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# EFFICIENT IMPLEMENTATION OF UNDECIMATED DIRECTIONAL FILTER BANKS

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## ABSTRACT

In this paper, an efficient method to implement undecimated directional filter banks (UDFBs) is proposed. The method is based on an observation that many non-separable two-dimensional two-channel FBs can be efficiently implemented by a separable structure if their polyphase components are separable. Therefore, with appropriate delay and advance blocks, undecimated non-separable FBs can be computed with a comparable computational complexity to the separable case. Structures for 2-, 4- and 8-channel UDFBs are presented to illustrate the idea.

## 1. INTRODUCTION

The directional filter bank (DFB), of which subband partitioning is presented in Fig. 1, was introduced by Bamberger and Smith [1]. A major property of the DFB is its ability to extract 2D directional information of an image, which is important in image analysis and other applications. The DFB is maximally decimated and perfect reconstruction (PR). This means that the total number of subbands' coefficients is the same as that of the original image, and they can be used to reconstruct the original image without error. The eight-channel DFB (Fig. 1(b)) can be implemented by a binary-tree structure consisting of three levels of two-channel systems. Each level can be implemented by using separable polyphase filters, which make the structure very computationally efficient.

One problem of image decomposition using decimated FBs is that the representations are not shift-invariant [2]. For many image analysis tasks, a critical representation of image is not necessary, and overcomplete decompositions are generally implemented. Directional filters employed in image analysis are usually non-separable, which are computationally expensive and difficult to implement. Thus, there exists a strong motivation for shift-invariant orientation filter with low computational complexity. Examples of undecimated DFBs (UDFB) are in [3, 4] for image enhancement and denosing applications, but their implementation is done by using two-channel non-separable FBs and has not taken advantage of the efficient structure in the conventional DFB [1]. The UDFB implementation proposed in [5] is based on the ladder structure for two-channel fan FBs. Since the polyphase realizations are separable, the orientation filters have much lower complexity than the non-separable ones. However, the framework is not optimal in the sense that the computation is carried out at twice the input rate and half of the computed outputs are discarded. Moreover, the extension from a fan FB to a $2^n$-channel DFB is not straightforward.

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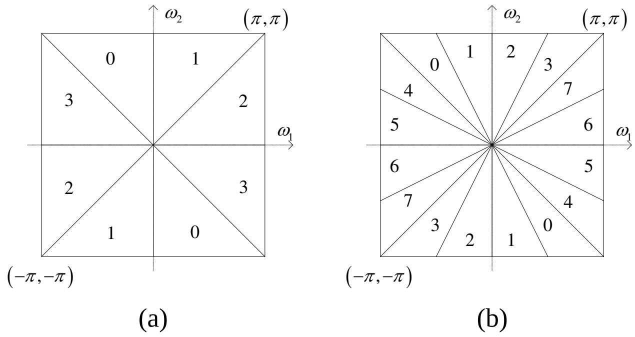


Figure 1: Frequency divisions of the conventional DFB [1] in case of: (a) four-channel DFB, and (b) eight-channel DFB.

*Paper outline*. An efficient structure to implement a 2-D FB is reviewed in the next section. Not all 2-D maximally decimated FB can be realized by the structure which requires only 1-D filtering. However, it is shown that many 2-D FBs with 'reasonable' frequency passband shapes are supported [6]. A structure for an undecimated 2-D FB is presented in Section 3 by using two copies of the separable polyphase block. Only one of these matrices is needed in the maximally-decimated case. The case of undecimated FBs having $2^n$ channels is considered in Section 4. The paper is concluded in Section 5.

*A note on notation*. Bold face letters represent vectors and matrices. The superscript $^T$ denotes the transpose operator. The matrix exponentials follow the notation used in [6], i.e.

$$[z_1, z_2]^T \begin{bmatrix} n_{00} & n_{01} \\ n_{10} & n_{11} \end{bmatrix} = (z_1^{n_{00}} z_2^{n_{10}}, z_1^{n_{01}} z_2^{n_{11}}). \quad (1)$$

$\mathbf{Q} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, $\mathbf{D}_0 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$, and $\mathbf{D}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$. Consequently, we have: $\mathbf{z}^{\mathbf{Q}} = \left([z_1, z_2]^T\right)^{\mathbf{Q}} = (z_1 z_2, z_1 z_2^{-1})$. The notation $\mathcal{N}(\mathbf{M})$ is defined as the set of integer vectors of the form $\mathbf{Mx}$ where $\mathbf{x} \in [0, 1)^2$.

## 2. SEPARABLE STRUCTURE FOR TWO-CHANNEL 2-D FBS

An example of a 2-D maximally decimated FB is in Fig. 2(a), where black and white regions indicate the stopband and passband in the 2-D frequency plane. The FB is called a fan or hourglass FB due to the shape of its passband supports. As one can easily see, these 2-D filters can not be realized by a separable structure; its $z$-transform is not the product of two polynomials of $z_1$ and $z_2$. In order to be separable, the passband shape must be rectangular and quadratically symmetric, which means that it has to be symmetric with respect to $\omega_1$ and $\omega_2$. However, in a critical sampling case, it is much more efficient to carry out the computation in polyphase domain (Fig. 2(b)) since the calculation is at a lower rate, and no computed coefficient is discarded. A 2-D FB is said to be *polyphase separable* if the components of the polyphase matrix are separable. For example, if the fan filters in Fig. 2(a) are given as

$$\begin{bmatrix} H_0^F(\boldsymbol{z}) \\ H_1^F(\boldsymbol{z}) \end{bmatrix} = \begin{bmatrix} H_{00}(\boldsymbol{z}^{\boldsymbol{Q}}) & H_{01}(\boldsymbol{z}^{\boldsymbol{Q}}) \\ H_{10}(\boldsymbol{z}^{\boldsymbol{Q}}) & H_{11}(\boldsymbol{z}^{\boldsymbol{Q}}) \end{bmatrix} \begin{bmatrix} 1 \\ z_1^{-1} \end{bmatrix}, = \boldsymbol{H}(\boldsymbol{z}^{\boldsymbol{Q}}) \begin{bmatrix} 1 \\ z_1^{-1} \end{bmatrix}, \quad (2)$$

where

$$H_i^F(z_1,z_2) = H_{i0}(z_1z_2, z_1z_2^{-1}) + z_1^{-1}H_{i1}(z_1z_2, z_1z_2^{-1}),$$

for $i=0,1$. The fan FB is polyphase separable iff each element $H_{ij}(z_1,z_2)$ of $\boldsymbol{H}(\boldsymbol{z})$ is a product of two 1-D filters, i.e. $H_{ij}(z_1,z_2) = \alpha_{ij}(z_1)\beta_{ij}(z_2), i,j = 0,1$. Therefore, it is interesting to see what passband shape can be polyphase separable. It is known that the polyphase components of a filter in a maximally decimated FB are approximately allpass [7]. Therefore $H_{ij}(\boldsymbol{z})$, $\alpha_{ij}(z_1)$, $\beta_{ij}(z_2)$, and $H_{ij}(\boldsymbol{z}^{\boldsymbol{Q}})$ are approximately allpass filters. Based on the possible phase functions of $H_{i0}(z_1z_2, z_1z_2^{-1})$ and $z_1^{-1}H_{i1}(z_1z_2, z_1z_2^{-1})$, one can show that the FB with diamond or fan-shape passband and decimation matrix $\boldsymbol{Q}$ can be implemented by separable polyphase components [8, 9].

After the determination of which 2-D FB can be implemented by 1-D polyphase, the next question is how to design the polyphase matrix. In the original DFB [1], the structure in Fig. 2(b) is used. This structure is a generalization of the quadrature mirror filters (QMF) to two-dimension. The main disadvantage of this method is that it is difficult to design the synthesis filters, since an FIR solution is not possible except for the trivial case of $\alpha_i(z)$, $\beta_i(z)$ being a delay. The most commonly-used method in realization of $\boldsymbol{H}(\boldsymbol{z})$ is the ladder structure [8](Fig. 2(c)). This method has lower complexity than the QMF approach, but the class of diamond FBs that can be implemented by this structure is rather limited. For example, one of the filters must be half-band, and one filter support is approximately twice the other one in the case of two ladder steps. The original construction of the structure in [8] is for 1-D and 2-D diamond FBs. It is generalized to quadrant FBs in [6] and fan FBs in [9]. Another method to implement $\boldsymbol{H}(\boldsymbol{z})$ is by using the lattice structure [7]. This method can yield exactly orthogonal FBs. However, it is difficult to design large filters with good passband and stopband characteristics because usually the objective function is highly nonlinear with respect to the lattice parameters [10].

Another important point that differentiate 2-D multirate systems from 1-D systems is that the signal can be resampled by a unimodular matrix (up or downsampled by matrices having determinant of $\pm 1$) and the signal contents remain unchanged. Therefore, by combining a resampling block with a 2-D FB, the effective frequency supports can be changed. For examples, the FB with parallelogram passband shape can be implemented by the diamond FB [1].

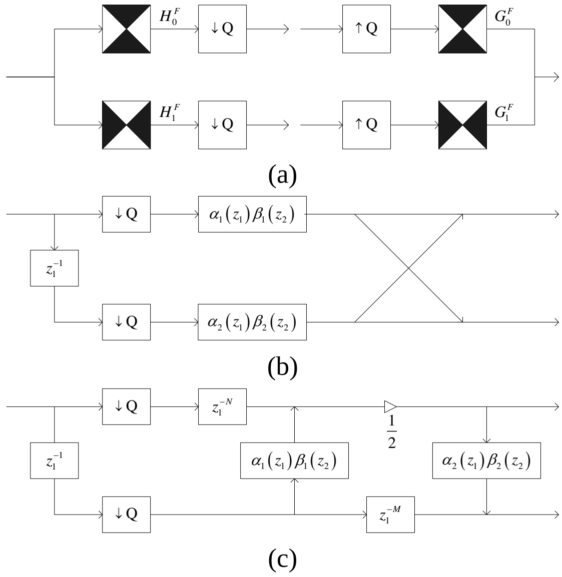


Figure 2: (a) Two-channel fan FB, (b) polyphase structure of a QMF FB and (c) polyphase structure using ladders.

## 3. EFFICIENT IMPLEMENTATION OF UNDECIMATED TWO-CHANNEL 2-D FBS

Let us consider an image $x(\mathbf{n})$, which is filtered by a filter $h_0(\mathbf{n})$. $X(\mathbf{z})$ and $H_0(\mathbf{z})$ can be written in the polyphase form as follows:

$$X(\mathbf{z}) = X_0(\mathbf{z}^{\mathbf{Q}}) + z_1X_1(\mathbf{z}^{\mathbf{Q}}), \quad (3)$$

and

$$H_0(\mathbf{z}) = H_{00}(\mathbf{z}^{\mathbf{Q}}) + z_1^{-1}H_{01}(\mathbf{z}^{\mathbf{Q}}). \quad (4)$$

The filtered signal $y_0(\mathbf{n})$ also has two polyphase components as in (5) at the top of next page. Let us define the following polyphase vectors as

$$\mathbf{x}(\mathbf{z}) = \begin{bmatrix} X_0(\mathbf{z}) \\ X_1(\mathbf{z}) \end{bmatrix}, \qquad \mathbf{x}^r(\mathbf{z}) = \begin{bmatrix} X_1(\mathbf{z}) \\ X_0(\mathbf{z}) \end{bmatrix},$$

$$\text{and} \quad \mathbf{h}_0(\mathbf{z}) = \begin{bmatrix} H_{00}(\mathbf{z}) \\ H_{01}(\mathbf{z}) \end{bmatrix}.$$

Then $Y_0(\mathbf{z})$ can be expressed as

$$Y_0(\mathbf{z}) = \mathbf{h}_0^T(\mathbf{z}^{\mathbf{Q}})\mathbf{x}(\mathbf{z}^{\mathbf{Q}}) + z_1\mathbf{h}_0^T(\mathbf{z}^{\mathbf{Q}})diag(1, z_1^{-\mathbf{Q}}z_2^{-\mathbf{Q}})\mathbf{x}^r(\mathbf{z}^{\mathbf{Q}}). \quad (6)$$

Similarly, the undecimated output $Y_1(\mathbf{z})$ in the second channel can be written in the same fashion. Therefore, the overcomplete two-channel FB can be implemented by the structure in Fig. 3

Intuitively, the structure in Fig. 3 can be viewed as follows. The upper polyphase block $\boldsymbol{H}(\boldsymbol{z})$ provides an output of a decimated FB, and we need to recover decimated coefficients in order to create undecimated images. Obviously, these lost coefficients will be obtained from the decimated FB if the input signal is appropriately shifted. That is precisely the reason for the delay block $z_1^{-1}z_2^{-1}$ before the lower polyphase matrix $\boldsymbol{H}(\boldsymbol{z})$ in Fig. 3. The outputs from the two

$$\begin{aligned} Y_0(\mathbf{z}) &= \left(X_0(\mathbf{z}^{\mathbf{Q}})H_{00}(\mathbf{z}^{\mathbf{Q}}) + X_1(\mathbf{z}^{\mathbf{Q}})H_{01}(\mathbf{z}^{\mathbf{Q}})\right) + \left(z_1^{-1}X_0(\mathbf{z}^{\mathbf{Q}})H_{01}(\mathbf{z}^{\mathbf{Q}}) + z_1X_1(\mathbf{z}^{\mathbf{Q}})H_{00}(\mathbf{z}^{\mathbf{Q}})\right) \\ &= \left(X_0(\mathbf{z}^{\mathbf{Q}})H_{00}(\mathbf{z}^{\mathbf{Q}}) + X_1(\mathbf{z}^{\mathbf{Q}})H_{01}(\mathbf{z}^{\mathbf{Q}})\right) + z_1\left(X_1(\mathbf{z}^{\mathbf{Q}})H_{00}(\mathbf{z}^{\mathbf{Q}}) + z_1^{-\mathbf{Q}}z_2^{-\mathbf{Q}}X_0(\mathbf{z}^{\mathbf{Q}})H_{01}(\mathbf{z}^{\mathbf{Q}})\right). \end{aligned} \quad (5)$$

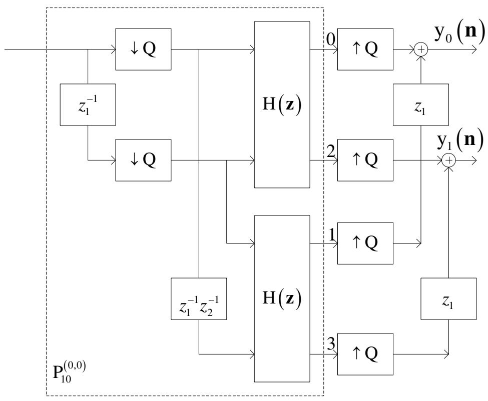


Figure 3: Quincunx UDFB.

polyphase matrices are then interlaced to form the two undecimated subband images $y_0(\boldsymbol{n})$ and $y_1(\boldsymbol{n})$.

Note that the implementation of the UDFB in [5] is also based on separable polyphase components of the decimated two-channel FBs. Instead of using double polyphase matrices, the input is upsampled by $\boldsymbol{Q}$ and processed in one polyphase matrix. The output signals are then decimated by $\boldsymbol{Q}$ to produce the desired subband images. Although the results of both methods are equivalent, the method in [5] discards half of the already computed coefficients. It is therefore concluded that the structure in Fig. 4 requires only half of the computation of that in [5]. Once this structure is used repeatedly in a binary-tree, the computational cost can be reduced even further.

## 4. TREE-STRUCTURE UDFB

One of the advantages of the conventional DFB is that it can be efficiently implemented by a binary-tree structure consisting of two-channel FBs with separable polyphase components. In fact, by cascading only 2-D FBs having that property, other type of FBs can be obtained, such as the nuqDFB [11]. Since it is possible to realize undecimated two-channel FBs by its separable polyphase structure, the undecimated version of a $2^n$-channel DFB can also be realized in a similar way.

### 4.1 Structure for undecimated four-channel DFB

A block diagram of the four-channel UDFB, whose frequency response is shown in Fig. 1(a), is presented in Fig. 4. The block $P_{10}^{(0,0)}$ is the same as that in Fig. 3. The blocks $P_{20}^{\boldsymbol{d}}$ and $P_{21}^{\boldsymbol{d}}$, where $\boldsymbol{d} \in \mathcal{N}(\boldsymbol{Q}) = \left\{[0,0]^T, [1,0]^T\right\}$, are similar to $P_{10}^{(0,0)}$ as they are the polyphase matrices of 2-nd level fan FBs. For simplicity, let $y_i(\boldsymbol{n}), i = 0,1,2,3$ be the four outputs of the four-band UDFB. Since the outputs 0 and 1 of $P_{10}^{(0,0)}$ in Fig. 3 are polyphase components of the first output of the two-channel FB, $y_0(\boldsymbol{n})$ must be obtained from the outputs of $P_{20}^{\boldsymbol{d}}$'s. It can be shown that first two outputs (0 and 1) of both $P_{20}^{\boldsymbol{d}}$ blocks are the four polyphase components of $y_0(\boldsymbol{n})$. Similarly, the other two outputs (2 and 3) correspond to $y_1(\boldsymbol{n})$. $y_2(\boldsymbol{n})$ and $y_3(\boldsymbol{n})$ can be obtained in the same fashion from both $P_{21}^{\boldsymbol{d}}$ blocks. Table 1 summarizes the polyphase components with their associated delays for each $y_i(\boldsymbol{n})$. In a maximally decimated case, the four subsampled subbands are at outputs 0 and 2 (marked by bold arrows in Fig. 4) of the top $P_{20}^{\boldsymbol{d}}$ and $P_{21}^{\boldsymbol{d}}$ blocks, which are the first polyphase components of $y_i(\boldsymbol{n})$.

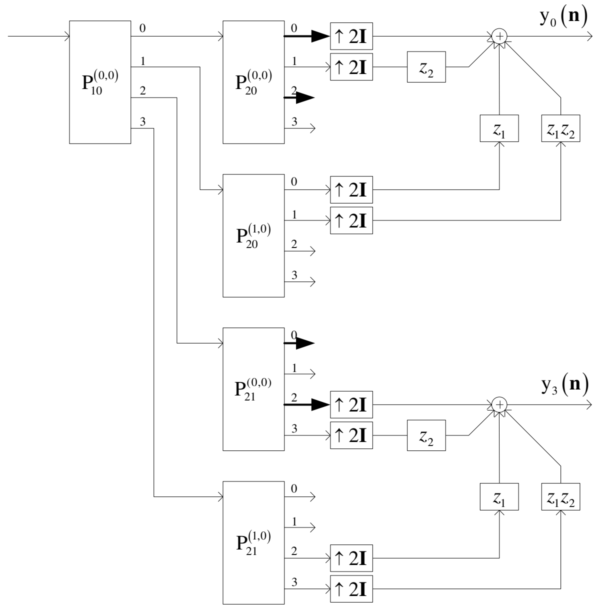


Figure 4: Four-channel UDFB.

### 4.2 Extension to $2^n$-channel UDFB

Generalization to $2^n$-channel can be done recursively by cascading the polyphase blocks $P_{nj}$ at the $4^{n-1}$ polyphase components (before upsampling) at level $n-1$. These new $4^n$ polyphase components are then upsampled by their corresponding (possibly different) decimation matrices, and followed by appropriate advances.

Let us consider the case of eight-channel UDFB. In the binary-tree structure of the eight-channel DFB in [1], whose subband frequency supports are presented in Fig. 1(b), four parallelogram two-channel FBs are used at the third level. The passbands of these two-channel FBs have parallelogram shapes, and the decimation matrices are $\boldsymbol{D}_0$'s for the first two FBs and $\boldsymbol{D}_1$'s for the others. Assume that these four FBs are polyphase separable, and let $P_{3j}^{\boldsymbol{d}}$ be their corresponding polyphase blocks, where $j = 0,\ldots,3$ and $\boldsymbol{d} \in \mathcal{N}(2\boldsymbol{I}) = \left\{[0,0]^T, [0,1]^T, [1,0]^T, [1,1]^T\right\}$. In order to create an un-

Table 1: The polyphase components at the output of $P_{2i}$ and its corresponding outputs in the four-channel UDFB in Fig. 4.

| | $P_{20}^{(0,0)}$ | $P_{20}^{(1,0)}$ | $P_{21}^{(0,0)}$ | $P_{21}^{(1,0)}$ |
|---|---|---|---|---|
| $y_0(\boldsymbol{n})$ | $0, 1(z_2)$ | $0(z_1), 1(z_1z_2)$ | | |
| $y_1(\boldsymbol{n})$ | $2, 3(z_2)$ | $2(z_1), 3(z_1z_2)$ | | |
| $y_2(\boldsymbol{n})$ | | | $0, 1(z_2)$ | $0(z_1), 1(z_1z_2)$ |
| $y_3(\boldsymbol{n})$ | | | $2, 3(z_2)$ | $2(z_1), 3(z_1z_2)$ |

Table 2: The polyphase components at the output of $P_{3i}$ and its corresponding outputs in the eight-channel UDFB in Fig. 5.

| $(i = 0, 1)$ | $P_{3i}^{(0,0)}$ | $P_{3i}^{(0,1)}$ | $P_{3i}^{(1,0)}$ | $P_{3i}^{(1,1)}$ |
|---|---|---|---|---|
| $y_{2i}(\boldsymbol{n})$ | $0$, | $0(z_2)$, | $0(z_1^2)$, | $0(z_1^2z_2)$, |
| | $1(z_1)$ | $1(z_1z_2)$ | $1(z_1^3)$ | $1(z_1^3z_2)$ |
| $y_{2i+1}(\boldsymbol{n})$ | $2$, | $2(z_2)$, | $2(z_1^2)$, | $2(z_1^2z_2)$, |
| | $3(z_1)$ | $3(z_1z_2)$ | $3(z_1^3)$ | $3(z_1^3z_2)$ |

| $(i = 0, 1)$ | $P_{3(i+2)}^{(0,0)}$ | $P_{3(i+2)}^{(0,1)}$ | $P_{3(i+2)}^{(1,0)}$ | $P_{3(i+2)}^{(1,1)}$ |
|---|---|---|---|---|
| $y_{2i+4}(\boldsymbol{n})$ | $0$, | $0(z_2^2)$, | $0(z_1)$, | $0(z_1z_2^2)$, |
| | $1(z_2)$ | $1(z_2^3)$ | $1(z_1z_2)$ | $1(z_1z_2^3)$ |
| $y_{2i+5}(\boldsymbol{n})$ | $2$, | $2(z_2^2)$, | $2(z_1)$, | $2(z_1z_2^2)$, |
| | $3(z_2)$ | $3(z_2^3)$ | $3(z_1z_2)$ | $3(z_1z_2^3)$ |

decimated version, four $P_{3j}^{\boldsymbol{d}}$ (of each $j$) are connected to the polyphase components corresponding to subband $j$ of the 2nd level (see Table 1). Fig. 5 shows the connection between the 2nd and 3rd levels of the tree using 16 blocks of $P_{3j}^{\boldsymbol{d}}$. This produces a total of $16\times 4 = 64$ polyphase components for the $8$ undecimated subbands. Thus, each subband image is composed of $8$ polyphase components. In general a $2^n$-channel UDFB would require $4^{n-1}$ blocks $P_{nj}^{\cdot}$, $0 \leq j < 2^{n-1}$, at the $n$-th level of the tree to provide $4^{n-1} \times 4 = 4^n$ polyphase components of the $2^n$ undecimated subbands. Note that filters used in each $P_{nj}^{\cdot}$ are the same as that of the decimated version, which can be polyphase separable for the DFB case.

According to Fig. 5, the polyphase outputs 0 and 1 of $P_{3j}^{\boldsymbol{d}}$ are components of the undecimated subband $2j$, and the polyphase outputs 2 and 3 belong to the undecimated subband $2j + 1$. These polyphase components are upsampled by appropriate decimation matrices. For the case of 8-channel DFB, the outputs of $P_{30}^{\boldsymbol{d}}$'s and $P_{31}^{\boldsymbol{d}}$'s are upsampled by $(2\boldsymbol{I})(\boldsymbol{D}_0) = 2\boldsymbol{D}_0$ whereas those of $P_{32}^{\boldsymbol{d}}$'s and $P_{33}^{\boldsymbol{d}}$'s are upsampled by $(2\boldsymbol{I})(\boldsymbol{D}_1) = 2\boldsymbol{D}_1$. These upsampled components are then shifted by different advances as shown in Fig. 5. Let $\boldsymbol{d}_0 = [1, 0]^T$ and $\boldsymbol{d}_1 = [0, 1]^T$. For each $\boldsymbol{d}$, it can be shown that:

1. outputs 0 and 2 of $P_{3j}^{\boldsymbol{d}}$ must be advanced by $\boldsymbol{z}^{\boldsymbol{D}_{\lfloor j/2 \rfloor}\boldsymbol{d}}$, and
2. outputs 1 and 3 of $P_{3j}^{\boldsymbol{d}}$ must be advanced by $\boldsymbol{z}^{\boldsymbol{D}_{\lfloor j/2 \rfloor}\boldsymbol{d}+\boldsymbol{d}_{\lfloor j/2 \rfloor}}$.

Table 2 summarizes the polyphase components with their associated delays for each $y_i(\boldsymbol{n})$.

Fig. 6 shows an example of a directional filter of an eight-channel UDFB. The filter is constructed using the proposed structure where the impulse responses in Fig. 6(a) are the eight polyphase components. The resulting impulse response obtained by interlacing these polyphase components is presented in Fig. 6(b) and its frequency response is in Fig. 6(c).

## 5. CONCLUSION

An efficient structure for the implementation of a shift-invariant directional analysis is presented in this paper. Although the discussion is limited to the case of UDFBs, the proposed approach can be applied to all undecimated FBs that have an efficient structure for their polyphase matrices. The directional filters implemented by the structure have computational complexity depending linearly on the size of the filters. The proposed structure reduces half of the computational compared to the previous approach in [5].

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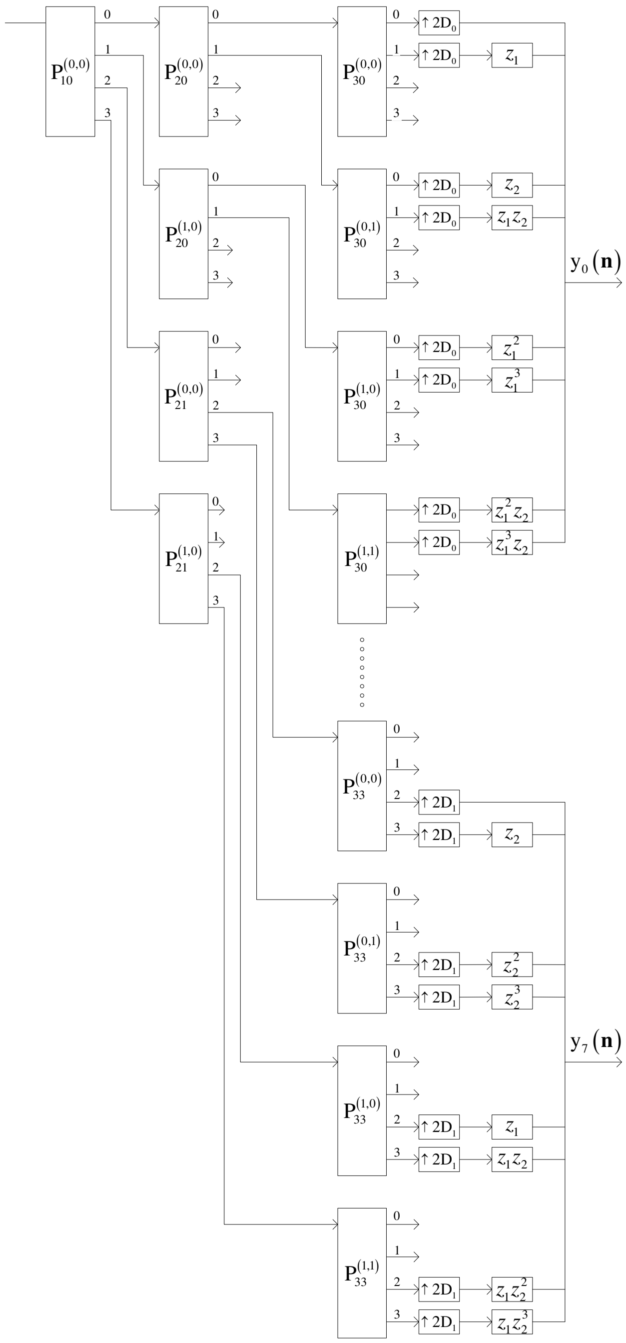
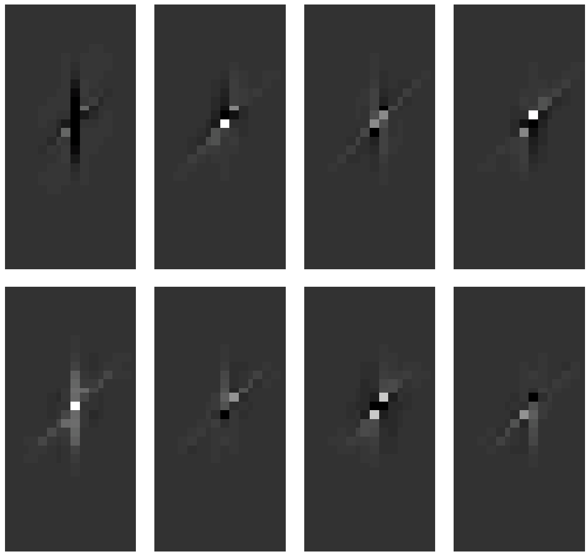
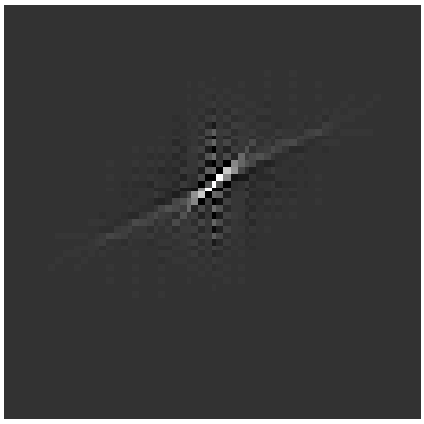


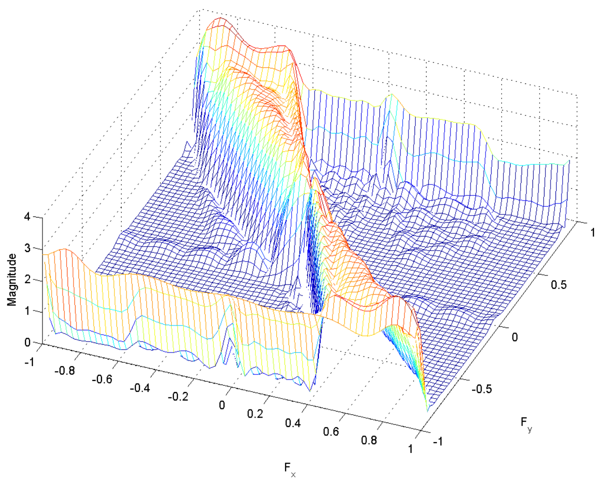

Figure 5: Eight-channel UDFB.



(a)



(b)




(c)

Figure 6: (a) Eight polyphase components of a filter in the conventional DFB.(b) The impulse response attained by up-sampling and interlacing the eight polyphase components and (c) The frequency response of the filter in (b).