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SUBCARRIER ALLOCATION IN A MULTIUSER MIMO CHANNEL USING LINEAR PROGRAMMING

Ari Hottinen

Nokia Research Center
P.O. Box 407,00045 Nokia Group,Finland
email:ari.hottinen@nokia.com

Tiina Heikkinen

University of Helsinki
P.O. Box 44, 00014 University of Helsinki,Finland
email:tiina.m.heikkinen@elisanet.fi

ABSTRACT

In this paper, we propose a solution to the subcarrier assignment problem in a high-rate channel-aware MIMO-OFDM system. Our model incorporates inter-stream interference of the MIMO channel. Optimization is applied to enforce efficient resource allocation in the presence of strict fairness constraints. Computational complexity of the optimization problem is reduced by approximating the original model with a modified model that has fewer decision variables. The proposed methods are shown to provide multi-user diversity gains.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) and Multiple-Input Multiple-Output (MIMO) modulation techniques have been recently incorporated in a number of wireless broadband standards, including the recent evolutions of IEEE 802.11 (WiFi) [3] and IEEE 802.16e (WiMax) [2] specifications. The peak data rates specified for these wireless standards (or standard proposals) require extensive use of MIMO technology and multi-stream modulation methods. WiMax supports also OFDMA, where the users or data streams can be assigned to different subcarriers.

A channel-aware OFDMA systems enables efficient exploitation of multiuser diversity in the frequency domain. Channel state information can be used in scheduling (or assigning) users to appropriate subcarriers. Such concepts were considered e.g. in [9, 10, 11, 12] in connection with SISO uplink. In the downlink direction, when the transmitter uses high rate MIMO modulators, algorithms and channel quality indicators for efficient temporal scheduling were considered in [6]. However, related frequency-domain MIMO scheduling solutions are less known. In particular, solutions that address scheduling with MIMO modulation, where the users have strict fairness constraints are called for.

In this paper, we propose a solution to a subcarrier assignment problem for use with high-rate (non-orthogonal) MIMO modulation matrices. With high-rate MIMO modulation interference generally prevails between transmitted and received symbols and this has to be accounted for when selecting the scheduling criteria. Here, we consider mostly the downlink direction where each user has access to the N_{t} transmit antennas and where the K receivers (users) have each N_{r} antennas. The MIMO modulation matrix that contains the symbols of user k is transmitted using a subset of the P available OFDMA subcarriers. The subcarrier assignment is solved by formulating a linear programming model [1] that enforces both a notion of fairness and total throughput (or performance) optimality.

The paper is structured as follows. Section II states the signal model for a single-user MIMO-OFDM system, a particular MIMO modulation method and related Channel Quality Indicators (CQI). The CQIs are used in Section III, where the subcarrier assignment problem is formulated for a channel-aware multiuser OFDMA system. Section IV shows simulation results for a multi-user MIMO-OFDMA system and show that significant performance improvement is attainable by using linear programming (LP) algorithms at the transmitter. The LP algorithm determines which user is assigned to which subcarrier, with implicit fairness and strict delay limits.

2. SIGNAL MODEL

2.1 SISO-OFDM

Let $\mathbf{F}$ denote a $P \times P$ inverse DFT (IDFT) matrix, where $[\mathbf{F}]_{p,q} = 1/\sqrt{P}\exp(j2\pi(p-1)(q-1)/P)$. The DFT matrix, applied at the OFDM receiver is given by $\mathbf{F}^{\dagger}$, the transpose conjugate of $\mathbf{F}$. We assume that the signal is transmitted through a finite impulse response (FIR) channel of length L and that a cyclic prefix of length $L_c > L$ is used at the transmitter.

The effective received signal model (after removing cyclic prefix at receiver) for SISO OFDM signal is

$$\mathbf{y} = \mathbf{F}^{\dagger}\mathbf{H}\mathbf{F}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{H}$ denotes a circulant convolution matrix with entries $[\mathbf{H}]_{p,q} = h((p-q) \bmod P)$, where $h(l)$ designates the lth channel tap. Throughout the paper, vector $\mathbf{x}$ represents the symbol vector and $\mathbf{n}$ complex iid gaussian noise. Since FFT diagonalizes a circulant matrix, the model can be written also as $\mathbf{y} = \mathbf{D}\mathbf{x} + \mathbf{n}$ where $\mathbf{D} = \mathrm{diag}(H(0), ..., H(P-1))$, with $H(p) = \sum_{l=0}^{L} h(l)\exp(-\mathrm{j}2\pi lp/P)$ [4].

2.2 MIMO-OFDM

The model above results in diagonal channel $\mathbf{D}$, i.e. the symbols remain orthogonal. In a multi-antenna context orthogonal modulation methods are available only for a limited set of antenna configurations and only when transmitting at most one symbol per channel use. As the symbol rate is further increased by using $N_{\mathrm{t}} > 1$ transmit and $N_{\mathrm{r}} > 1$ receive antennas the symbols generally interfere with each other. For example, a conventional MIMO-OFDM system applies vector modulation by transmitting simultaneously N_{t} symbol vectors, with $\mathbf{F}\mathbf{x}_n$ transmitted from antenna n, where $\mathbf{x}_n$ is P dimensional symbol vector. With N_{r} receiver antennas the received baseband frequency-domain signal, extending model

(1), is of form

$$\mathbf{y} = \begin{bmatrix} \mathbf{D}_{1,1} & \cdots & \mathbf{D}_{1,N_t} \\ \vdots & & \vdots \\ \mathbf{D}_{N_r,1} & \cdots & \mathbf{D}_{N_r,N_t} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_{N_t} \end{bmatrix} + \mathbf{n} \tag{2}$$

where $\mathbf{D}_{m,n} = \mathrm{diag}(H_{m,n}(0),...,H_{m,n}(P-1))$, with $H_{m,n}(p) = \sum_{l=0}^{L} h_{m,n}(l)\exp(-\mathrm{j}2\pi lp/P)$. The model may be converted with appropriate permutations into a block diagonal form, where each block contains the symbols received by subcarrier p. Then, the received signal model for the pth $N_t \times N_r$ block is

$$\mathbf{y}[p] = \mathbf{E}[p]\mathbf{x}[p] + \mathbf{n}[p] \tag{3}$$

where $\mathbf{E}[p]$ is a (non-orthogonal) $N_t \times N_r$ MIMO channel matrix, as perceived at the output of pth frequency bin at receiver.

2.3 Equivalent channel and CQI

The model above is covers conventional vector modulation (a.k.a BLAST). For many high-rate high-diversity MIMO-OFDM systems, the vector $\mathbf{x}$ is replaced by an $N_t \times$ T modulation matrix $\mathbf{X}$, where T is the block length. With Q input symbols the symbol rate is thus Q/T. Thus, the signal model is

$$\mathbf{Y} = \mathbf{E}\mathbf{X} + \mathbf{N}, \tag{4}$$

where matrix $\mathbf{N}$ contains noise terms for T channel uses. For the purposes of decoding, or for defining channel quality indicators, it is convenient to vectorize the model, by explicitly taking into account the structure of the MIMO modulation matrix $\mathbf{X}$. In this subsection we omit the subcarrier index to simplify notations. Namely, for the frequency-domain MIMO channel we write $\mathbf{E} \doteq \mathbf{E}[p]$ and similarly for the other symbols.

As an example, consider the vectorized model for Double ABBA modulator [5] that embeds $\mathbf{X}_A, \mathbf{X}_B, \mathbf{X}_C$ and $\mathbf{X}_D$ as four 2×2 STTD (Alamouti) blocks, encoding the symbol pairs (x_1,x_2), (x_3,x_4), (x_5,x_6) and (x_7,x_8), respectively. The symbol rate two modulator ($Q = 8, T = 4$) is

$$\mathbf{X}_{DABBA} = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{X}_A + \mathbf{X}_C & \mathbf{X}_B + \mathbf{X}_D \\ \mathbf{X}_B - \mathbf{X}_D & \mathbf{X}_A - \mathbf{X}_C \end{bmatrix} \tag{5}$$

We convert the signal model with matrix symbols $\mathbf{X}$ into an equivalent model comprising vector symbol $\mathbf{x}$, and state the related equivalent channel correlation matrix $\mathbf{R}_{eq} = \mathbf{E}_{eq}^{\dagger}\mathbf{E}_{eq}$ that arises when rewriting model (4) as

$$\mathbf{y} = \mathbf{E}_{eq}\mathbf{x} + \mathbf{n}. \tag{6}$$

The correlation matrix reads [5]

$$\mathbf{R}_{eq} = \begin{bmatrix} P_1+P_2 & S_1 & P_1-P_2 & S_2^{\dagger} \\ S_1 & P_1+P_2 & S_2 & P_2-P_1 \\ P_1-P_2 & S_2^{\dagger} & P_1+P_2 & S_1 \\ S_2 & P_2-P_1 & S_1 & P_1+P_2 \end{bmatrix} \tag{7}$$

where

$$P_1 = (\sum_{j=1}^{N_r} |e_{j,1}|^2 + |e_{j,2}|^2)\mathbf{I}_2, \tag{8}$$

$$P_2 = (\sum_{j=1}^{N_r} |e_{j,3}|^2 + |e_{j,4}|^2)\mathbf{I}_2, \tag{9}$$

where $S_1 = S + S^{\dagger}, S_2 = S - S^{\dagger}$, with

$$S = \begin{bmatrix} \alpha & -\beta^* \\ \beta & \alpha^* \end{bmatrix}, \tag{10}$$

where $\alpha = \sum_{j=1}^{N_r} e_{j,3}^* e_{j,1} + e_{j,2}^* e_{j,4}$ and $\beta = \sum_{j=1}^{N_r} e_{j,4}^* e_{j,1} + e_{j,2}^* e_{j,3}$.

The equivalent channel can be used to define simple performance metrics that can be exploited by the scheduler. One possible way of determining the merit of assigning a subcarrier to a given user is to compute the effective signal-to-noise ratio at the output of a MIMO equalizer or filter. Here, the model (for representative user and subcarrier) is

$$\mathbf{z} = \mathbf{R}_{eq}\mathbf{x} + \mathbf{n} \tag{11}$$

as given in the previous section. Here, due to matched-filtering with $\mathbf{E}_{eq}^{\dagger}$ at the receiver, the noise term is correlated. The symbol vector $\mathbf{x}$ is detected with a linear filter $\mathbf{L}$, operating on eq. (11), for which a simple performance estimate was derived in [8] by invoking the Gaussian approximation using coefficients

$$\gamma_{k',j} = (\mathbf{L}^{\dagger}\mathbf{R}_{eq})_{k',j},$$

$$\beta_{k'} = \frac{(\mathbf{L}^{\dagger}\mathbf{R}_{eq})_{k',k'}}{\sigma\sqrt{(\mathbf{L}^{\dagger}\mathbf{R}_{eq}\mathbf{L})_{k',k'}}},$$

and

$$\lambda_k'^2 = \frac{\beta_k'^2 \sum_{j \neq k'} \gamma_{k',j}^2}{\gamma_{k',k'}},$$

where k' is the symbols index. Using these notations, a computationally attractive and accurate approximation to the average error probability for a given subcarrier is

$$P_b = \frac{1}{Q}\sum_{k'=1}^{Q} Q(\frac{\beta_{k'}}{\sqrt{1+\lambda_{k'}^2}}). \tag{12}$$

Here, the fraction $\gamma_{k',j}/\gamma_{k',k'}$ quantifies interference leakage between the k'th and jth symbol. As is well known, this vanishes for the decorrelating detector, $\lambda_{k'}^2 = 0, \forall k'$. The signal-to-noise-ratio (SNR) approximation is

$$\mathrm{SNR}_{k'} = \frac{\beta_{k'}^2}{1+\lambda_{k'}^2} \tag{13}$$

For full diversity modulators where each symbol is treated equally (as in DABBA) and all symbols attain the same SNR and we can parameterize performance for each model in equation (14) with one number, e.g. aggregateSNR. With BLAST, each symbol generally has different SNR and it is more difficult to obtain a simple performance indicator.

3. OPTIMIZATION MODEL FOR MULTIUSER CHANNEL

In an OFDMA system different subcarriers may be assigned to different users. Considering the downlink, i.e. a point-to-multipoint link with K receivers, we have K different MIMO channels. Naturally, the benefit of assigning subcarrier p to user K depends on the user-specific channel realization, which we assume to be known at transmitter (e.g. via feedback channels as defined in [3]). The input to the subcarrier optimization algorithm is a set of feasible linear models

$$\mathbf{z}_k[p] = \mathbf{R}_{k,eq}[p] + \mathbf{n}_k[p],\ k = 1,..,K\ p = 1,..,P, \quad (14)$$

where the subscript k designates that the model is related to user ks channel. We assume that the same subcarrier can be assigned to only one user, and compress each linear model in (14) into one channel quality indicator (CQI), as detailed in previous section. The CQIs are then used as an input to a linear program, as given below.

3.1 Assignment problem

For notational convenience, we let $c_{k,p}$ designate the CQI in assigning subcarrier p to user k, and these are captured in matrix $\mathbf{C} = [c_{k,p}]$. As an example, we can let $c_{k,p} \doteq \mathrm{SNR[k,p]}$, where $\mathrm{SNR}[k,p]$ is defined as in eq. (13) for the corresponding model in (14). Alternatively, the CQI elements of the cost matrix may be defined as $-Q(\sqrt{SNR_{k,p}})$, if an allocation that achieves minimum bit-error-rate is of interest, or $\log_2(1+\sqrt{SNR_{k,p}})$ if mutual information is to be maximized. If $K < P$ we copy the rows of the matrix so that the cost matrix becomes square. If P is a multiple of K this can be done symmetrically for all rows. A matrix formed in this way is denoted below as $\tilde{\mathbf{C}}$.

Let $x_{k,p} = 1$ if subcarrier p is assigned to user k, otherwise, $x_{k,p} = 0$. Having selected the appropriate CQI, the assignment problem is posed as

$$\max \sum_p \sum_k c_{k,p} x_{k,p} \quad (15)$$

subject to

$$\sum_p x_{k,p} = 1, \forall k \quad (16)$$

$$\sum_k x_{k,p} = 1, \forall p, \quad (17)$$

$$x_{k,p} \geq 0, \forall p, k \quad (18)$$

Although the decision variables above are continuous, the optimal solution is known to be integral, where $x_{k,p} \in \{0,1\}$ $\forall k,p$ [1]. The constraints thus formalize the requirement that each subcarrier is assigned to exactly one user.

Due to integrality, the objective can be interpreted as the sum of CQIs, with the assumption that each subcarrier is used only once, and that all K channels (users) are assigned one subcarrier within a symbol period. Thus, all users get deterministically the same 'delay' and the number of channel uses, as these constitute our strict fairness criteria. Despite these constraints, the performance of the K users (channels) can be naturally still somewhat different, since the objective is to maximize CQIs over all users, subject to constraints. Without the fairness constraints, all subcarriers could be given to just one user. The constraints can be relaxed e.g. so that the users can be assigned any number of subcarriers. In this case we formulate a *transportation problem* [1], that holds the assignment problem as a special case.

Clearly, if the channel is flat, the CQI values are identical in each column, though generally different in each row. In this case, all assignments are equally good and one could simply select the subcarriers at random or sequentially in a round-robin fashion. Then, the benefit from channel-awareness is lost as the fairness constraints dominate the assignment solution. In effect, the optimization model, if the strict fairness constraints are maintained, is beneficial only in a frequency-selective channel.

3.2 Complexity reduction

In converting the problem to a square matrix the problem dimension remains at $P \times P$. Since the computational complexity of finding the optimal solution is a high order polynomial (approximately $O(P^4)$, depending on the algorithm [1]), it is important to reduce the problem dimensionality. The dimensionality is reduced via an approximation, which has to defined so that the performance or capacity loss remains tolerable.

A viable approximate solution can be obtained by utilizing correlations between different (e.g. neighboring) elements of the cost matrix, in analogy with [7]. The CQI correlations are largely due to channel correlations when considering subcarriers within channel coherence bandwidth. Using the correlations, we replace the original cost matrix with an approximate cost matrix. The approximate cost matrix may be computed as a (weighted) average the values of the utilities of c neighboring subcarriers. Algorithmically, this is implemented by defining a matrix

$$\mathbf{U} = \mathbf{I}_{P/c} \otimes \mathbf{1}_c \quad (19)$$

and forming a reduced dimensional model

$$\bar{\mathbf{C}} \leftarrow \mathbf{U}^T \tilde{\mathbf{C}} \mathbf{U} \quad (20)$$

If the c neighboring the values of the cost matrix are similar the performance loss is expected to be marginal. However, in practice the number c has to be carefully selected and matched to reflect the frequency-selectivity of the channel.

Clearly, as the matrix $\bar{\mathbf{C}}$ is compressed from the original cost matrix, the optimization solution will also relate to indices that correspond to the c subcarriers that were averaged when forming the lower-dimensional cost matrix. Traversing these indices back to the original indices is trivial.

4. PERFORMANCE

The use of the assignment algorithm results in fair channel-aware frequency domain scheduling. In this section we quantify the performance improvement for MIMO modulators in both uplink and downlink using a CQI that models BER. The transmission rate is 4 bps/Hz, since symbol rate 2 modulator (DABBA) is used with QPSK symbol constellation.

We evaluate the performance with and without complexity (dimension) reduction, and compare the results with round-robin scheduling (TDMA), where each user is assigned all subcarriers when accessing the channel. From the results, it will be seen that the multiuser diversity gain is apparent in all these cases, despite the strict fairness constraints.

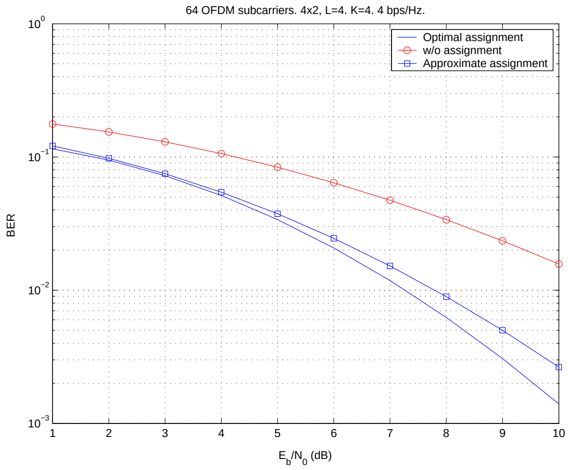

Figure 1: Subcarrier assignment in MIMO system using DABBA modulator (4 tx-2rx) in OFDMA with 4 users, 64 subcarriers in a four path channel. ”Downlink”

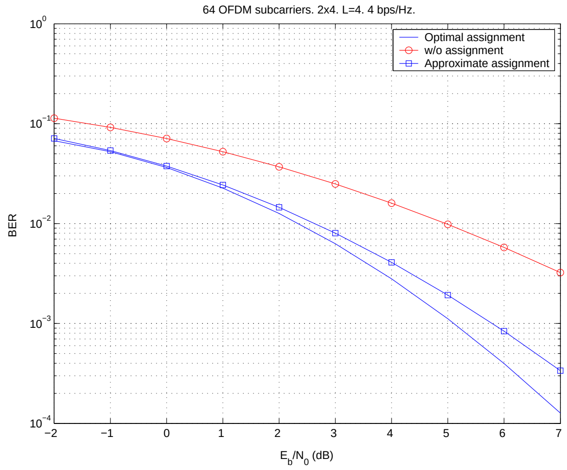

Figure 2: Subcarrier assignment in MIMO system using punctured DABBA modulator (2 tx-4rx) in OFDMA with 64 subcarriers in a four path channel. ”Uplink”

4.1 Downlink

The first example depicts the benefit of subcarrier assignment in conjunction with MIMO (DABBA) modulation in downlink. Here, we have 64 subcarriers, 4 users, 4 path iid rayleigh channel with minimal delay spread, statistically identical for all users. The subcarriers are assigned to these 4 users adaptively, using the assignment algorithm. Figure 1 shows the results using a decorrelating detector (both the defining the elements of the assignment matrix and in detection) for a case with 4 tx in the transmitter and 2 rx antennas in each of the 4 terminals.

The use of approximate assignment using complexity reduction, via assigning simultaneously a set of neighboring subcarriers, is seen to deteriorate performance only sightly when compared to the case where the assigned subcarrier need not be next to each other in frequency domain.

4.2 Uplink

Figure 2 shows the results using a decorrelating detector for a case with 2 tx in each of the 4 transmitters, and 4 rx antennas in the 4 receiver, corresponding to the uplink case. The MIMO transmission matrix for two tx antennas is formed by puncturing even columns from DABBA transmission matrix (”punctured DABBA”).

As in downlink, the use of approximate assignment using complexity reduction, via assigning simultaneously a set of neighboring subcarriers, is seen to deteriorate performance only sightly. The performance gain, when compared to downlink, is due to greater number of receive antennas.

5. CONCLUSION

Linear programming algorithms were used to determine optimal subcarrier assignments in a channel-aware MIMO-OFDMA system. The inter-stream interference was captured by appropriate channel quality indicators and applied in formulating the linear program. Methods to reduce the dimensionality of linear programming model (and complexity) were described. These methods offer significant computational savings at the expense of marginal performance deterioration.

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