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A SIMPLE DECOUPLED ESTIMATION OF DOA AND ANGULAR SPREAD FOR SINGLE SPATIALLY DISTRIBUTED SOURCES

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ABSTRACT

In wireless communications, local scattering in the vicinity of the mobile station results in angular spreading. A few estimators (of direction of arrival (DoA) and angular spread) have already been developed, but they suffer from high computational load or are developed in very specific contexts. In this paper, we present a new simple low-complexity decoupled estimator of DoA and angular spread for a spatially dispersed source. The proposed MDS (Music for Dispersed Sources) algorithm does not assume any particular sensor array geometry nor temporal independence hypothesis. Moreover, the estimation of DoA and angular spread does not require knowledge of the angular and temporal distribution shapes of the sources. In addition, the low computational cost of this method makes it attractive.

1. INTRODUCTION

The problem of estimating the Direction of Arrival (DoA) of multiple sources impinging on an antenna array has been intensively studied these last decades. The subspace-based algorithm, MUSIC [1] is one of the algorithms developed for the DoA estimation.

These algorithms all rely on the “point scatterer” hypothesis. This hypothesis is usually not satisfied in real conditions [2]. Indeed, distributed sources arise in many different domains such as wireless communications, radar, radio astronomy, etc.. The source is thus no longer viewed by the array as a point source but as a spatially distributed source with mean DoA and angular spread. Due to the mismatch between the true effect of a distributed source and the effect of the underlying point-scatterer model, the conventional DoA estimators may result in poor performance (see for example [3]). Furthermore, these techniques do not provide any estimate of the angular spread. As a consequence, the problem of estimating the DoA and angular spread of spatially dispersed sources has experienced a growing development in the last few years.

Numerous models have been developed (see for example [4]). The signal received at the array is generally described as the sum of numerous angularly spread signals that are phase-delayed and amplitude weighted. The signal can then be seen as a continuous distribution of DoA, described by its probability density function. Different probability distributions of the angular deviation have been proposed, the most used being the von-Mises distribution [5].

There have been numerous studies on estimation of distributed sources. Since a direct application of conventional subspace-based algorithms may lead to inaccurate estimators, some algorithms such as DSPE [6] and DISPARE [7] have been introduced that are based on an augmentation of the signal subspace dimension. Nevertheless, these two methods require a 2D-optimization and are not consistent (see [8]). Bengtsson proposed in [8] a generalization of the WSF algorithm for distributed sources and developed a consistent WPSF algorithm. Unfortunately, these algorithms suffer from a very high computational load. Other proposed methods exhibit a very low computational cost but are developed in a single source model for particular sensor array geometry [9] or do not provide an estimator of the angular spread [10]. Furthermore, based on the generalized array manifold (GAM) model proposed in [10], different techniques are proposed for the estimation of nominal azimuth of arrival and azimuth spread of the spatially dispersed source [11]. However, since the Taylor-series expansion on which the GAM model relies is fixed to be the first-order, these techniques are applicable only in the case of slightly distributed sources.

In this paper, we present a new simple low-cost decoupled estimator of DoA and angular spread for spatially dispersed sources. The proposed MDS (MUSIC for Dispersed Sources) algorithm does not assume any particular sensor array geometry nor temporal independence hypothesis. Moreover, the estimation of DoA and angular spread does not require knowledge of the shape of the angular distribution of the sources. The low computational cost of this method makes it very attractive.

This paper is organized as follows: Section 2 describes the signal model and assumptions. In Section 3, we present the MDS algorithm. The MDS algorithm is validated by Monte Carlo simulations in Section 4 and we conclude in Section 5.

2. SIGNAL MODEL, ASSUMPTIONS AND PROBLEM FORMULATION

Consider a single narrowband, far-field, spatially dispersed source with wave-fields that impinge on a uniform linear array of M sensors. We consider horizontal-only propagation. The received signal at the output of the array can be seen as the contribution of multiple sub-

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sources within a spatially distributed source [12] with nominal azimuth of arrival θ_0 and angular spread σ_0.

In this paper, we propose a new approximation model which characterizes the impinging wavefront originating from a spatially dispersed source as the superposition of L discrete delayed, attenuated, eventually correlated wavefronts that are regularly spaced out within the cone of arrival.

The $M \times 1$ vector $\mathbf{x}(t)$ of sensors' outputs is given by the following equation

$$\mathbf{x}(t) = \sum_{k=1}^{L} \mathbf{a}\left(\theta_0 + \widetilde{\theta}_k\right) s_k(t) + \mathbf{n}(t)$$

with

$$s_k(t) = \rho_k(t)\, s(t - \tau_k), \; \rho_k(t) = \rho_k e^{j2\pi f_k (t-\tau_k)}. \quad (1)$$

In the above expression,

- $s(t)$ is the stochastic signal emitted by the source at time t,
- $s_k(t)$ is the stochastic signal which interacts with the k^{th} sub-source in the spatially dispersed source,
- $\mathbf{a}(\theta)$ is the $M \times 1$ point source steering vector,
- θ_0 is the nominal azimuth of arrival of the spatially dispersed source,
- $\theta_0 + \widetilde{\theta}_k$ is the azimuth of arrival of the k^{th} wavefront with attenuation ρ_k, delay $\tau_k = \tau(\theta_0 + \widetilde{\theta}_k)$ and Doppler frequency f_k,
- $\mathbf{n}(t)$ is the $M \times 1$ additive Gaussian noise vector with component variance σ_n^2.

The correlation between the i^{th} and j^{th} scatterers is

$$r(\tau_i - \tau_j) = E\left[s(t - \tau_i)\, s(t - \tau_j)^*\right],$$

where $r(u) = E\left[s(t)\, s(t-u)^*\right]$ is the correlation function of $s(t)$, $E[.]$ denotes expectation and $(\cdot)^*$ is the complex conjugate of the given argument.

In this paper, we do not assume any prior information about the shape of the distribution of the angular spread. The angular spread σ_0 is directly obtained from the underlying model of (1) according to

$$\sigma_0 = \sqrt{\sum_{k=1}^{L} \frac{\gamma_k}{\gamma} \widetilde{\theta}_k^2} \quad (2)$$

with $\gamma_k = E[|s_k(t)|^2]$ and $\gamma = \sum_{k=1}^{L} \gamma_k$. The term $\frac{\gamma_k}{\gamma}$ appears since the sub-path might have different power.

Assuming that the individual sources are equally spaced and the distance between two adjacent individual sources is ε, we have

$$\widetilde{\theta}_k = (k-1)\,\varepsilon - \left(\frac{L-1}{2}\right)\varepsilon = \xi(k, L)\,\varepsilon. \quad (3)$$

In the following, we use $\mathbf{R}_x = E\left[\mathbf{x}(t)\mathbf{x}(t)^{\mathrm{H}}\right]$ with $(\cdot)^{\mathrm{H}}$ representing the Hermitian transpose to denote the covariance matrix of $\mathbf{x}(t)$, K the rank of $\mathbf{R}_x$ and $(\lambda_i, \mathbf{e}_i)$ the i^{th} eigenvalue and eigenvector of $\mathbf{R}_x$ respectively, with $\lambda_1 \geq \ldots \geq \lambda_K \geq \ldots\ldots \geq \lambda_M$. The matrices $\mathbf{E}_s = [\mathbf{e}_1, \ldots \mathbf{e}_K]$ and $\mathbf{E}_n = [\mathbf{e}_{K+1}, \ldots \mathbf{e}_M]$ are respectively associated with the signal and noise subspaces of $\mathbf{R}_x$ and the diagonal matrices $\mathbf{\Lambda}_s = \mathrm{diag}(\lambda_1, \ldots, \lambda_K)$ and $\mathbf{\Lambda}_n = \mathrm{diag}(\lambda_{K+1}, \ldots, \lambda_M)$ (see [13] for more details). Here $\mathrm{diag}(\cdot)$ is the diagonal matrix with diagonal elements equal to the components listed as an argument.

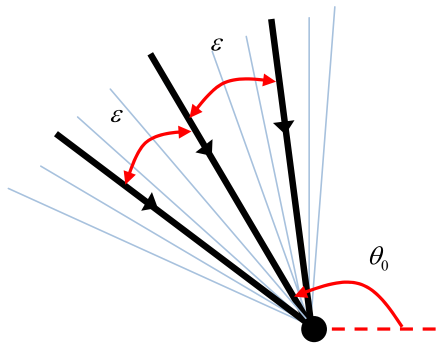

Figure 1: Characterization of a spatially dispersed source by three sub-sources

Note that the rank K of $\mathbf{R}_x$ satisfies: $1 \leq K \leq \min(M, L)$ with $\min(\cdot)$ denoting the minimum of the given argument. The case $K = 1$ corresponds to coherently distributed scatterers satisfying $r(\tau_i - \tau_j) = E[|s(t)|^2], \forall (i,j) \in [1, L]$. The case $K = \min(M, L)$ is holding for distributed scatterers satisfying $r(\tau_i - \tau_j) \neq E[|s(t)|^2], \forall (i,j) \in [1, L]$. Notice that the special case where distributed scatterers are totally incoherent, i.e. $r(\tau_i - \tau_j) = 0, \forall (i,j) \in [1, L]$, results in $K = \min(M, L)$.

3. THE MDS ALGORITHM

3.1 Estimation of DoA θ_0

The K eigenvectors $\mathbf{e}_i$, $i = 1, \ldots, K$, of the signal subspace can be written as a linear combination of vectors $\mathbf{a}(\theta_0 + \widetilde{\theta}_k)$, i.e.

$$\begin{aligned} \mathbf{e}_i &\simeq \sum_{k=1}^{L} \alpha_k^i \mathbf{a}(\theta_0 + \widetilde{\theta}_k) \\ &= \mathbf{A}(\theta_0, \widetilde{\boldsymbol{\theta}}, L)\boldsymbol{\alpha}^i \\ &= \mathbf{e}\left(\boldsymbol{\eta}_0^i\right), \quad i \in [1, K] \end{aligned} \quad (4)$$

with

$$\widetilde{\boldsymbol{\theta}} = \left[\begin{array}{ccc} \widetilde{\theta}_1 & \ldots & \widetilde{\theta}_L \end{array}\right]^{\mathrm{T}} \quad (5)$$

$$\boldsymbol{\alpha}^i = \left[\begin{array}{ccc} \alpha_1^i & \ldots & \alpha_L^i \end{array}\right]^{\mathrm{T}} \quad (6)$$

$$\mathbf{A}(\theta_0, \widetilde{\boldsymbol{\theta}}, L) = \left[\begin{array}{ccc} \mathbf{a}(\theta_0 + \widetilde{\theta}_1) & \ldots & \mathbf{a}(\theta_0 + \widetilde{\theta}_L) \end{array}\right] \quad (7)$$

$$\boldsymbol{\eta}_0^i = \left[\theta_0, \widetilde{\boldsymbol{\theta}}^{\mathrm{T}}, \left(\boldsymbol{\alpha}^i\right)^{\mathrm{T}}, L\right]^{\mathrm{T}}. \quad (8)$$

Here, $(\cdot)^{\mathrm{T}}$ denotes the transpose of the given argument.

The parameter estimation can be conducted in the framework of subspace-based methods. In this contribution, we propose a novel MDS algorithm which is based on a minimization of the following function

$$J(\boldsymbol{\eta}) = \frac{\mathbf{e}^{\mathrm{H}}(\boldsymbol{\eta})\,\mathbf{\Pi}_n \mathbf{e}(\boldsymbol{\eta})}{\mathbf{e}^{\mathrm{H}}(\boldsymbol{\eta})\,\mathbf{e}(\boldsymbol{\eta})} \text{ with } \boldsymbol{\eta} = \left[\theta, \widetilde{\boldsymbol{\theta}}^{\mathrm{T}}, \boldsymbol{\alpha}^{\mathrm{T}}, L\right], \quad (9)$$

which gives the K estimates $\widehat{\boldsymbol{\eta}}_0^i$, $i = 1 \ldots K$.

The minimization of the function in (9) requires a highly non-linear multidimensional optimization, which is of high computational load. In order to reduce the computional load, in the MDS algorithm the parameters θ_0 and σ_0 are estimated in two successive steps. In the first step, the only parameter to be estimated is the DoA θ_0, the other parameters being considered as nuisance parameters. This is obtained by reformulating $J(\boldsymbol{\eta})$ to a function of θ_0. For this, the steering vector $\mathbf{a}(\theta_0+\widetilde{\theta})$ is factorized using a P^{th}-order Taylor expansion according to

$$\begin{aligned} \mathbf{a}(\theta_0+\widetilde{\theta}) &\approx \mathbf{a}(\theta_0) + \sum_{p=1}^{P} \frac{\partial^p \mathbf{a}(\theta_0)}{\partial \theta^p} \frac{\left(\widetilde{\theta}\right)^p}{p!} \\ &= \mathbf{D}(\theta_0)\,\Delta\left(\widetilde{\theta}\right) \end{aligned} \quad (10)$$

with

$$\begin{aligned} \mathbf{D}(\theta_0) &= \left[\begin{array}{cccc} \mathbf{a}(\theta_0) & \frac{\partial \mathbf{a}(\theta_0)}{\partial\theta} & \ldots & \frac{\partial^P \mathbf{a}(\theta_0)}{\partial\theta^P} \end{array}\right], \quad (11) \\ \Delta\left(\widetilde{\theta}\right) &= \left[\begin{array}{cccc} 1 & \widetilde{\theta} & \ldots & \frac{(\widetilde{\theta})^P}{P!} \end{array}\right]^{\mathrm{T}}. \quad (12) \end{aligned}$$

It is worth mentioning that inserting (10) with $P = 1$ in (1) yields the GAM model proposed in [10].

Inserting the approximation (10) into (7) yields $\mathbf{A}(\theta, \widetilde{\boldsymbol{\theta}}, L) = \mathbf{D}(\theta)\boldsymbol{\Delta}(\widetilde{\boldsymbol{\theta}})$ with $\boldsymbol{\Delta}(\widetilde{\boldsymbol{\theta}}) = [\ \Delta(\widetilde{\theta}_1)\ \ldots\ \Delta(\widetilde{\theta}_L)\ ]$ and

$$J(\boldsymbol{\eta}) = \frac{\boldsymbol{\beta}^{\mathrm{H}}\left(\widetilde{\boldsymbol{\theta}}, \boldsymbol{\alpha}, L\right) \mathbf{Q}_1(\theta)\, \boldsymbol{\beta}\left(\widetilde{\boldsymbol{\theta}}, \boldsymbol{\alpha}, L\right)}{\boldsymbol{\beta}^{\mathrm{H}}\left(\widetilde{\boldsymbol{\theta}}, \boldsymbol{\alpha}, L\right) \mathbf{Q}_2(\theta)\, \boldsymbol{\beta}\left(\widetilde{\boldsymbol{\theta}}, \boldsymbol{\alpha}, L\right)}, \quad (13)$$

with

$$\begin{aligned} \boldsymbol{\beta}(\widetilde{\boldsymbol{\theta}}, \boldsymbol{\alpha}, L) &= \boldsymbol{\Delta}(\widetilde{\boldsymbol{\theta}})\boldsymbol{\alpha}, \\ \mathbf{Q}_1(\theta) &= \mathbf{D}^{\mathrm{H}}(\theta)\mathbf{\Pi}_n\mathbf{D}(\theta), \\ \mathbf{Q}_2(\theta) &= \mathbf{D}^{\mathrm{H}}(\theta)\mathbf{D}(\theta). \end{aligned} \quad (14)$$

Using the classical results on quadratic forms [14], Criterion (13) can be reformulated as a function with respect to the parameter θ

$$J_{\mathrm{DoA}}(\theta) = \lambda_{\min}\left\{\mathbf{Q}_2(\theta)^{-1}\mathbf{Q}_1(\theta)\right\}, \quad (15)$$

where $\lambda_{\min}\{\cdot\}$ is the smallest eigenvalue of the matrix given as an argument. Note that (15) is well defined only in the case where $\mathbf{Q}_2(\theta)^{-1}$ exists. This condition can be satisfied in real applications by adjusting P in such a way that $\mathbf{Q}_2(\theta)$ is non-singular.

Note that the estimation of DoA using (15) does not require the estimation of L.

Remark 1 *According to [15], since $J_{\mathrm{DoA}}(\theta_0) = 0$ then $J_c'(\theta_0) = \det\left\{\mathbf{Q}_2(\theta_0)^{-1}\mathbf{Q}_1(\theta_0)\right\} = 0$ and from (14), the function in (15) can also be replaced by*

$$J_{\mathrm{DoA}}'(\theta) = \frac{\det(\mathbf{Q}_1(\theta))}{\det(\mathbf{Q}_2(\theta))}. \quad (16)$$

Minimization of (16) with respect to θ requires lower computational cost than the minimization of (15).

In practise, we only get an estimate $\widehat{\mathbf{\Pi}}_n = \widehat{\mathbf{E}}_n\widehat{\mathbf{E}}_n^{\mathrm{H}}$ of $\mathbf{\Pi}_n$, where $\widehat{\mathbf{E}}_n$ is estimated from the eigen decomposition of the sample covariance matrix

$$\widehat{\mathbf{R}}_x = \frac{1}{T/T_e}\sum_{k=1}^{T/T_e} \mathbf{x}(kT_e)\,\mathbf{x}(kT_e)^{\mathrm{H}} = \widehat{\mathbf{E}}_s\widehat{\mathbf{\Lambda}}_s\widehat{\mathbf{E}}_s^{\mathrm{H}} + \widehat{\mathbf{E}}_n\widehat{\mathbf{\Lambda}}_n\widehat{\mathbf{E}}_n^{\mathrm{H}}, \quad (17)$$

where T_e is the sampling period, $\widehat{\mathbf{E}}_s$ and $\widehat{\mathbf{E}}_n$ are the estimate of the signal $\mathbf{E}_s$ and noise $\mathbf{E}_n$ subspace respectively and $\widehat{\mathbf{\Lambda}}_s$ and $\widehat{\mathbf{\Lambda}}_n$ are the estimates of the diagonal matrices $\mathbf{\Lambda}_s$ and $\mathbf{\Lambda}_n$.

3.2 Estimation of angular spread σ_0

The purpose of this section is to provide an estimation $\widehat{\sigma}_0$ of the angular spread σ_0 without knowledge of the shape of the angular distribution, under the hypothesis that $P \geq 2$. In the following, we fix the value of L to $L = 2$. Consequently, from (2) and (3), the estimation of σ_0 requires estimating ε with $\sigma_0 = \frac{\varepsilon}{\sqrt{2}}$. Inserting (3) respectively in (4) and (10) yields

$$\mathbf{e}(\theta_0, \varepsilon, \boldsymbol{\alpha}) = \alpha_1\mathbf{a}\left(\theta_0 - \frac{\varepsilon}{2}\right) + \alpha_2\mathbf{a}\left(\theta_0 + \frac{\varepsilon}{2}\right),$$

where

$$\mathbf{a}(\theta + \xi(k, L)\,\varepsilon) = \mathbf{D}(\theta)\,\boldsymbol{\Delta}_k(\varepsilon), \quad k = 1, 2 \quad (18)$$

with

$$\boldsymbol{\Delta}_k(\varepsilon) = \left[\begin{array}{cccc} 1 & (-1)^k\frac{\varepsilon}{2} & \ldots & (-1)^{Pk}\frac{\varepsilon^P}{2.P!} \end{array}\right]^{\mathrm{T}}. \quad (19)$$

Thus, (18) can be written

$$\mathbf{e}(\theta, \varepsilon, \boldsymbol{\alpha}) = \mathbf{D}(\theta)\,\boldsymbol{\beta}(\varepsilon, \boldsymbol{\alpha}) \quad (20)$$

with

$$\boldsymbol{\beta}(\varepsilon, \boldsymbol{\alpha}) = \sum_{k=1}^{2}\alpha_k\boldsymbol{\Delta}_k(\varepsilon) = \mathbf{U}_2(\varepsilon)\,\boldsymbol{\alpha}. \quad (21)$$

Here, $\mathbf{U}_2(\varepsilon) = [\ \boldsymbol{\Delta}_1(\varepsilon)\ \ \boldsymbol{\Delta}_2(\varepsilon)\ ]$.

Using the estimate $\widehat{\theta}_0$ of the source, the estimate $\widehat{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}$ can be calculated to be proportional to the eigenvector $\mathbf{v}_{\min}$ of $\mathbf{Q}_2(\widehat{\theta}_0)^{-1}\mathbf{Q}_1(\widehat{\theta}_0)$ associated with the smallest eigenvalue [15]. Assuming that $P \geq 2$, the parameters $(\varepsilon, \boldsymbol{\alpha})$ are given by the minimum of

$$J_\sigma(\varepsilon, \boldsymbol{\alpha}) = \frac{\boldsymbol{\beta}(\varepsilon, \boldsymbol{\alpha})^{\mathrm{H}}\,\mathbf{\Pi}\left(\widehat{\boldsymbol{\beta}}\right)\boldsymbol{\beta}(\varepsilon, \boldsymbol{\alpha})}{\boldsymbol{\beta}(\varepsilon, \boldsymbol{\alpha})^{\mathrm{H}}\,\boldsymbol{\beta}(\varepsilon, \boldsymbol{\alpha})}, \quad (22)$$

with respect to σ. Here, $\mathbf{\Pi}\left(\widehat{\boldsymbol{\beta}}\right) = \mathbf{I} - \widehat{\boldsymbol{\beta}}\widehat{\boldsymbol{\beta}}^{\dagger} = \mathbf{I} - \mathbf{v}_{\min}\mathbf{v}_{\min}^{\mathrm{H}}$ with $\widehat{\boldsymbol{\beta}}^{\dagger}$ representing the Moore Penrose pseudo-inverse of $\widehat{\boldsymbol{\beta}}$. According to (21) and using the results of [15], (22) can finally be concentrated with respect to the parameters ε

$$J_{\sigma}(\varepsilon) = \lambda_{\min}\left\{\mathbf{Q}_2'(\varepsilon)^{-1}\mathbf{Q}_1'(\varepsilon)\right\}, \qquad (23)$$

with

$$\begin{aligned} \mathbf{Q}_1'(\varepsilon) &= \mathbf{U}_2(\varepsilon)^{\mathrm{H}}\mathbf{\Pi}\left(\widehat{\boldsymbol{\beta}}\right)\mathbf{U}_2(\varepsilon), \\ \mathbf{Q}_2'(\varepsilon) &= \mathbf{U}_2(\varepsilon)^{\mathrm{H}}\mathbf{U}_2(\varepsilon). \end{aligned}$$

3.3 Selection of P

In Subsections 3.1 and 3.2, the estimation of θ_0 and σ_0 relies on a P^{th}-order Taylor expansion of $\mathbf{a}(\theta_0 + \widetilde{\theta})$, whose order P has to be estimated. Because of lack of space, we only present in this subsection the idea used for obtaining the estimate $\widehat{P}$ of P. The derivation of the method will be reported in a forth-coming paper. We first determine a threshold value ξ satisfying

$$J(N) = \frac{\mathbf{a}^{\mathrm{H}}\widehat{\mathbf{\Pi}}_n(N)\mathbf{a}}{\mathbf{a}^{\mathrm{H}}\mathbf{a}} > \xi$$

with a false alarm probability P_{fa}. Here, N is the number of snapshots.

Under the hypothesis of Gaussian noise $\mathbf{n}(t)$ and stochastic source $s(t)$, we can demonstrate that $\frac{N\gamma}{{\sigma_n}^2}J(N)$ follows a central chi-square distribution with $2(M-1)$ degrees of freedom provided the order P is correctly selected. Consequently, the threshold value ξ corresponding to the false alarm probability P_{fa} is given by

$$\xi = \frac{{\sigma_n}^2}{2MN\gamma}\chi^2\left(2(M-1), P_{fa}\right).$$

Given a false alarm probability P_{fa}, we may test different values of P in ascending order.The estimate $\widehat{P}$ corresponds to the first value of P for which the minimum of $J(N)$ is below the threshold value ξ.

4. SIMULATIONS

4.1 Signal generation

The purpose of the simulations is to estimate the parameters (θ_0, σ_0) of a uniform law without prior information about the shape of the distribution of the angular spread. An 8-element uniform linear array is used with inter-element spacing d satisfying $d/\lambda = \frac{1}{2}$. Sources are NRZ BPSK of symbol duration $T_s = 10T_e$. This implies that the signals $s(t-\tau_i)$ and $s(t-\tau_j)$ are incoherent when $|\tau_j - \tau_i| > T_s$. In the simulations, the signal is generated according to

$$\mathbf{x}(t) = \frac{\rho_0}{\sqrt{200}}\sum_{k=1}^{200}\mathbf{a}\left(\theta_0 + \widetilde{\theta}_k\right)s(t-\tau_k) + \mathbf{n}(t),$$

where $\widetilde{\theta}_k$ is uniformly distributed between $-\Delta\theta/2$ and $\Delta\theta/2$. The relation between the angular spread σ_0 and $\Delta\theta$ is calculated to be $\sigma_0 = \Delta\theta/\sqrt{12}$. The signal-to-noise ratio (SNR) is $20\log 10(\rho_0/\sigma_n)$. The covariance matrix of the noise $\boldsymbol{n}(t)$ reads $E[\mathbf{n}(t)\mathbf{n}(t)^{\mathrm{H}}] = {\sigma_n}^2\mathbf{I}_M$. Ray tracing considerations lead us to the following approximate linear relation between the angular and temporal spread

$$\tau_k = \tau(\theta_0 + \widetilde{\theta}_k) = \widetilde{\theta}_k\frac{\Delta\tau}{\Delta\theta}.$$

In all the simulations, the temporal spread is fixed at $\Delta\tau = T_s/10$ leading to $|\tau_i - \tau_j| < T_s/20$. In this condition, for NRZ modulation, it is easy to verify that the correlation $r_{ij} = E[s(t-\tau_i)\,s(t-\tau_j)^*]/E[|s_m(t)|^2]$ is $19/20 < |r_{ij}| < 1$. Note that this is an intermediate case between the coherent and incoherent distributed case that corresponds to the severe scenario in urban propagation context.

4.2 Simulation results

In order to demonstrate the importance of a good estimation of the parameter P, we first study the behavior of the MDS algorithm when P is fixed at different values $P \in \{1, 2, 3, 4\}$. Secondly we compare these performance with the MDS algorithm including the automatic determination of P described in Subsection 3.3.

Fig. 2 is devoted to the performance of the estimation of θ_0, whereas Fig. 3 presents that of the angular spread σ_0. The nominal azimuth of arrival of the source is $\theta_0 = 0°$. The SNR at the input of the antenna array equals 20 dB.

- For a given value of P, the accuracy of the estimation of θ_0 and $\Delta\theta$ depends on the value of angular spread σ_0. For instance, for $P = 2$, the performance is optimal around $\sigma_0 = 15°$. Indeed, in the region of $\sigma_0 \leq 10°$, due to the model mismatch the impact of the noise becomes significant and consequently the performance of the estimators degrade. For high σ_0, the value of P is too small to match correctly the true effect of the spatially dispersed scatterer, which explains the increase of the curves in this area.
- As σ_0 increases the optimal value P_{opt}, for which the RMS (root mean square) error is minimal, increases. This observation highlights the important role of parameter P in the accuracy of the estimations. We can observe that the MDS curve provides a quite perfect fit to the optimal performance of the estimators with fixed P. This demonstrates the good behavior of this algorithm and the quasi-optimality of the determination of P.

5. CONCLUSION

We have introduced a new simple low-cost decoupled MUSIC-based approach for estimation of DoA and angular spread of spatially dispersed sources. This technique is derived using a generalized array manifold model based on Taylor-series expansion of the steering vector. We also propose a novel method that allows automatic determination of the expansion order. The proposed MDS (MUSIC for Dispersed Sources) algorithm does not assume any particular sensor array geometry nor temporal independence hypothesis. Moreover, the

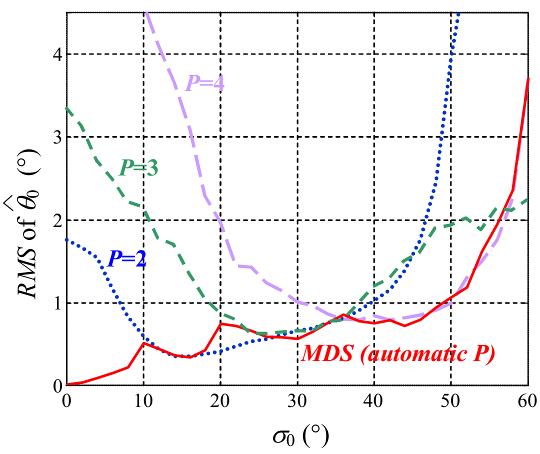

Figure 2: RMS error on the estimation of θ_0 as a function of σ_0

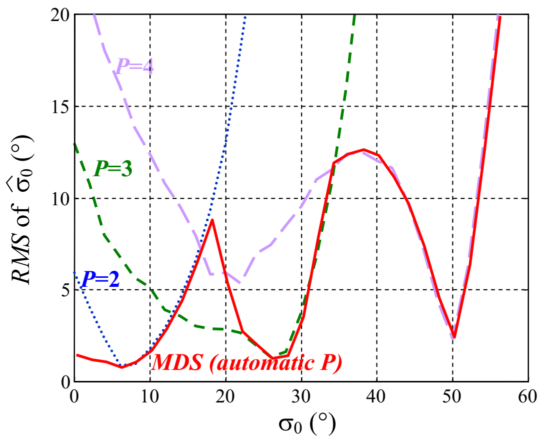

Figure 3: RMS error on the estimation of σ_0 as a function of σ_0

estimation of DoA and angular spread does not require knowledge of the shape of the angular distribution of sources. Simulations showed that this algorithm provides accurate estimates even in the case where the signal of the distributed source is highly correlated. In addition, significant performance improvement is observed for the MDS algorithm when it is applied in combination with a scheme performing automatic determination of the Taylor-series expansion order.

REFERENCES

[1] G. Bienvenue and L. Kopp, "Principes de la goniométrie passive adaptative," in *GRETSI*, pp. 106–116, 1979.

[2] K. I. Pederson, P. E. Mogensen, and B. H. Fleury, "Spatial channel characteristics in outdoors environments and their impact on BS antenna system performance," *Proc. IEEE Veh. Technol. Conf.*, pp. 719–724, May 1998.

[3] D. Asztely and B. Ottersten, "The effects of local scattering on Direction Of Arrival estimation with MUSIC," *IEEE Trans. Signal Processing*, vol. 47, pp. 3220–3224, Dec. 1999.

[4] P. Zetterberg, *Mobile cellular communications with base station antenna arrays: Spectrum efficiency, algorithms and propagation models.* PhD thesis, R. Inst. Technol. Stokholm, Sweden, 1997.

[5] X. Yin, B. Fleury, T. Pedersen, and J. Nuutinen, "Low complexity nominal azimuth and azimuth spread estimation for slightly distributed scatterers," *Submitted to IEEE Transactions on Signal Processing*, 2005.

[6] S. Valaee, B. Champagne, and P. Kabal, "Parametric localization of distributed sources," *IEEE Trans. Signal Processing*, vol. 43, pp. 2144–2153, Sept. 1995.

[7] Y. Meng, P. Stoica, and K. Wong, "Estimation of the Direction Of Arrival of spatially dispersed signals in array processing," in *Proc. Inst. Elect. Eng. Radar. Sonar. Navigat.*, vol. 143, pp. 1–9, Feb. 1996.

[8] M. Bengtsson and B. Ottersten, "A generalization of Weighted Subspace Fitting to full-rank models," *IEEE Trans. on Signal Processing*, vol. 49, pp. 1002–1012, May 2001.

[9] O. Besson and P. Stoica, "Decoupled estimation of DOA and angular spread for a spatially distributed source," *IEEE Trans. Signal Processing*, vol. 48, pp. 1872–1882, July 2000.

[10] D. Asztely, B. Ottersten, and A. Lee Swindlehurst, "A Generalized Array Manifold Model for local scattering in wireless communications," in *Proceedings ICASSP*, (Munich, Germany), pp. 4021–4024, 1997.

[11] X. Yin, B. Fleury, T. Pedersen, and J. Nuutinen, "Azimuth spread estimation for slightly distributed scatterers using the generalized array manifold model," *Proc. of the IEEE Int. Symp. on Personal, Indoor and Mobile Radio Communications (PIMRC '05), Berlin, Germany*, 2005.

[12] M. Bengtsson and B. Ottersten, "Low-complexity estimators for distributed sources," *IEEE Trans. on Signal Processing*, vol. 48, pp. 2185–2194, Aug. 2000.

[13] R. O. Schmidt, "A signal subspace approach to multiple emitter location and spectral estimation," *IEEE Trans. on Ant. and Propag.*, vol. 43, pp. 817–821, Apr. 1986.

[14] F. Gantmacher, *The theory of matrices.* Chelsa: Vol I-II, 1959.

[15] A. Ferréol, E. Boyer, and P. Larzabal, "Low cost algorithm for some bearing estimation methods in presence of separable nuisance parameters," *IEE Elec. Letters*, vol. 40, pp. 966–967, July 2004.