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ROBUST ADAPTIVE BEAMFORMER WITH FEASIBILITY CONSTRAINT ON THE STEERING VECTOR

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ABSTRACT

The standard MVDR beamformer has high resolution and interference rejection capability when the array steering vector is accurately known. However, it is known to degrade if steering vector error exists. Motivated by recent work in robust adaptive beamforming, we develop variants of the constrained robust adaptive beamformer that attempt to limit the search in the underlying optimization problem to a feasible set of steering vectors thereby achieving improved performance. The robustness against steering vector error is provided through a spherical uncertainty set constraint, while a set of magnitude constraints is enforced on each element of the steering vector to better constrain the search in the space of feasible steering vectors. By appropriately changing the variables, the optimization problem is modified such that the need for the magnitude constraints are avoided. The developed algorithm is tested in the context of speech enhancement using a microphone array and shown to be superior to existing algorithms.

1. INTRODUCTION

The standard MVDR beamformer has high resolution and interference rejection capability when the array steering vector is accurately known [1]. However, the performance of traditional adaptive beamformer can degrade severely in practice when the Signal Of Interest (SOI) steering vector errors exist, which may be due to look direction error, array sensor position error, and small mismatches in the sensor responses. In such cases, the SOI might be mistaken as an interference signal and be suppressed. Many robust beamforming algorithms have been proposed to address this problem. Derivative constraint in the look direction is proposed in [2, 3]. Er and Cantoni proposed a robust beamforming algorithm which restricts the error between the desired and actual beam pattern of the array over a small spatial region around the array's look direction, allowing for uncertainty in the look direction [4]. Norm constrained and white noise gain constrained adaptive beamformer is studied in [5, 6] and widely used thereafter.

Recently some interesting robust adaptive beamformers have been proposed. Robust adaptive beamforming using worst-case performance optimization is proposed in [7, 8]. The problem is formulated as minimizing a quadratic function subject to infinitely many quadratic constraints. It is reduced to a second-order cone programming problem which can be solved by interior point methods. Li and Stoica proposed the robust Capon beamformer (RCB) [9] where a spherical uncertainty set constraint is enforced on the array steering vector. They also developed a doubly constrained robust Capon beamformer (DCRCB) [10] based on RCB, wherein a norm constraint on the beamformer steering vector is added. A comparison of these two beamformers is given in [11] and a geometrical explanation is provided.

In this paper, motivated by the constrained robust adaptive beamformer developed by Li and Stoica. We develop variants that attempt to limit the search in the underlying optimization problem to a feasible set of steering vectors thereby achieving improved performance.The robustness against steering vector error is provided through a spherical uncertainty set constraint, while a set of magnitude constraints is enforced on each element of the steering vector to better constrain the search to the space of feasible steering vectors. By appropriately changing the variables, the optimization problem is modified such that the need for the magnitude constraints are avoided. The developed algorithm is tested in the context of speech enhancement using a microphone array and shown to be superior to existing algorithms.

2. BACKGROUD

2.1 Standard MVDR Beamforming (MVDR)

The MVDR beamforming is also called Capon beamforming [1]. The problem is formulated as minimizing the output energy of the beamformer while maintaining a constant response in the look direction, i.e.

$$\min_{\mathbf{w}} \mathbf{w}^H R \mathbf{w}, \quad \text{s.t.} \quad \mathbf{w}^H \mathbf{a} = 1. \tag{1}$$

where R is the signal correlation matrix. $\mathbf{a}$ is the SOI steering vector. $\mathbf{w}$ is the beamformer weight vector. The solution to this optimization problem is given by

$$\mathbf{w} = \frac{R^{-1}\mathbf{a}}{\mathbf{a}^H R^{-1}\mathbf{a}}. \tag{2}$$

2.2 Robust Capon Beamforming (RCB)

The Robust Capon Beamforming (RCB) is proposed in [9]. Suppose $\mathbf{a}_0$ is the true SOI steering vector and $\overline{\mathbf{a}}$ is the assumed steering vector. $\mathbf{a}_0$ is assumed to be in the vicinity of $\overline{\mathbf{a}}$. This can be expressed mathematically by the following inequality

$$\|\mathbf{a}_0 - \overline{\mathbf{a}}\|^2 \le \varepsilon, \tag{3}$$

where ε is a bound controlling the uncertainty in the assumed look direction.

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The Capon beamforming problem can be reformulated as

$$\max_{\sigma^2} \sigma^2, \quad \text{s.t.} \quad R - \sigma^2 \mathbf{a}\mathbf{a}^H \geq 0. \tag{4}$$

where R is the signal correlation matrix. σ^2 is the signal power to be estimated.

Use the new formulation, one can write the RCB problem as

$$\max_{\sigma^2, \mathbf{a}} \sigma^2, \quad \text{s.t.} \quad R - \sigma^2 \mathbf{a}\mathbf{a}^H \geq 0 \quad \text{and} \quad \|\mathbf{a} - \bar{\mathbf{a}}\|^2 \leq \varepsilon. \tag{5}$$

Using the fact that, for any fixed $\mathbf{a}$, the solution to (4) with regard to σ^2 is obtained by

$$\hat{\sigma}^2 = 1/(\mathbf{a}^H R^{-1} \mathbf{a}) \tag{6}$$

the optimization problem (5) can be written as

$$\min_{\mathbf{a}} \mathbf{a}^H R^{-1} \mathbf{a}, \quad \text{s.t.} \quad \|\mathbf{a} - \bar{\mathbf{a}}\|^2 \leq \varepsilon. \tag{7}$$

The solution can be found using Lagrange multiplier method as

$$\hat{\mathbf{a}}_0 = \bar{\mathbf{a}} - U(I + \lambda\Gamma)^{-1} U^H \bar{\mathbf{a}} \tag{8}$$

where $R = U\Gamma U^H$ is the eigenvalue decomposition of R, and λ is the Lagrange multiplier. Once the SOI steering vector is estimated, the signal power can be estimated as in (6) and the beamformer weight vector is easily obtained as in MVDR beamforming (2).

One difficulty with this approach is that it tends to over-estimate the signal power σ^2, because both the SOI power and the SOI steering vector are taken as unknowns in problem (5). Thus, $(\sigma^2, \mathbf{a})$ and $(\sigma^2/\alpha, \alpha^{1/2}\mathbf{a}), \forall \alpha > 0$ will give the same item $\sigma^2 \mathbf{a}\mathbf{a}^H$. Suppose $(\sigma_0^2, \mathbf{a}_0)$ is the true solution to be found, the formulation of (5) will prefer the pair $(\sigma_0^2/\alpha, \alpha^{1/2}\mathbf{a}_0)$ if only $\alpha < 1$ and $\alpha^{1/2}\mathbf{a}_0$ is still in the uncertainty set. By the deduction above, we can be certain that the solution to (5) will make the inequality constraint in (5) active, i.e. $\|\hat{\mathbf{a}}_0 - \bar{\mathbf{a}}\|^2 = \varepsilon$. This problem is solved in [9] by a normalization step such that $\|\hat{\mathbf{a}}_0\|^2 = N$, where N is the number of sensor elements.

2.3 Doubly Constrained Robust Capon Beamforming (DCRCB)

To avoid the signal power overestimation problem discussed above in section 2.2, the Doubly Constrained Robust Capon Beamforming (DCRCB) is proposed [10]. The problem is formulated in a similar way as in (7) except that an extra norm constraint on the steering vector $\mathbf{a}$ is added.

The problem is formulated as

$$\min_{\mathbf{a}} \mathbf{a}^H R^{-1} \mathbf{a}, \quad \text{s.t.} \quad \|\mathbf{a} - \bar{\mathbf{a}}\|^2 \leq \varepsilon \quad \text{and} \quad \|\mathbf{a}\|^2 = N \tag{9}$$

The solution can be found using the Lagrange multiplier method

$$\hat{\mathbf{a}} = (N - \frac{\varepsilon}{2}) \frac{U(I + \lambda\Gamma)^{-1} U^H \bar{\mathbf{a}}}{\bar{\mathbf{a}}^H U(I + \lambda\Gamma)^{-1} U^H \bar{\mathbf{a}}} \tag{10}$$

where $R = U\Gamma U^H$ is the eigenvalue decomposition of R, and λ is the Lagrange multiplier. Then the beamformer weight vector is easily obtained as in MVDR beamforming (2).

In both RCB and DCRCB, the bound ε is chosen such that all possible SOI steering vectors $\mathbf{a}_0$ is included in the uncertainty set described by (3).

3. MAGNITUDE CONSTRAINED ROBUST MVDR BEAMFORMER

The RCB (section 2.2) and DCRCB (section 2.3) beamforming algorithms may fail because the optimum solution $\hat{\mathbf{a}}$ to the optimization problem (7) or (9) may not be a valid steering vector. A valid steering vector is usually structured and is not any arbitrary element in the constrained set (3). We develop variants of the constrained robust adaptive beamformer that attempt to limit the search in the underlying optimization problem to a feasible set of steering vectors thereby achieving improved performance. For an array with identical omni-directional sensors, a valid steering vector $\mathbf{a}$ can be expressed as $\mathbf{a} = [e^{-j\omega\tau_1}, e^{-j\omega\tau_2}, ..., e^{-j\omega\tau_N}]^T$ for the far field sources. We observe that each element of the the steering vector $\mathbf{a}$ has magnitude 1. Therefore an option is to enforce a set of magnitude constraints on each element of the steering vector $\mathbf{a}$ based on RCB (7) thereby making the search space smaller and more feasible. The new optimization problem can be formulated as

$$\min_{\mathbf{a}} \mathbf{a}^H R^{-1} \mathbf{a}, \quad \text{s.t.} \quad \|\mathbf{a} - \bar{\mathbf{a}}\|^2 \leq \varepsilon \quad \text{and} \quad |a_k| = 1, k = 1..N \tag{11}$$

where a_k is the k_{th} element of the steering vector $\mathbf{a}$, i.e. $\mathbf{a} = [a_1, a_2, ..., a_N]^T$. Unfortunately, a closed form solution to this optimization problem is not available and an optimization routine has to be utilized.

3.1 Time Delay Based Robust MVDR Beamformer (robMVDRtd)

By manipulating the variables, we can create a robust beamforming problem similar to problem (11). In particular, we use the form of the steering vector $\mathbf{a}_i$ for a specific frequency ω_i as

$$\mathbf{a}_i = [e^{-j\omega_i\tau_1}, e^{-j\omega_i\tau_2}, ..., e^{-j\omega_i\tau_N}]^T \tag{12}$$

As $|e^{j\omega_i\tau_k}| \equiv 1$, optimizing over the time delay variables τ_i ensures the magnitude constraint in (11) is automatically satisfied and thus need not be explicitly enforced. The new robust beamforming problem is formulated as

$$\min_{\tau} \mathbf{a}_i^H R_i^{-1} \mathbf{a}_i, \quad \text{s.t.} \quad |\tau_k - \bar{\tau}_k| \leq \delta_k, \quad k = 1..N \tag{13}$$

where R_i is the signal correlation matrix for frequency ω_i. $\tau = [\tau_1, \tau_2, ..., \tau_N]^T$, and $\bar{\tau} = [\bar{\tau}_1, \bar{\tau}_2, ..., \bar{\tau}_N]^T$ is the assumed look direction time delay vector. $\delta_k, k = 1..N$ is a set of bounds controlling the uncertainty in the look direction. The new problem (13) can be solved by using an appropriate optimization routine.

We use a subspace trust region method which is based on interior-reflective Newton algorithm to find the solution to problem (13). We need the gradient and Hessian of the objective function $h(\tau) = \mathbf{a}_i^H R_i^{-1} \mathbf{a}_i$, where $\mathbf{a}_i$ is specified by (12). It is straightforward to obtain gradient as

$$\nabla_\tau h = A R_i^{-T} \mathbf{a}_i^* + A^* R_i^{-1} \mathbf{a}_i \tag{14a}$$

$$= real(A^* R_i^{-1} \mathbf{a}_i) \tag{14b}$$

where $(.)^*$ denotes conjugate and $(.)^T$ denotes transpose.

$$A = (-j\omega_i) \begin{bmatrix} a_1 & 0 & \cdots & 0 \\ 0 & a_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_N \end{bmatrix} \tag{15}$$

where a_k is the k_{th} element of the steering vector $\mathbf{a}_i$, i.e. $\mathbf{a}_i = [a_1, a_2, ..., a_N]^T$.

Also, the Hessian is obtained as

$$\nabla_\tau^2 h = real((-j\omega_i) diag(\mathbf{a}_i^H R_i^{-1}) A + A R_i^{-T} A^*) \tag{16}$$

In the context of broadband signals, for each frequency component ω_i of the signal one has to solve a problem like (13). However, the objective minimizer $\hat{\tau}$ is the true time delay from the SOI to each microphone element, which doesn't depend on the frequency ω_i. In other words, we want to find a common minimizer $\hat{\tau}$ that is valid for all the frequencies. This is not automatic and has to be enforced. It can be achieved by combining the series of beamforming problems on individual frequency bins into a single problem to provide robustness. The broadband beamforming problem can be formulated as

$$\min_\tau \sum_i \mathbf{a}_i^H R_i^{-1} \mathbf{a}_i, \quad \text{s.t.} \quad |\tau_k - \overline{\tau}_k| \le \delta_k, \quad k = 1..N \tag{17}$$

3.2 Angle Based Robust MVDR Beamformer (robMVDRangle)

The RCB (section 2.2), DCRCB (section 2.3) and robMVDRtd (section 3.1) algorithms assume uncertainty in the steering vector, which takes both the SOI look direction error and the array sensor's position error into consideration. The problem can be simplified when only SOI look direction error exists. For instance, in the case of 2-dimensional space the sources' incidence directions can be represented by only one parameter θ. Hence, we can use $\mathbf{v}(\theta)$ to substitute for the steering vector $\mathbf{a}$ in (13). The new robust beamforming problem can be written as

$$\min_\theta \mathbf{v}(\theta)^H R^{-1} \mathbf{v}(\theta), \quad \text{s.t.} \quad |\theta - \overline{\theta}| \le \varepsilon \tag{18}$$

where $\mathbf{v}(\theta) = [e^{-j\omega\tau_1}, e^{-j\omega\tau_2}, ..., e^{-j\omega\tau_N}]^T$, and $\tau_i, i = 1, .., N$ is functions of θ based on the geometry of the array. $\overline{\theta}$ is the assumed look direction. ε is a bound controlling the uncertainty in the assumed look direction. The problem (18) can be solved by one dimensional numerical optimization algorithm such as the golden section search method.

4. SIMULATION

4.1 Beamforming Algorithms Notation

We use the following notation for each beamforming algorithm.

- OMVDR: the ideal MVDR beamforming which assumes we know the true SOI steering vector
- MVDR: standard MVDR beamforming
- DS: conventional delay and sum beamforming
- RCB: robust Capon beamforming (section 2.2)
- DCRCB: doubly constrained robust Capon beamforming (section 2.3)
- robMVDRtd: time delay based robust MVDR beamforming (section 3.1)
- robMVDRangle: angle based robust MVDR beamforming (section 3.2)

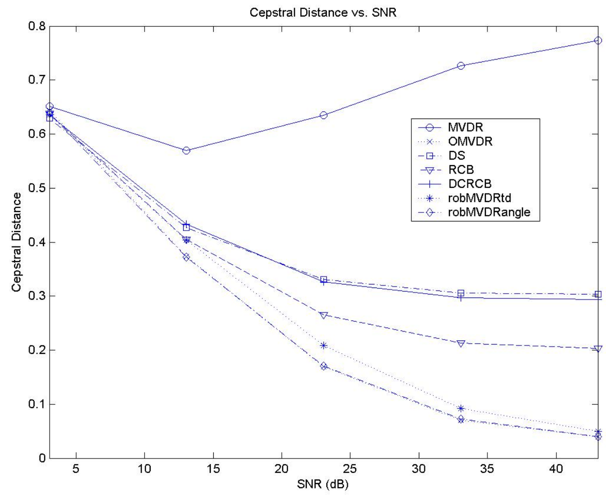

Figure 1: Cepstral distance between recovered signal's spectrum and the SOI's spectrum, only look direction error exists.

4.2 Simulation Scenario

In this section, we provide numerical examples on speech enhancement using a microphone array to compare the performances of various beamformers. We assume a circular microphone array with 8 sensors. The 8 sensors are equally distributed on a 20cm diameter circle and indexed counter clockwise. The sources, both the SOI and interference signals, are plane waves which exist in the same plane as the circular array. We define the origin of the coordinate system to be the center of the circular microphone array, and define angle 0° to be the direction of the 8_{th} microphone. The angle increases counter clockwise, which means the 1_{st} microphone is at angle 45°, the 2_{nd} microphone is at angle 90°, and so on. In the simulation, every source signal is a single channel speech sentence, which is around 1s in duration. The sampling rate is 8kHz. Short Time Fourier Transform (STFT) is used to transform the multichannel data into the frequency domain and the narrowband beamforming algorithms are then applied. The frame length is 0.25s (200 samples), with a step length of 0.125s (100 samples). A 256 points FFT is used on each frame.

4.3 Simulation Results

The performance of various beamformers is measured by the cepstral distance between the recovered signal's spectrum and the original SOI's spectrum. The cepstral distance is used because it is a perceptual metric commonly used in speech processing to measure distortion. Fig.1 shows the beamformers' performance versus SNR, which is signal to white noise ratio. Only one SOI and one interference signal exist in this experiment. The interference signal and SOI has the same level of energy. The interference signal come from direction 90°. The assumed look direction is 180°, while the true SOI direction is 178°, which means a 2° look direction error. There's no sensor position error in this experiment.

Fig.2 shows the beamformers' performance by cepstral distance when there exist not only the look direction error, but also sensor position error. The displacement error for each sensor is generated by an uniformly distributed random

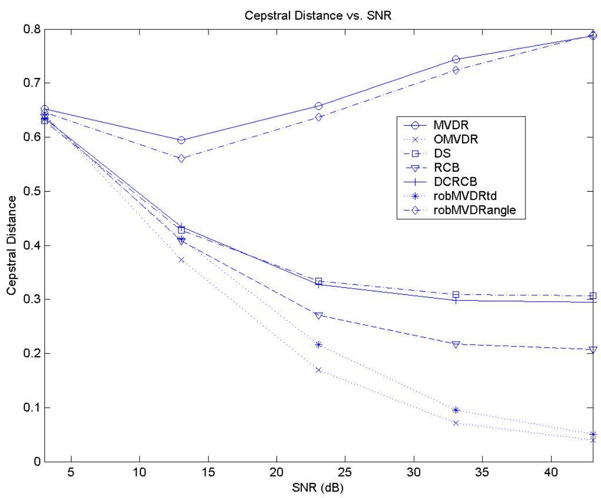

Figure 2: Cepstral distance between recovered signal's spectrum and the SOI's spectrum, both look direction error and sensor position error exist.

variable whose maximum value is 3mm. All the other settings are the same as those of the aforementioned experiment.

The OMVDR beamformer gives the optimal performance and bounds the performance that can be attained by these class of adaptive beamformers. Our simulation results clearly demonstrate that the proposed robMVDRtd beamformer consistently performance well and is very close in performance to the OMVDR beamformer. The robMVDRtd beamformer outperforms the conventional fixed DS beamformer and other adaptive beamformers such as MVDR, RCB and DCRCB. The proposed robMVDRangle beamformer works extremely well when only look direction error exists. It has the same performance as the optimal OMVDR beamformer in this condition. However, it deteriorates and has performance comparable to the standard MVDR beamformer when sensor position error exists. This highlights the sensitivity of angle based formulation. Although the norm constraint on steering vector is introduced in the DCRCB method to prevent overestimation of signal power in the RCB method, our simulations show that the DCRCB method has worse performance than the RCB method. This can be explained by noting that even when an extra norm constraint on steering vector is added, the minimizer to the optimization problem (9) is still not a valid steering vector. This phenomenon can be observed by examining the beampatterns in Fig.3. Listening to the reconstructed speech indicates the output of the RCB/DCRCB to be better than the DS beamformer even though it is not evident from the cepstral distance measure employed.

Fig.3 shows the magnitude beam pattern of various beamformers on one sample data. This sample data is selected from the data set used to generate Fig.1. The SNR is 43dB. It is evident that the beam pattern of the robMVDRtd method is close to that of the OMVDR beamformer. The MVDR beamformer forms two deep nulls, one in the interference direction, the other in the SOI direction. The RCB and DCRCB method can steer a null in the interference direction (90°) at low frequency range, while at middle to high frequency range, their beam patterns are similar to that of DS beamformer. This can be explained by the choice of uncertainty bound ε. The bound ε is chosen such that all possible SOI steering vectors $\mathbf{a}_0$ are included in the prescribed uncertainty set. This usually brings on a big value of ε at high frequency, which results in many infeasible steering vectors being included in the uncertainty set (3). Thereby the minimizer to the optimization problem (7) and (9) is no longer a valid steering vector in the high frequency range. A close observation of the steering vector which minimizes the optimization problem (7) and (9) at high frequency range confirms the above reasoning. The element magnitudes of those steering vectors have been found to be far away from 1.

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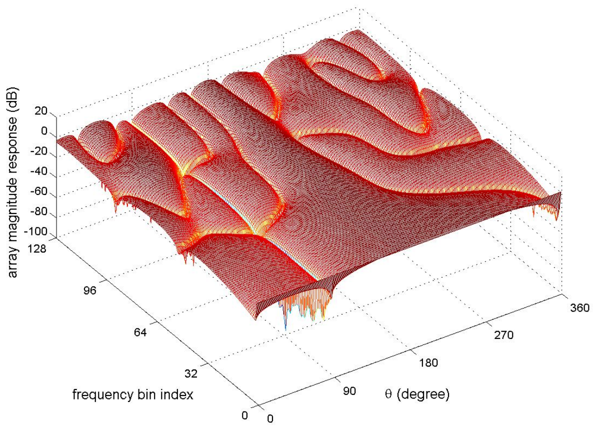
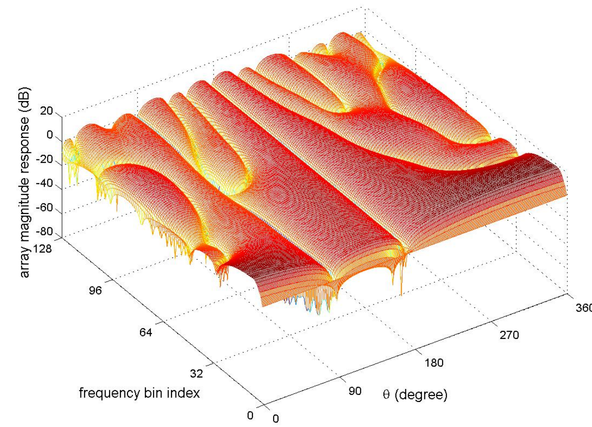
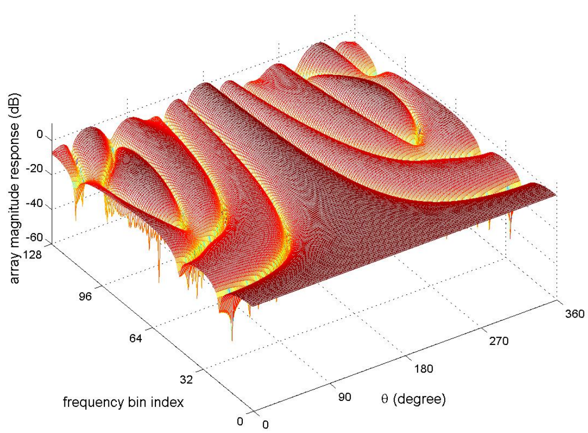


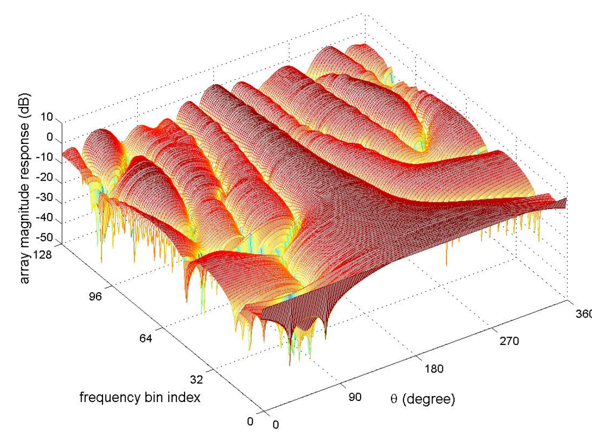
(a) OMVDR beamformer



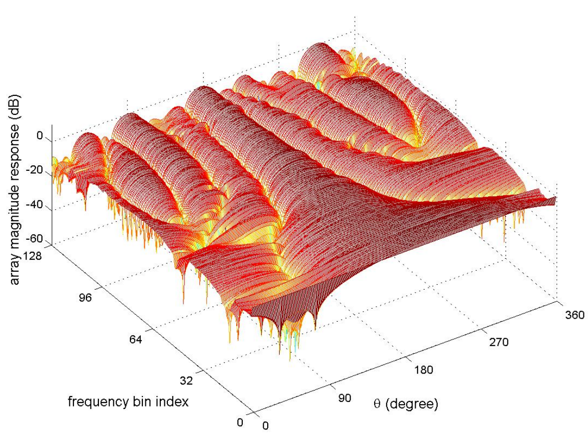
(b) standard MVDR beamformer



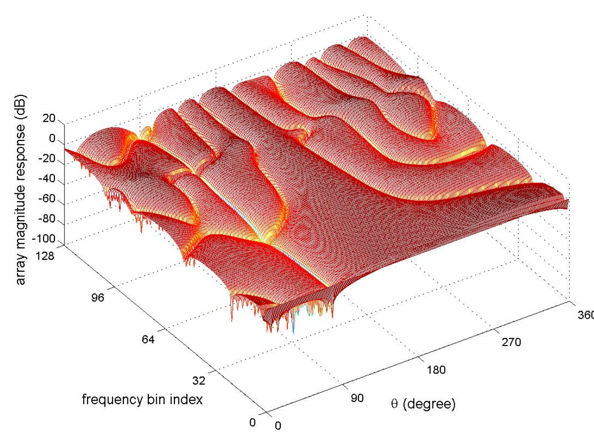
(c) DS beamformer



(d) RCB beamformer



(e) DCRCB beamformer



(f) robMVDRtd beamformer

Figure 3: Beam pattern of various beamformers over angle θ and frequency bins. The look direction is 180°, the true SOI direction is 178°, and the interference direction is 90°.