tail becomes thicker and the frequency of occurrence of large-amplitude noise realizations increases. The dispersion γ plays a role that is equivalent to variance. In fact, when $\alpha=2$, the dispersion γ equals half the variance.

For a complex isotropic $S\alpha S$ random variable $Z=X_1+jX_2$, where X_1 and X_2 are jointly SαS, the characteristic function is $\phi(\omega)=\exp(j(\mathrm{Re}(\omega Z^*)))=\exp(-\gamma|\omega|^{\alpha})$, for which $\omega=\omega_1+j\omega_2$ and '*' represents the complex conjugate.

By taking the Fourier transform of $h(t)$ in (1), the frequency domain channel response becomes

$$H(f)=\sum_{k=0}^{P-1} b_k e^{-j2\pi f\tau_k}. \tag{3}$$

If we exchange the roles of time and frequency in (3), we obtain a harmonic signal model as in [4]. Most time-domain spectral estimation techniques are also applicable to the frequency response of the multipath indoor radio channel. The sampled frequency response has M coefficients at M equally spaced frequencies. Assigning Δf as the frequency increment, the mth sample is

$$H(m\Delta f)=\sum_{k=0}^{P-1} b_k e^{-j2\pi(f_0+m\Delta f),\tau_k} \tag{4}$$

where $m=0,1,\ldots,M-1$. If the channel response is low-pass equivalent, then $f_0=0$ as it is assumed in this work.

The observed channel response at sampled discrete frequencies includes environmental noise, modeled as a complex isotropic SαS distributed random process. Therefore $x(m)=H(m\Delta f)+n(m)$, i.e.,

$$x(m)=\sum_{k=0}^{P-1} b_k e^{-j2\pi m(\Delta f)\tau_k}+n(m). \tag{5}$$

This can also be written in vector form as

$$\begin{aligned}\mathbf{x} &= \mathbf{H}+\mathbf{n} \\ &= \mathbf{A}\mathbf{b}+\mathbf{n}.\end{aligned} \tag{6}$$

where

$$\begin{array}{rcllll}
\mathbf{x} &=& [x(0) & x(1) & \cdots & x(M-1)]^T \\
\mathbf{H} &=& [H(0) & H(\Delta f) & \cdots & H((M-1)\Delta f)]^T \\
\mathbf{n} &=& [n(0) & n(1) & \cdots & n(M-1)]^T \\
\mathbf{A} &=& [\mathbf{a}(\tau_0) & \mathbf{a}(\tau_1) & \cdots & \mathbf{a}(\tau_{P-1})] \\
\mathbf{a}(\tau_k) &=& [1 & e^{-j2\pi\Delta f\tau_k} & \ldots & e^{-j2\pi(M-1)\Delta f\tau_k}]^T \\
\mathbf{b} &=& [b_0 & b_1 & \cdots & b_{P-1}]^T
\end{array}$$

where T denotes the transpose.

Under the infinite variance model, which is a consequence of the SαS model, MUSIC cannot be directly applied using the second order statistics of a covariance matrix. Instead, FLOMs are used to form a covariation matrix, the FLOM-based covariation matrix [8]. The basic assumptions of in the derivation of the proposed TOA estimation algorithm are as follows:

- A1: In $b_k=|b_k|e^{j\theta_k}$, the amplitude $|b_k|$ and the phase θ_k are independent and identically distributed (iid) statistically independent real random variables, with the former Rayleigh distributed while the latter uniformly distributed in $[0,2\pi)$.
- A2: $n(m)$ is the sequence of zero-mean iid complex isotropic SαS distributed variables with $1<\alpha\le 2$.
- A3: $\mathbf{A}$ has full rank.

Note from A1 and A2 that $\mathbf{x}$ given by (6) is zero-mean.

The problem addressed under this model is that of the estimation of the TOA of the first arrival in the impulse response of the channel assuming that this impulse corresponds to the LOS path.

3. TOA ESTIMATION IN IMPULSIVE NOISE

In this section, MUSIC-based TOA estimation under SαS noise is implemented by utilizing the FLOM-based covariation matrix instead of the correlation matrix, which is composed of second order moments. The covariation matrix $\mathbf{C}$ is defined as

$$\mathbf{C}=E[\mathbf{x}(|\mathbf{x}|^{p-2}\odot\mathbf{x})^{\dagger}] \tag{7}$$

where $p<\alpha$, '$\odot$' represents elementwise product and '$\dagger$' represents conjugate transpose [8]. The covariation coefficients λ_{XY}, whose estimation leads to the covariation matrix of FLOM-MUSIC, can be determined through FLOMs as $\lambda_{XY}=E[XY^{<p-1>}]/E[|Y|^p]$ for any $1\le p<\alpha$ where $Y^{<p-1>}=|Y|^{p-1}\mathrm{sgn(Y)}$.

Based on the signal model in the preceding section, it can be shown that

$$\mathbf{C}=\mathbf{A}\mathbf{S}\mathbf{A}^{\dagger}+\gamma\mathbf{I} \tag{8}$$

with $\mathbf{A}$ as in (6), and $\mathbf{S}$ a positive real diagonal matrix [8]. The values of the diagonal of $\mathbf{S}$ and γ are complicated functions of the fractional lower order statistics of $\mathbf{x}$ as in [8] with the following properties:

- Its elements are finite since $p < \alpha$ except when $\alpha < 1$.
- It is Hermitian symmetric and the elements of its diagonal are identical.
- $\mathbf{S}$ is nonsingular.

Based on the covariation matrix $\mathbf{C}$, the FLOM-MUSIC algorithm is implemented as follows:

1. Build a frequency domain data matrix from N snapshots of received data. The nth snapshot $\mathbf{x}_n$ is a vector modelled as in (6) and it has length M. Compose a data matrix $\mathbf{X} = [\mathbf{x}_0 \cdots \mathbf{x}_{N-1}]$ is $M \times N$ where each column corresponds to a snapshot.(In practice, snapshot n of data is obtained by measuring, at time $t = nT_s$, a set of simultaneously emitted narrowband pulses at frequencies $f_0 + m\Delta f$, $m = 1, \cdots, M$. Then, the mth component of the nth snapshot $[\mathbf{x}_n]_m$ corresponds to the measurement at the frequency $f_0 + m\Delta f$. Note that T_s should be larger than $1/(\Delta f)$.)
2. Obtain the $M \times M$ FLOM-based covariation matrix, $\mathbf{C}$ which replaces the correlation matrix in MUSIC. The covariation matrix estimate is given by $\hat{\mathbf{C}} = (1/N)\mathbf{X}(|\mathbf{X}|^{p-2} \odot \mathbf{X})^\dagger$.
3. Using singular value decomposition (SVD), perform the eigenanalysis of $\mathbf{C}$,

$$\mathbf{C} = \mathbf{P}\mathbf{D}\mathbf{P}^\dagger \tag{9}$$

where $\mathbf{D}$ is an $M \times M$ diagonal matrix, $\mathbf{P}$ is the $M \times M$ matrix whose mth column is an eigenvector whose corresponding eigenvalue is the mth element of the diagonal of $\mathbf{P}$.

The number of frequency samples M must be chosen to be greater than the number of paths P for the last $M - P$ eigenvalues to correspond to noise only. MUSIC splits the space of $\mathbf{x}$ into two subspaces: a signal subspace which is spanned by the eigenvectors of $\mathbf{C}$ corresponding to the largest P eigenvalues, and a noise subspace spanned by the remaining $M - P$ eigenvectors. Let

$$\mathbf{K}_s = [\mathbf{e}_0\ \mathbf{e}_1 \cdots \mathbf{e}_{P-1}] \tag{10}$$

be the matrix whose P columns correspond to signal subspace eigenvectors $\mathbf{e}_0, \ldots, \mathbf{e}_{P-1}$, and

$$\mathbf{K}_n = [\mathbf{e}_P\ \mathbf{e}_{P+1} \cdots \mathbf{e}_{N-1}] \tag{11}$$

be the matrix whose $N - P$ columns correspond to the noise subspace eigenvectors. Note that $\mathbf{P} = [\mathbf{K}_s\ \mathbf{K}_n]$. The multipath delays τ_k, $k = 0, \ldots, P-1$ are then obtained from the peaks of the FLOM-MUSIC pseudospectrum

$$S_{\text{FLOM-MUSIC}}(\tau) = \frac{1}{\mathbf{a}(\tau)^\dagger \mathbf{K}_n \mathbf{K}_n^\dagger \mathbf{a}(\tau)}, \tag{12}$$

where $\mathbf{a}(\tau)$ is defined in (6). The first (leftmost) peak of this pseudospectrum is considered to be the TOA estimate.

The covariation matrix $\hat{\mathbf{C}}$ estimated from (9) does not satisfy the Toeplitz requirement. There are two variations on MUSIC that take into account the structural properties of the covariation matrix [11]: The first and simplest, the so-called general improvement method simply ensures the Hermitian symmetry of the covariation matrix estimate:

$$\hat{\mathbf{C}}_{\text{G}} = \frac{1}{2}(\hat{\mathbf{C}} + \hat{\mathbf{C}}^\dagger). \tag{13}$$

The second one sets an additional the persymmetry structure to this matrix estimate:

$$\hat{\mathbf{C}}_{\text{FB}} = \frac{1}{2}(\hat{\mathbf{C}} + \mathbf{J}\hat{\mathbf{C}}^\dagger \mathbf{J}) \tag{14}$$

where $\mathbf{J}$ is the anti-diagonal $M \times M$ exchange matrix. This is called FLOM-MUSIC with forward-backward (FB) improvement [11], which is applicable to both the classical covariance and the new covariation matrix estimates and is expected to enhance performance by increasing the SNR of the correlation matrix.

4. SIMULATION RESULTS

The simulations in this work compare the TOA estimation performance of FLOM-MUSIC to SOS-MUSIC (second order statistics-MUSIC) under SαS noise. Both FLOM-MUSIC and SOS-MUSIC are implemented with and without their FB matrix modification. For a given α, complex isotropic SαS noise is generated according to [6] and [12].

The observation data vector $\mathbf{x}$ represents the channel frequency response sampled uniformly over a frequency band as in (3). For indoor multipath channels, it can be assumed that the maximum delay $\tau_{\max}$ is less than 500 ns [13]. Therefore, the frequency sampling interval Δf is chosen to be $1/2\tau_{\max} =$ 1 MHz. The bandwidth is chosen as 100 MHz and this results in a snapshot size of 200 points. Note that the complex low-pass equivalent of the transfer function is simulated.

Two performance criteria are considered: (1) success rate which is the rate at which the first measured arriving peak falls in the interval $[\tau_0 - \Delta\tau/2, \tau_0 + \Delta\tau/2]$; (2) the mean-absolute error (MAE) of the first measured arriving path, which is taken as the TOA estimate.

Since noise is SαS, the signal-to-noise ratio (SNR) parameter, which requires finite noise variance, is invalid. Therefore a generalized signal to noise ratio (GSNR) is defined as [8]

$$\text{GNSR} = 10\log_{10}\frac{E[|h(t)|^2]}{\sigma^\alpha} \tag{15}$$

where $\sigma = 2(\gamma^{1/\alpha})$.

The resulting pseudospectra of our TOA estimation algorithm have peaks around the delays of the indoor radio channel paths if the estimation is successful. The locations of those peaks are registered as estimates of path delays.

In the simulations, the channel contains two pulses at 100 ns and 200 ns corresponding to a two-path indoor radio channel. The parameters of stable noise are assumed known considering that there are several parameter estimation methods for stable random processes [5]. In each graph, 4000 realizations are used to generate the data.

In Figures 1 and 2, the effect of noise dispersion on the algorithm is studied by varying the GSNR while keeping the characteristic exponent α and the fractional order p of the moment fixed at 1.5 and 0.75, respectively. With this α, noise is moderately impulsive. FLOM-MUSIC clearly outperforms SOS-MUSIC for GSNR ≥ 7 dB with or without FB. While the general FLOM-MUSIC (without FB) outperforms FB-FLOM-MUSIC marginally in terms of the success rate, the latter seems to have better MAE performance for GSNR ≤ 19 dB. This can be interpreted as general FLOM-MUSIC having very slightly better peak-finding success than its FB version, while FB provides reduced estimation variance until GSNR = 19 dB. For GSNR $\geq$ 20 dB, the FB improvement does not yield any tangible performance gain. A similar relationship between the use and non-use of FB can be observed in the SOS-MUSIC case.

In Figures 3 and 4, the same analysis is repeated, but this time with $\alpha = 1.8$ and $p = 0.9$. This corresponds to a scenario where noise is slightly impulsive. While general FLOM-MUSIC's success rate is higher up to GSNR = 15-16 dB, especially with FB improvement, the conventional SOS-MUSIC takes over above 16 dB, and it has also the lowest variance for GSNR $\geq$ 20 dB. The FB modification in FLOM-MUSIC provides less MAE only below 10-dB GSNR.

The more the noise distribution deviates from the Gaussian, the harder it becomes for the SOS-based TOA estimators to catch up with the FLOM-MUSIC performance. When $\alpha = 1.5$, a minimum GSNR of 30 dB is required for the SOS-MUSIC success rate to approach that of FLOM-MUSIC whereas GNSR = 15 dB is sufficient for $\alpha = 1.8$.

In Figures 5 and 6, α is varied for a fixed GSNR (20 dB) and the fractional order moment p is systematically chosen to be half of α. Therefore, the impact of the frequency of outliers in noise on the algorithm's performance is considered. The lower the value of α, the more frequent the impulsive components. The moment order p is chosen to provide near-maximal FLOM performance for each α. For $\alpha < 1.8$, the general FLOM-MUSIC has the highest success rate, and with FB is also has uniformly lower MAE than the general SOS-MUSIC. The noise distribution starts becoming Gaussian-like when $\alpha \geq 1.8$, in which case the SOS-MUSIC begins to overtake the FLOM-based estimators.

5. CONCLUSION

FLOM-MUSIC and its modified FB version are proposed for the TOA estimation of the first arriving path of a radio channel whose impulsive noise is modelled as complex SαS. The effects of GSNR, the characteristic exponent α, and the fractional lower order moment p on the performances of the estimators are studied. The use of FLOM-MUSIC is clearly advantageous for scenarios with moderate-to-high impulsiveness ($\alpha < 1.8$), and the FB modification to FLOM-MUSIC provides even lower estimation variance at the expense of a little drop in the success rate for moderate GSNR and low α.

This work is supported under contract number 06A207 by the Boğaziçi University Research Fund.

REFERENCES

[1] H. Zaghloul, G. Morrison, and M. Fattouche, “Frequency response and path loss measurements of indoor channel,” *IEE Electronic Letters*, vol. 27, pp. 1021–1022, June 1991.

[2] X. Li, K. Pahlavan, and J. Beneat, “Performance of TOA estimation techniques in indoor multipath channels,” in *Proceedings of the 13th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, vol. 2, pp. 911–915, September 2002.

[3] G. Carter, “Time delay estimation,” *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. 29, pp. 461–462, June 1981.

[4] X. Li and K. Pahlavan, “Super-resolution TOA estimation with diversity for indoor geolocation,” *IEEE Transactions on Wireless Communications*, vol. 3, pp. 224–234, January 2004.

[5] C. L. Nikias and M. Shao, *Signal Processing with Alpha-Stable Distributions and Applications*. Wiley, 1995.

[6] G. Samarodnitsky and M. S. Taqqu, *Stable Non-Gaussian Random Processes*. Chapman & Hall, 1994.

[7] P. Tsakalides and C. L. Nikias, “The robust covariation-based MUSIC (ROC-MUSIC) algorithm for bearing estimation in impulsive environments,” *IEEE Transactions on Signal Processing*, vol. 44, pp. 1623–1633, July 1996.

[8] T. H. Liu and J. M. Mendel, “A subspace-based direction finding algorithm using fractional lower order statistics,” *IEEE Transactions on Signal Processing*, vol. 49, pp. 1605–1613, August 2001.

[9] M. A. Altınkaya, H. Deliç, B. Sankur, and E. Anarım, “Subspace-based frequency estimation of sinusoidal signals in alpha-stable noise,” *Signal Processing*, vol. 82, pp. 1807–1827, December 2002.

[10] A. A. M. Saleh and R. A. Valenzuela, “A statistical model for indoor multipath propagation,” *IEEE Journal on Selected Areas in Communications*, vol. 5, pp. 128–137, February 1987.

[11] P. Stoica and R. Moses, *Introduction to Spectral Analysis*. Prentice-Hall, 1997.

[12] P. Tsakalides, *Array Signal Processing with Alpha-Stable Distributions*. PhD thesis, University of Southern California, 1995.

[13] K. Pahlavan and P. Krishnamurthy, *Principles of Wireless Networks: A Unified Approach*. Prentice-Hall, 2002.

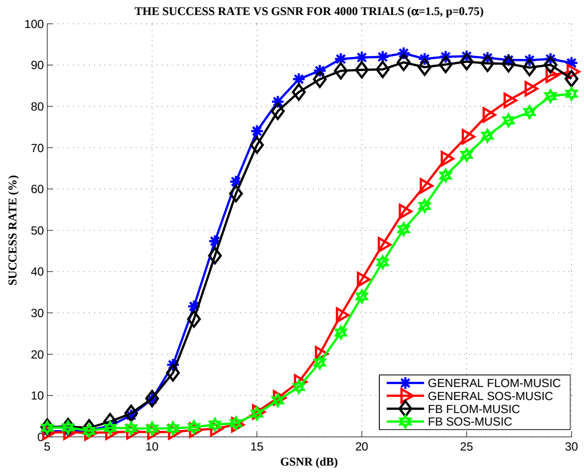

Figure 1: Effect of GSNR on the success-rate for moderately impulsive noise ($\alpha = 1.5$, $p = 0.75$, 4000 realizations).

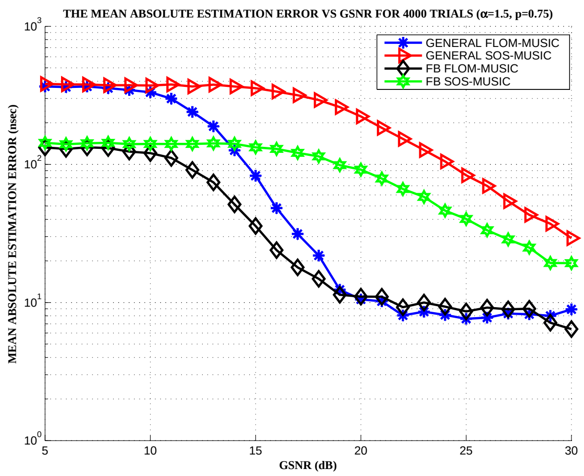

Figure 2: Effect of GSNR on the mean-absolute error for moderately impulsive noise. ($\alpha = 1.5, p = 0.75$, 4000 realizations).

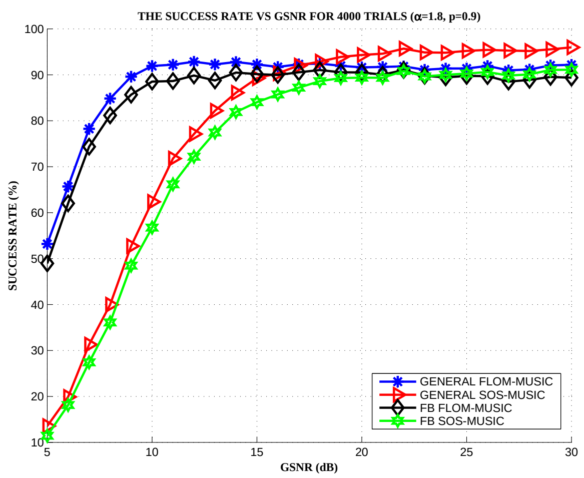

Figure 3: Effect of GSNR on the success-rate for slightly impulsive noise ($\alpha = 1.8$, $p = 0.9$, 4000 realizations).

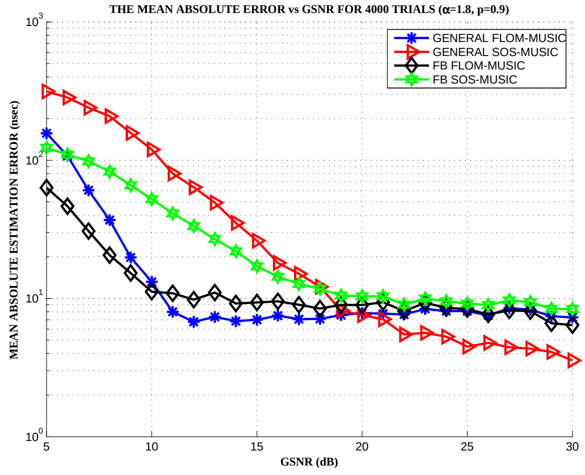

Figure 4: Effect of GSNR on the mean-absolute error for slightly impulsive noise ($\alpha = 1.8$, $p = 0.9$, 4000 realizations).

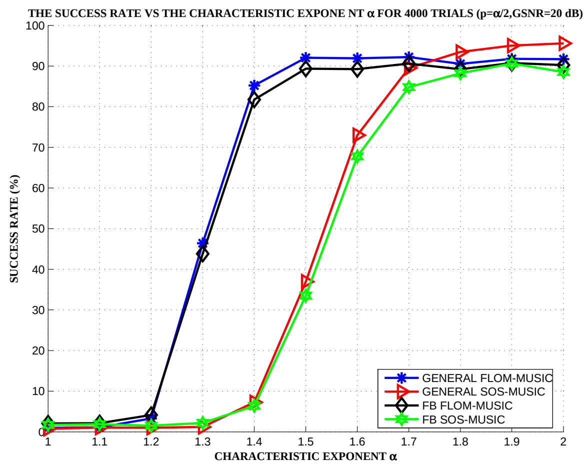

Figure 5: Effect of α on the success-rate for 20 dB GSNR ($p = \alpha/2$, 4000 realizations). The lower the α-level, the higher the frequency of impulsive components.

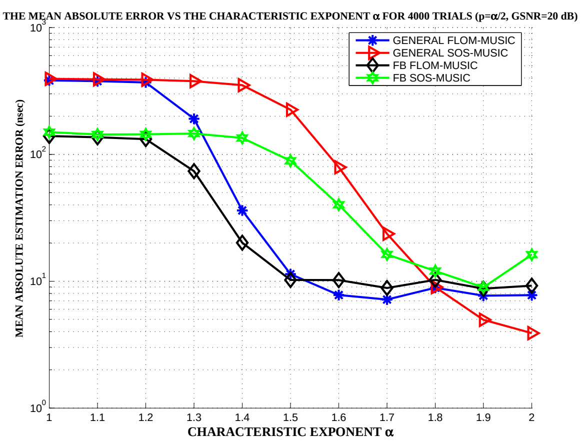

Figure 6: Effect of α on the mean-absolute error for 20 dB GSNR ($p = \alpha/2$, 4000 realizations). The lower the α-level, the higher the frequency of impulsive components.