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# EFFICIENT IMAGE REGISTRATION WITH SUBPIXEL ACCURACY

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## ABSTRACT

The contribution of this paper is twofold. First, a new spatial domain image registration technique with subpixel accuracy is presented. This technique is based on a double maximization of the correlation coefficient and provides a closed-form solution to the subpixel translation estimation problem. Second, an efficient iterative scheme for integer registration is proposed, which reduces significantly the number of searches, as compared to the exhaustive search. This scheme can be used as a pre-processing step in the sub-pixel accuracy technique, leading to lower computational complexity. Extensive simulation results have shown that the performance of the proposed technique compares very favorably with respect to existing ones.

## 1. INTRODUCTION

Many image processing applications require image registration in order to estimate the correspondence between two or more images. The image registration techniques proposed in literature can be classified in feature-based and intensity-based [1, 2]. Intensity-based techniques are more computationally demanding, but avoid the sensitivity of the feature extraction stage. Intensity-based registration can be achieved minimizing the compared images' squared error, maximizing their correlation [3], their mutual information [4] or their phase-correlation [5], e.t.c. Some methods provide pixel-level registration, which may be satisfactory for some applications, but other applications require subpixel accuracy. Such an accuracy is usually provided by methods based on interpolation [6] and depends highly on the interpolation algorithms' quality. Other approaches are based on the differential properties of the image sequences [6], or formulate the subpixel registration as an optimization problem [7].

A Fourier-based algorithm for image registration with subpixel accuracy is presented in [8], where the image differences are restricted to translations and uniform changes of illumination. The algorithm detects and removes the frequency components that might cause errors in the shift estimation due to aliasing. In [9], it is shown that the signal power in the phase correlation corresponds to the polyphase transform of a filtered unit impulse centered at the point of registration. Recently, a frequency domain technique has been proposed for the registration of aliased images, based on their low-frequency, aliasing-free (or marginally affected by aliasing) part [10]. Frequency domain is suitable for describing and handling aliasing and is also suitable for global motion models. On the other hand, spatial domain methods generally lend themselves for more general motion models. In [11], an iterative scheme based on Taylor expansion is presented and a pyramidal scheme is used to increase the precision for large motion parameters. The authors of [12] use sparsely sampled regional correlation, providing accuracy better than 0.2 pixels. In [13], an error function linear in the model parameters is minimized using least-squares.

The technique proposed here has been motivated by the approach suggested recently in [14, 15], where an enhanced correlation-based method for stereo correspondence is presented. Based on [14, 15], we present here a new spatial domain technique, suitable for subpixel image registration, which aims to the maximization of the correlation coefficient. In contrast to the interpolation based techniques, the proposed one does not require the reconstruction of the intensity values and provides a closed-form solution. Extensive simulation results have shown that the performance of the proposed technique compares very favorably with respect to existing ones. The optimum subpixel translation is found with a slight increase in the computational cost as compared to the integer registration problem, but, if the compared images are first registered up to a pixel level, this cost is significantly reduced. Commonly, the pixel-level registration is obtained by maximizing the correlation coefficient for $N^2$ integer translations, where $N$ is the number of searches in each dimension. We propose here a new efficient iterative scheme, which reduces the number of searches to $2aN$, where $a$ is the number of iterations, which, in most cases, is much smaller than $N/2$. This scheme converges fast to the correct solution if the initial translation estimation lies in the main lobe of the correlation coefficient function (i.e., the lobe around the global maximum). Otherwise, proper re-initializations of the algorithm might be needed and the number of searches becomes $2N \cdot \sum_{i=1}^{k} a_i$, where $k$ is the total number of the algorithm initializations and $a_i$ the number of algorithm's iterations for each initialization $i$. Note that $\sum_{i=1}^{k} a_i$ also remains much smaller than $N/2$, in most cases.

In Section 2 the problem is formulated and the proposed subpixel image registration technique is described. In Section 3 the efficient iterative scheme for pixel-level registration is presented. Experimental results are provided in Section 4 and in Section 5 the work is concluded.

## 2. SUBPIXEL IMAGE REGISTRATION

### 2.1 Problem Formulation

Let $f$ be a reference image and $f_t$ a translated version of $f$. Let also $w$ be a window in $f$, with dimensions $n \times n$ and $g$ a search area in $f_t$, with dimensions $m \times m$ (where $m > n$). All the possible positions of window $w$ in the search area $g$ take values in the set

$$\mathscr{I} = [0, N-1] \times [0, N-1], \quad N = m - n + 1. \tag{1}$$

Let now $s(x,y)$ be a window of the search area $g$ that has the same size with $w$ and $(x,y)$ denotes the coordinates of its upper left corner. Then, image registration is equivalent to searching

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for a $(x_0, y_0) \in \mathscr{I}$ such that the following relation holds

$$s(x_0, y_0) = w, \quad (x_0, y_0) \in \mathscr{I}. \tag{2}$$

The correlation coefficient of windows $w$ and $s(x,y)$ is used as similarity measure, which has the advantage of being invariant to linear photometric distortions, and is defined as

$$C(x,y) = \bar{\mathbf{w}}^T \bar{\mathbf{s}}(x,y) \tag{3}$$

where $\bar{\mathbf{w}} = \mathbf{w}/||\mathbf{w}||$ and $\bar{\mathbf{s}} = \mathbf{s}/||\mathbf{s}||$ stand for the zero mean, Euclidean normalized versions of vectors $\mathbf{w} = vec\,(w - m_w)$ and $\mathbf{s}(x,y) = vec\left[s(x,y) - m_{s(x,j)}\right]$, with $m_w$ and $m_{s(x,y)}$ denoting the mean values of windows $w$ and $s(x,y)$, respectively, and operator *vec* stacking each window's columns in a column vector of length $n^2$.

The translation $(x_0, y_0)$ that maximizes (3) is found by computing the correlation coefficient for all the possible positions of $w$ in $g$. In fact, the following maximization problem has to be solved

$$\max_{(x,y) \in \mathscr{I}} C(x,y). \tag{4}$$

Solving this problem, we can register images with pixel accuracy, but since in many applications subpixel accuracy is required, we extend (in the next subsection) in the two dimensions' case the similarity measure proposed in [14, 15] and formulate an appropriate maximization problem in order to obtain the desired sub-pixel accuracy.

### 2.2 A New Measure for Subpixel Accuracy

For computing translations with subpixel accuracy, the correlation coefficient in (3) has to be redefined as follows

$$C(x+t_x, y+t_y) = \bar{\mathbf{w}}^T \bar{\mathbf{s}}(x+t_x, y+t_y) \tag{5}$$

where $t_x$ and $t_y$ are continuous variables, which stand for the subpixel translations. The corresponding maximization problem is

$$\max_{(x,y) \in \mathscr{I}} \max_{(t_x,t_y)} C(x+t_x, y+t_y) \tag{6}$$

which involves a maximization with respect to the integer translation $(x,y)$ and a maximization related to the subpixel translation $(t_x, t_y)$.

The form of the above correlation coefficient and the computational cost of the maximization problem depend on the interpolation used for the windows reconstruction. Moreover, if the maximization of (6) with respect to $(t_x, t_y)$ does not have a closed form solution, a maximization algorithm is needed, increasing significantly the computational cost. Otherwise, the cost will increase only slightly as compared to that of the integer translation estimation problem. Note that changing the order of maximizations in (6) and sampling the continuous variables, the total registration problem can be solved at the expense of increased cost and bounded accuracy. In order to avoid these problems, for a given integer translation, we incorporate in (5) the following first order interpolation kernel

$$\begin{aligned}\mathbf{s}(x+t_x, y+t_y) &\approx \mathbf{s}(x,y) + t_x\left[\mathbf{s}(x,y) - \mathbf{s}(x-1,y)\right] \\ &+ t_y\left[\mathbf{s}(x,y) - \mathbf{s}(x,y-1)\right] = \mathbf{s} + t_x \Delta\mathbf{s}_{1,0} + t_y \Delta\mathbf{s}_{0,1}\end{aligned} \tag{7}$$

which is, in fact, the first order Taylor approximation of the translated window $\mathbf{s}(x+t_x, y+t_y)$. The vectors $\mathbf{s}(x-1,y)$ and $\mathbf{s}(x,y-1)$ correspond to the one-pixel translated versions of window $s(x,y)$ on axes $x$ and $y$, respectively, and $\Delta\mathbf{s}_{1,0} = \mathbf{s}(x,y) - \mathbf{s}(x-1,y)$, $\Delta\mathbf{s}_{0,1} = \mathbf{s}(x,y) - \mathbf{s}(x,y-1)$ are approximations of the spatial derivatives in directions $x$, $y$.

Let us now define the following quantities

$$\begin{aligned}&\rho_{k,\,l} = \bar{\mathbf{w}}^t \bar{\mathbf{s}}(x-k, y-l), \;\; \lambda_{k,\,l} = \frac{||\mathbf{s}(x-k,y-l)||}{||\mathbf{s}(x,y)||} \\ &r_{k,\,l} = \bar{\mathbf{s}}^t(x-k,l)\bar{\mathbf{s}}(x,y-l)\end{aligned} \tag{8}$$

where $\rho_{k,\,l}$ corresponds to the correlation coefficient of window $w$ with window $s(x,y)$ (for $k = l = 0$) or its translated versions $s(x-k, y-l)$, $r_{k,\,l}$ is similarly defined, and $\lambda_{k,\,l}$ is the norm of $\mathbf{s}(x-k,y-l)$ to the norm of $\mathbf{s}(x,y)$ ratio. Let us also define the following auxiliary quantities

$$\begin{aligned}&a_0 = \rho_{0,0} \quad a_1 = \rho_{0,0} - \rho_{1,0}\lambda_{1,0} \quad\; a_2 = \rho_{0,0} - \rho_{0,1}\lambda_{0,1} \\ &b_0 = 1 \qquad b_1 = 2\,(1 - r_{1,0}\lambda_{1,0}) \quad b_2 = 2\,(1 - r_{0,1}\lambda_{0,1}) \\ &b_3 = 2\,(1 - r_{1,0}\lambda_{1,0} - r_{0,1}\lambda_{0,1} + r_{1,1}\lambda_{1,0}\lambda_{0,1}) \\ &b_4 = 1 + \lambda_{1,0}^2 - 2r_{1,0}\lambda_{1,0} \quad b_5 = 1 + \lambda_{0,1}^2 - 2r_{0,1}\lambda_{0,1}.\end{aligned} \tag{9}$$

Incorporating (7) in (5), the correlation coefficient becomes a function of the continuous translation parameters $t_x$ and $t_y$. Then, for a given integer translation $(x,y)$, we have to find the solution $(t_x^{'}, t_y^{'})$ that maximizes the following function

$$C(x+t_x, y+t_y) = \frac{a_0 + a_1 t_x + a_2 t_y}{\sqrt{b_0 + b_1 t_x + b_2 t_y + b_3 t_x t_y + b_4 t_x^2 + b_5 t_y^2}} \tag{10}$$

which implies that for a given $(x_0, y_0)$ of $(x,y) \in \mathscr{I}$, we have to solve the following maximization problem

$$\max_{(t_x,t_y)} C(x_0 + t_x, y_0 + t_y). \tag{11}$$

The maximization of (10) results in a closed form solution given in the next theorem.

***Theorem 1**: Let $(x_0, y_0) \in \mathscr{I}$ be given, the correlation coefficient be defined as in equation (10) and the denominator of (10) be different from zero. Then, $C(x_0 + t_x, y_0 + t_y)$ attains its unique extremum for*

$$t_x^{'} = \frac{(a_2 b_1 - a_1 b_2)(a_1 b_2 - a_0 b_3) + (2a_1 b_0 - a_0 b_1)(2a_1 b_5 - a_2 b_3)}{(a_1 b_3 - 2a_2 b_4)(a_1 b_2 - a_0 b_3) - (a_1 b_1 - 2a_0 b_4)(2a_1 b_5 - a_2 b_3)} \tag{12}$$

*and*

$$t_y^{'} = \frac{(a_2 b_1 - a_1 b_2)(2a_0 b_4 - a_1 b_1) + (2a_1 b_0 - a_0 b_1)(2a_2 b_4 - a_1 b_3)}{(a_1 b_3 - 2a_2 b_4)(a_1 b_2 - a_0 b_3) - (a_1 b_1 - 2a_0 b_4)(2a_1 b_5 - a_2 b_3)}. \tag{13}$$

*The extremum is a maximum, if and only if the Hessian matrix of $C(x_0 + t_x, y_0 + t_y)$ evaluated at $(t_x^{'}, t_y^{'})$ is negative definite.*

Thus, using the above theorem, which is given without proof due to space limitations, we can find the optimum subpixel translation with a slight increase in the computational cost. However, if the compared images are first registered up to a pixel level, that cost can be significantly reduced. In order to achieve low cost pixel-level registration, we propose in the next section a new efficient iterative algorithm.

## 3. EFFICIENT ITERATIVE SCHEME FOR PIXEL-LEVEL REGISTRATION

The computationally intensive part of a registration process is the evaluation of the involved similarity measure for different relative image positions. Due to the computational cost of spatial domain convolution, several fast, but approximate, spatial domain matching methods have been developed [16]. Since the correlation coefficient cannot be computed via a simple and efficient frequency domain procedure [17], we propose in this section the use of an efficient spatial-domain iterative algorithm for image registration with pixel accuracy.

Such a registration can be obtained by restricting the correlation coefficient maximization to integer translations. The number of the required searches is $N^2$, where $N$ is the number of searches in each dimension, and the aim of the proposed algorithm is to reduce this number by searching in columns and rows successively. Following this searching strategy, the number of searches is reduced to $2aN$, where $a$ is the number of iterations, which is, in most cases, much smaller than $N/2$. In the next paragraph we give the outline of the proposed algorithm.

### 3.1 Algorithm Description

Let $f$, $f_t$, $w$, $g$ and $s$ be as defined in Section 2.1. The proposed iterative algorithm for pixel level registration is as follows.

*function [Newf,Newf$_t$] = iterative_ registration (reference_image, translated_ image, threshold)*
*f = reference_ image, $f_t$ = translated_ image, $C_{ff_t}$ = threshold* -1
*while* $C_{ff_t} < threshold$
  *Determine w and g*
  *for* $i = 0 : N-1$
    $C(i,0) = corr2\,[w, s(i,0)]$
  *end*
  $x = find[C(x,0) == max(C(x,0))]$
(∗) *Register f, $f_t$ according to x → Intermediate images $f'$ and $f_t'$*
  *Determine $w'$ and $g'$*
  *for* $j = 0 : N-1$
    $C(0,j) = corr2\left[w', s'(0,j)\right]$
  *end*
  $y = find\,[C(0,y) == max\,(C(0,y))]$
  *Register $f'$, $f_t'$ according to y → New images Newf, Newf$_t$*
  $C_{ff_t} = corr2(Newf, Newf_t)$
  *if* $\left((x \neq 0)\ OR\ (y \neq 0)\right)\ AND\ (C_{ff_t} < threshold)$
    $f = Newf$, $f_t = Newf_t$
  *elseif* $(x == 0)\ AND\ (y == 0)\ AND\ (C_{ff_t} < threshold)$
    *f=reference_ image, $f_t$=translated_ image*
    *Give a new random value for column translation, Newx*
    $x = Newx$
    *goto* (∗)
  *end*
*end*

The above function takes as input the initial reference and translated images, along with a threshold value associated to the correlation coefficient, $C_{ff_t}$, between the registered images. The variables $f$ and $f_t$ are set equal to the input images, while the correlation coefficient, $C_{ff_t}$, is set equal to $threshold - 1$. Subsequently, the *while* loop can be initiated. We must stress on this point that the convergence as well as the efficiency of the proposed algorithm heavily depends on the value of the above mentioned threshold. The optimal adaptive update of this threshold is currently under investigation. Hence in what follows, we consider that there exists at least one location in the search area, where the value of the correlation coefficient is greater than the threshold value.

Under this assumption, as long as the condition $C_{ff_t} < threshold$ is true, the correlation coefficient, $C(x,0)$, is computed for $N$ vertical translations of the window $w$ in the search area $g$. After finding the translation, $x$, that maximizes the correlation coefficient, the images $f$ and $f_t$ are registered according to $x$ and their common areas $f'$ and $f_t'$ are kept. Then, new window and search areas, $w'$ and $g'$, are determined, which are used for searching in rows. The horizontal translation, $y$, is computed and the resulting images are $Newf$ and $Newf_t$. The correlation coefficient, $C_{ff_t}$, between these images is also computed.

If translation $x$ or translation $y$ is different from zero and the correlation coefficient $C_{ff_t}$ is smaller than the predetermined threshold, the next iteration is performed, with $Newf$ and $Newf_t$ being the initial images. If translations $x$ and $y$ are equal to zero, but the correlation coefficient $C_{ff_t}$ is smaller than the threshold, this implies that the algorithm has been trapped in a local maximum and needs a perturbation in order to escape from this. As expected, the speed of the algorithm's convergence depends mainly on the form of the correlation function and the initial translation estimate. If this lies in the main lobe around the maximum of the correlation coefficient function (when it is computed for all possible translations), the algorithm converges to the global maximum, since the shape of the correlation coefficient function around its global maximum value resembles a concave function. Thus, we may overcome the algorithm's trapping by simply re-initializing the algorithm with a randomly selected initial estimate. Note, that in any case, the aim is the proposed algorithm to achieve the performance of the exhaustive search scheme, but with smaller computational cost. In order to achieve our goal, first, the variables $f$ and $f_t$ are set equal to the function's input images, a randomly selected value for the initial vertical translation $x$, $Newx$, is given and the algorithm continues performing the next operations in the *while* loop. Note finally that *goto* is used in order to bypass the operations that correspond to the initial vertical translation estimation. The algorithm converges if the translations $x$ and $y$ become zero and the correlation coefficient $C_{ff_t}$ between the registered images is equal or higher than the predetermined threshold.

Note that in case the algorithm needs re-initializations, the number of searches becomes $2N \cdot \sum_{i=1}^{k} a_i$, where $k$ is the total number of the algorithm initializations, i.e., the number of re-initializations plus one (the initialization when the algorithm is executed for the first time). Moreover, $a_i$ is the number of iterations for each initialization and $\sum_{i=1}^{k} a_i$ is the total number of algorithm's iterations that, in most cases, is much smaller than $N/2$. Examples that verify the above analysis are given in the next section.

## 4. EXPERIMENTAL RESULTS

### 4.1 Experimental Convergence Study of the Pixel-Level Registration Scheme

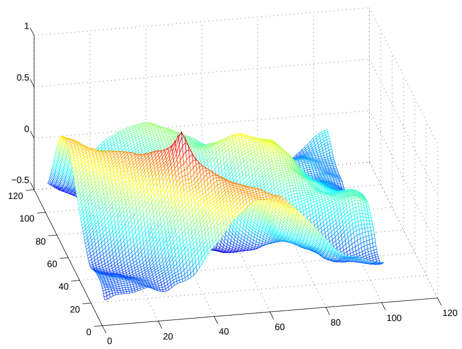
The iterative algorithm's convergence was studied through extensive experiments, but due to space limitations only results concerning the image of Lenna are presented here. Three window cases were examined, with dimensions $128 \times 128$, $64 \times 64$ and $32 \times 32$. The corresponding search areas were $228 \times 228$, $164 \times 164$ and $132 \times 132$, which means that the number of searches was $101 \times 101$ in all the above cases. We used 400

Table 1: 400 Experiments For Lenna

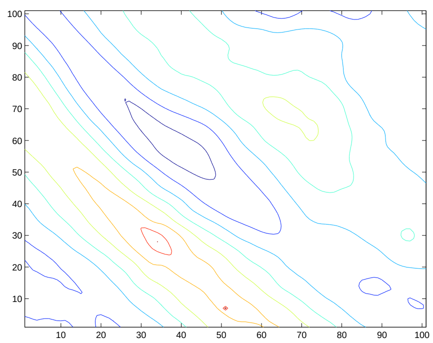
| **Window Size:** | **128×128** | **64×64** | **32×32** |
|---|---|---|---|
| *Correct Registrations* | 400 | 400 | 400 |
| *Re-Initializations* | 3 | 2 | 597 |
| *Max. Re-Initializations* | 1 | 1 | 15 |
| *Experiments with re-initialization* | 3 | 2 | 152 |
| *Mean Iterations (per experiment)* | 3.2200 | 4.7075 | 23.5675 |
| *Mean Searches (per experiment)* | $6.440 \times 101$ | $9.415 \times 101$ | $47.135 \times 101$ |

Table 2: Performance Evaluation Of The Compared Methods (Axis X, Axis Y)

| **S. Lenna** | **Mean Error** | **Error's STD** | **Max. Error** | **Min. Error** |
|---|---|---|---|---|
| *Proposed method* | $(-0.0015,\ 4.7575 \cdot 10^{-5})$ | $(0.0317,\ 0.0248)$ | $(0.1111,\ 0.1094)$ | $(-0.0861, -0.0914)$ |
| *FD method (radial=1)* | $(-0.0109, -0.0044)$ | $(0.0508,\ 0.0264)$ | $(2.1742 \cdot 10^{-5},\ 0.0044)$ | $(-0.5032, -0.2707)$ |
| *FD method (radial=2)* | $(0.0023, -0.0015)$ | $(0.0166,\ 0.0462)$ | $(0.1722,\ 0.2928)$ | $(-0.0934, -0.4896)$ |
| *FD method (radial=3)* | $(4.6096 \cdot 10^{-4},\ 7.49 \cdot 10^{-6})$ | $(0.0159,\ 0.0287)$ | $(0.1267,\ 0.1804)$ | $(-0.1411, -0.3010)$ |
| *FD method (radial=4)* | $(-0.0556, -0.0398)$ | $(0.7542,\ 0.8621)$ | $(0.0978,\ 4.3022)$ | $(-13.8289, -16.1364)$ |




(a)




(b)

Figure 1: Correlation coefficient function (a) and its contour (b) for window of size $64 \times 64$.

Figure 2: Unregistered images generated as in [10].

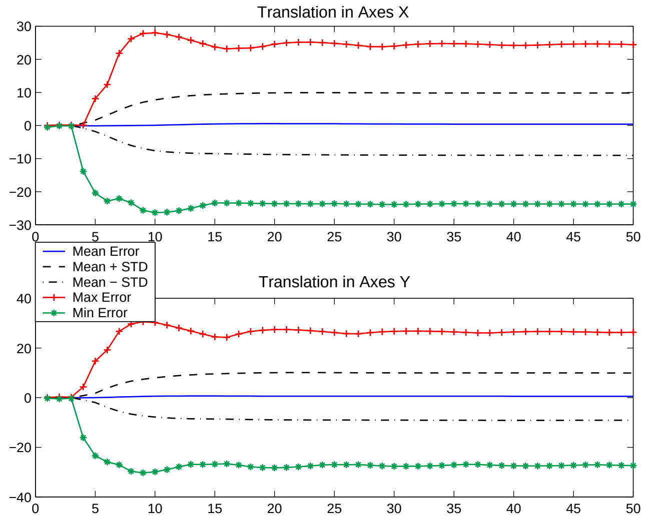
different translations in our experiments, produced by a generator of normally distributed random numbers with standard deviation equal to 10. The resulting numbers were rounded towards the nearest integers. The proposed iterative algorithm converged in all cases, but some re-initializations were needed. Note, in Fig. 1, where a correlation coefficient function along with its contour plot are shown for the case of window size $64 \times 64$, that the position of the first estimate lies in the main lobe.

The results for all the experiments we performed are shown in Table 1. For the $128 \times 128$ and $64 \times 64$ windows, only 3 and 2 re-initializations are needed, the mean numbers of iterations per experiment, i.e., $mean\left(\sum_{i=1}^{k} a_i\right)$, are 3.2200 and 4.7075, which are much smaller than $N/2$, and the mean numbers of searches, i.e., $2N \cdot mean\left(\sum_{i=1}^{k} a_i\right)$, are $6.4400 \times 101$ and $9.4150 \times 101$, respectively. In case a $32 \times 32$ window is used, the algorithm needs to be re-initialized in 152 experiments. The maximum number of re-initializations for one experiment was equal to 15 and the total number of re-initializations was 597. Note, though, that this case is rather difficult, since the tested translations take values comparable to the used window size ($32 \times 32$). However, the mean number of iterations per experiment remains small (23.5675, which is much smaller than $N/2$), while the mean number of searches is $47.1350 \times 101$.

### 4.2 Subpixel registration

The new subpixel registration technique was compared to a recent frequency domain based method [10] that outperforms existing techniques [11, 18, 19]. In [10], the translation is computed based on the compared images low frequency, aliasing-free (or marginally affected by aliasing) part. The phase difference between the compared images is computed and for the aliasing-free frequencies the corresponding linear equations are written. Then, the shift parameters are found as the least squares solution of these equations.

In [10], the test images are obtained via down-sampling. As a result, integer shifts in the high-resolution images correspond to subpixel (non-integer) shifts in the resulting images. Moreover, a lowpass filter is used prior to down-sampling in order to control the amount of aliasing. The initial images are multiplied by a Tukey window in order the images to be circularly symmetric and avoid boundary effects. A pair of such images is shown in Fig. 2. Similar approaches for producing




Sampled Lenna

Figure 3: Performance evaluation for the frequency domain method.

images with subpixel translations were used in [8, 20].

We produced such pairs of images (down-sampled by a factor 4) for 400 different translation pairs, derived by a generator of normally distributed random numbers with standard deviation equal to 10. For the two methods under comparison (i.e., the proposed one and that of [10]) we computed the error between the actual translations and their estimations. In Table 2, the errors' mean values, standard deviations, as well as maximum and minimum values, are shown.

As we mentioned above, in [10], the translation's computation is based on a low-frequency part of the phase difference between the compared images. We computed the error for many different sizes (radials from 1 to 50) of the remaining low-frequency part and the results are shown in Fig. 3. Note, that the error remains relatively small, for small radials. Otherwise, it significantly increases.

We obtained similar results in a large number of experiments (we present here results only for Lenna) indicating that the proposed technique is more accurate.

## 5. CONCLUSION

A new technique for subpixel image registration, based on the correlation coefficient maximization, is proposed that provides a closed form solution. Extensive simulation results have shown that the performance of the proposed technique compares very favorably with respect to existing ones. The optimum subpixel translation is found with a slight increase in the computational cost, but using the new efficient scheme for pixel level registration as a pre-processing step, the computational complexity of the whole problem is significantly reduced.

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