

MITIGATION OF NARROWBAND INTERFERENCE USING ADAPTIVE EQUALIZERS

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ABSTRACT

It has previously been shown that a decision-feedback filter can mitigate the effect of narrowband interference. An adaptive implementation of the filter was seen to converge relatively quickly for mild interference. It is shown here, however, that in the case of severe narrowband interference, the decision-feedback equalizer (DFE) requires a convergence time that makes it unsuitable for some types of communication systems. The introduction of a linear predictor, as a pre-filter to this equalizer, greatly reduces the total convergence time. There is a trade-off, however, between convergence time and steady-state performance, and that is evaluated in this paper.

1. INTRODUCTION

A wireless communication channel can be severely degraded in the presence of severe narrowband interference (NBI). Li and Milstein discuss the use of decision-feedback filters [1] to mitigate the effects of narrowband interference in spread spectrum communications systems. The least mean-square (LMS) adaptive algorithm is used to approach the optimal Wiener filter and it is shown that a reasonable convergence time is achievable for a modest signal-to-interference ratio (SIR) [1]. It is discussed below that SIR governs the convergence of the adaptive algorithm.

Although it has been shown that alternate adaptive algorithms, such as the recursive least squares (RLS) algorithm [2], provide improved convergence relative to the LMS algorithm in cases of high eigenvalue disparity, there are many reasons why LMS is chosen for practical communications system applications. Hassibi discusses [3] some of the fundamental differences in the performance of gradient based estimators such as the LMS algorithm and time averaged recursive estimators such as the RLS algorithm in the cases of modeling errors and incomplete statistical information concerning the input signal, interference, and noise parameters. Hassibi [3] examines the conditions for which LMS can be shown to be more robust to variations and uncertainties in the signaling environment than RLS. LMS has also been shown to track more accurately than RLS because it is able to base the filter updates on the instantaneous error rather than the time averaged error [4, and references therein]. The improved tracking performance of LMS over RLS for a linear chirp input is well established [2,4]. In [5] it is shown that an extended RLS filter that estimates the chirp rate of the input

signal can minimize the tracking errors associated with the RLS algorithm and provide performance that exceeds that of LMS. It should be noted however, that the improved tracking performance requires a significant increase in computational complexity and knowledge that the underlying variations in the input signal can be accurately modeled by a linear FM chirp. For cases where the input is not accurately represented by the linear chirp model, performance can be expected to be significantly worse than simply using an LMS estimator, for the reasons discussed in [3]. The computational complexity of RLS, in particular for high order systems, favors the use of LMS. The latter is also more robust in fixed-point implementations. In addition the LMS estimator has been shown to provide nonlinear, time-varying weight dynamics that allow the LMS filter to perform significantly better than the time-invariant Wiener filter in several cases of practical interest [6–8]. It is further shown that the improved performance associated with these non-Wiener effects is difficult to realize for RLS estimators due to the time-averaging that is inherent in the estimation process [9].

In this paper we provide an alternate two-stage structure that employs an LMS linear predictor in conjunction with the LMS decision-feedback equalizer (DFE) to provide significant improvements in convergence rate for the case of large eigenvalue disparity. This behavior stems from the fact that the equalizer does not have a true reference for the interference. For strong interference the predictor generates a direct reference for the interference and mitigates it prior to equalization. This is shown to provide a dramatic reduction in convergence time. It is also shown, however, that there is a slight loss in absolute BER performance after convergence due to residual errors introduced in the prediction stage. The trade-off between convergence time and BER performance for the two-stage system is investigated. It is shown that excellent BER performance can be achieved in reasonable convergence times with the two-stage system in the presence of severe interference.

2. SYSTEM MODEL

A single-carrier communication system in complex baseband representation is depicted in Fig. 1. While emerging multicarrier technologies such as orthogonal frequency division multiplexing (OFDM) have become prevalent, this work is intended for implementation in pre-existing single-carrier systems. The input signal, d_k , is defined to be *i.i.d.* BPSK symbols. The input signal is passed through a pulse shaping filter that is necessary for bandlimited transmission. This signal is corrupted by narrowband interference that is modeled as a pure complex exponential and additive white Gaus-

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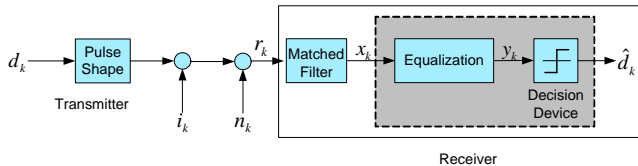


Figure 1: System model.

sian noise. A matched filter is employed at the receiver to maximize the signal-to-noise ratio (SNR) at the output of the filter. Note that the overall frequency response of the pulse shape and the matched filter is assumed to satisfy Nyquist's criterion for no intersymbol interference (ISI).

The signal at the input to the equalizer, x_k , is defined as

$$x_k = \sqrt{S}d_k + \sqrt{J}e^{j(\Omega kT + \theta)} + n_k, \quad (1)$$

where T is the symbol duration, S is the signal power, J is the interferer power, Ω is the angular frequency of the interferer, and θ is a random phase that is uniformly distributed between 0 and 2π . The additive noise, n_k is modeled as a zero-mean Gaussian random process with variance σ_n^2 . It is assumed that the communication signal, interferer, and noise are uncorrelated to each other. Note that the filters operate at the symbol rate.

3. ADAPTIVE ALGORITHM

The LMS algorithm [2] is defined by the following three equations:

$$\hat{d}_k = \mathbf{w}_k^H \mathbf{x}_k, \quad (2)$$

$$e_k = d_k - \hat{d}_k, \quad (3)$$

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mu e_k^* \mathbf{x}_k, \quad (4)$$

where \mathbf{x}_k is the N -tap input vector defined as

$$\mathbf{x}_k = [x_k \quad x_{k-1} \quad \cdots \quad x_{k-(N-1)}]^T \quad (5)$$

and N is the filter order, \mathbf{w}_k is the vector of adaptive tap weights, d_k is the desired signal, \hat{d}_k is the estimate of the desired signal, e_k is the error signal, and μ is the step-size parameter. Finally, $(\cdot)^*$ indicates conjugation, $(\cdot)^T$ is the transpose operator and $(\cdot)^H$ represents conjugate (Hermitian) transpose.

3.1 LMS Convergence

The step-size parameter is chosen in a manner to guarantee convergence in the mean-square sense, namely

$$0 < \mu < \frac{1}{\lambda_{\max}}, \quad (6)$$

where λ_{\max} is the maximum eigenvalue of the input autocorrelation matrix.

Shensa [10] showed that the convergence of the weight vector can be reflected as

$$\|\mathbf{w}_{\text{opt}} - \mathbf{E}[\mathbf{w}_k]\|^2 = \sum_{i=1}^N (1 - \mu \lambda_i)^{2k} |\mathbf{v}^{iH} \mathbf{w}_{\text{opt}}|^2, \quad (7)$$

where λ_i are the eigenvalues and \mathbf{v}^i are the eigenvectors of the input autocorrelation matrix. The optimal Wiener solution is represented by \mathbf{w}_{opt} . A similar equation arises for the

convergence of the mean-square error (MSE) [11], when gradient noise (on the order of $L\mu E[e_{\min}^2]$) is neglected

$$\|E[e_k^2] - E[e_{\min}^2]\|^2 = \sum_{i=1}^N (1 - \mu \lambda_i)^{2k} \lambda_i |\mathbf{v}^{iH} \mathbf{w}_{\text{opt}}|^2. \quad (8)$$

A time constant for each mode [2] is defined as

$$\tau_i \simeq \frac{1}{2\mu \lambda_i}. \quad (9)$$

The maximum modal time constant will be associated with the minimum eigenvalue,

$$\tau_{\max} \simeq \frac{1}{2\mu \lambda_{\min}}. \quad (10)$$

This maximal time constant can be seen to be a conservative estimate by examining (7) more closely. The convergence will be influenced only by those eigenvalues from which the projection of the corresponding eigenvector on the optimal weights is large. Lastly, it can be seen for the case of $\lambda_i \ll 1$, that it is possible for the convergence of the filter output (mean-square error) to be faster than the convergence of the filter weights. This is because there will be fewer modes controlling the MSE convergence (i.e. $\lambda_i |\mathbf{v}^{iH} \mathbf{w}_{\text{opt}}| < |\mathbf{v}^{iH} \mathbf{w}_{\text{opt}}|$).

3.2 Eigenvalues

The eigenvalues for the correlation matrix given by (1) can be found [10, 12, 13] to be equal to

$$\lambda = \begin{cases} S + NJ + \sigma_n^2, & 1 \text{ eigenvalue} \\ S + \sigma_n^2, & N-1 \text{ eigenvalues} \end{cases} \quad (11)$$

4. EQUALIZER STRUCTURE

The DFE is composed of a transversal feedforward filter with $M + 1$ taps (one main tap and M side taps) and a feedback filter that has M taps. The feedback taps allow the equalizer to cancel out postcursor ISI caused by the feedforward taps. A block diagram of the DFE is shown in Fig. 2. The output of the filter, y_k , with input x_k is

$$y_k = \sum_{l=0}^M w_l x_{k-l} + \sum_{l=1}^M f_l \hat{d}_{k-l}, \quad (12)$$

where \hat{d}_k is the estimate of the symbol d_k out of the decision device. Note that w_l are the tap weights associated with the feedforward filter, and f_l are the tap weights associated with the feedback filter.

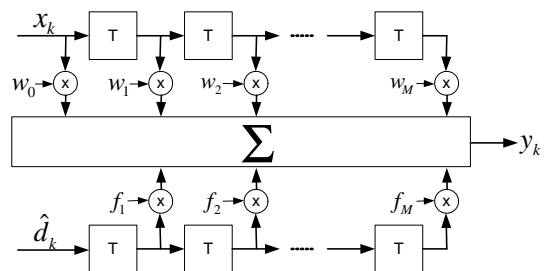


Figure 2: Decision-feedback equalizer block diagram.

The optimal weights under the minimum mean-square error criterion can be found using the orthogonality principle [2]. $2M + 1$ equations are obtained, and the weights can be solved for using the method described in [1, 14]. The optimal main tap weight, feedforward side tap weights, and feedback tap weights are respectively,

$$w_0 = \frac{(\sigma_n^2 + MJ)S}{(S + \sigma_n^2)(\sigma_n^2 + MJ) + \sigma_n^2 J}, \quad (13)$$

$$w_l = \frac{-JS}{(S + \sigma_n^2)(\sigma_n^2 + MJ) + \sigma_n^2 J}, \quad l = 1, \dots, M, \quad (14)$$

$$f_l = \frac{JS}{(S + \sigma_n^2)(\sigma_n^2 + MJ) + \sigma_n^2 J}, \quad l = 1, \dots, M. \quad (15)$$

The weight of the feedback taps is the negative of the feedforward side taps. This implies that if the data fed back is perfect, the ISI caused by the feedforward filter will be completely canceled. The BER analysis of the DFE with error propagation utilizes Markov chains to model the term $[d_{k-l} - \hat{d}_{k-l}]$ as contents of a shift register and the assumption that the decisions fed back are perfect [1, 15–17].

Finally, we look at the convergence properties of the DFE when implemented using the LMS algorithm. As seen in [1], we can obtain a recursive equation for the evolution of the tap weights starting from the weight update equation under the assumption of a small step-size (μ) [2]. The mean weight evolution of the main tap, feedforward side taps ($l = 1, \dots, M$), and the feedback taps ($l = 1, \dots, M$) are given respectively by,

$$u_0(k+1) = [1 - \mu(S + \sigma_n^2 + J)]u_0(k) - \mu MJu_l(k) + \mu S, \quad (16)$$

$$u_l(k+1) = [1 - \mu(S + \sigma_n^2 + MJ)]u_l(k) - \mu Ju_0(k) - \mu S v_l(k), \quad (17)$$

$$v_l(k+1) = [1 - \mu S]v_l(k) - \mu S u_l(k). \quad (18)$$

As in [1], the difference equations of (16)-(18) can be solved using the Z-transform and the final value theorem to yield the steady-state value results given in (13)-(15).

We remark that the step-sizes for the feedforward filter and the feedback filter can be different.

4.1 Equalizer Convergence in Severe NBI

Li and Milstein demonstrate that the feedforward and feedback taps of a decision-feedback filter converge relatively quickly, in on the order of 800 iterations, for the case of SIR = 0 dB, $E_b/N_0 = 20$ dB, $\mu = 0.01$ and $M = 3$ [1]. Results for these conditions can be seen in Fig. 3 when $E_b/N_0 = 10$ dB and $M = 6$.

Fig. 4 illustrates the required convergence time of the equalizer for the case of SIR = -20 dB. For this case convergence could not be achieved with $\mu = 0.01$ and it was necessary to decrease μ to 10^{-4} or less. The convergence time shown in Fig. 4 is seen to be approximately 60,000 iterations for this case. The time constant (10) is proportional to the minimum eigenvalue (i.e. $\tau_{DFE} \approx 1/2\mu(S + \sigma_n^2)$) because the projection of all the eigenvectors on the optimal weight vector is nonzero. The delay in convergence can be attributed to the fact that the DFE does not have a direct reference for the interferer during adaptation and is thus forced to

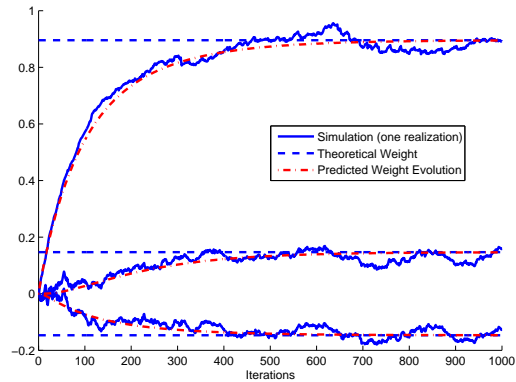


Figure 3: Decision-feedback equalizer tap weight evolution for $M = 6$, SIR = 0 dB, and $\mu = 0.01$.

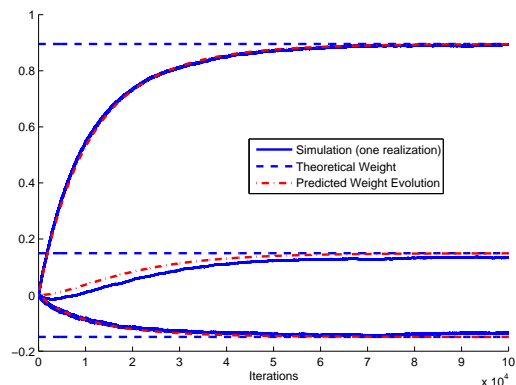


Figure 4: Decision-feedback equalizer tap weight evolution for $M = 6$, SIR = -20 dB, and $\mu = 0.0001$.

converge on the basis of the training data only. The feedback taps converge slower than the feedforward taps due to the fact that the DFE is designed such that the interferer is canceled by the feedforward taps, while the feedback taps attempt to cancel out the signal distortion caused by the feedforward taps [1].

5. PREDICTION FILTER

The linear predictor (LP) was introduced as a technique to remove narrowband interference in many applications [13, 14, 17–19]. The filter is able to predict the interferer well, due to its narrowband properties. A block diagram of the LP is shown in Fig. 5. The LP is a transversal filter with L taps. The delay by Δ ensures that the signal of interest at the current sample is decorrelated from the samples in the filter when calculating the error term. The decorrelation delay (Δ) is set to one for all the results to follow. The linear combination of the weighted input samples is an estimate of the interferer, while the error term retains the signal of interest (albeit with some distortion). The output of the filter, y_k , with input x_k is defined as

$$y_k = x_k - \sum_{l=0}^{L-1} c_l x_{k-\Delta-l}, \quad (19)$$

where c_l are the tap weights of the predictor.

The optimal tap weights can be found in a way similar

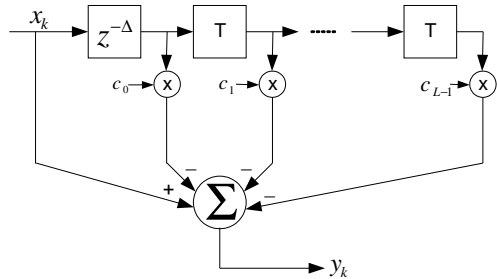
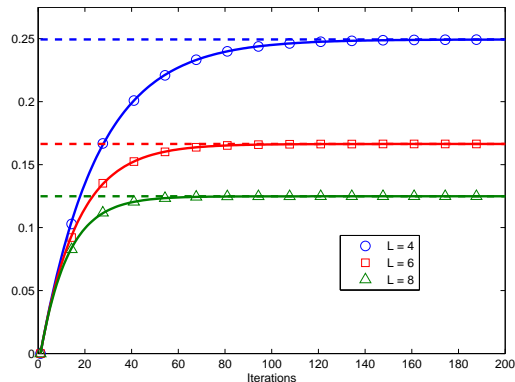


Figure 5: Linear predictor block diagram.


 Figure 6: Linear predictor tap weight evolution for $SIR = -20$ dB and $\mu = 0.0001$.

to those for the equalizers above. Using the orthogonality principle, L equations are obtained. The optimal tap weights are found to be

$$c_l = \frac{J}{S + \sigma_n^2 + LJ} e^{j\Omega(l+\Delta)T} \quad l = 0, \dots, L-1. \quad (20)$$

5.1 Predictor Convergence in Severe NBI

Using the LMS weight update equation, a recursive function of the weights can be derived. The mean tap weight evolution for the LP is found to be

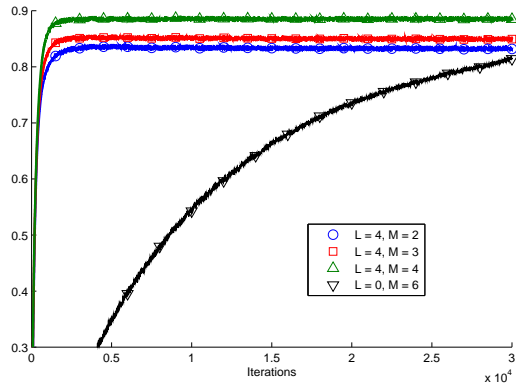
$$z_l(k+1) = [1 - \mu(S + \sigma_n^2 + J)]z_l(k) + \mu J e^{j\Omega\Delta T} \quad \forall l. \quad (21)$$

The steady-state value of (21) can be shown to be that given in (20).

The LP is shown to converge very rapidly in Fig. 6 for $L = 8, 6$, and 4 taps. The convergence times for the given cases range, respectively from approximately 65 iterations to about 120 iterations. The time constant for this structure is shown [10] to be dependent only upon the maximum eigenvalue (i.e. $\tau_{LP} \approx 1/2\mu(S + LJ + \sigma_n^2)$). This result arises because the $L-1$ eigenvectors corresponding to the minimum eigenvalues are orthogonal to the optimal weight vector, hence these eigenvalues do not affect the convergence properties.

6. TWO-STAGE SYSTEMS

The rapid convergence of the LP filter suggests the use of a two-stage system to allow improved performance. We will now consider the performance of the LP followed by the DFE, which will be abbreviated as LP+DFE. It is necessary


 Figure 7: Decision-feedback equalizer main tap weight evolution after linear prediction for $L = 4$, $SIR = -20$ dB, $\mu_{LP} = 0.0001$, and $\mu_{DFE} = 0.005$.

to determine the optimal weights of the equalizer in order to examine the convergence time of the two-stage structure. The optimal weights of the DFE are found by solving the Wiener-Hopf equations [2, 7]. The feedforward weights are equal to $\mathbf{w}_{LP+DFE} = (R - Q^H Q)^{-1} p$, where R is defined as

$$R = E \left\{ \begin{bmatrix} y(k) \\ \vdots \\ y(k-M) \end{bmatrix} \begin{bmatrix} y(k) \\ \vdots \\ y(k-M) \end{bmatrix}^H \right\}, \quad (22)$$

Q is defined as

$$Q = E \left\{ \begin{bmatrix} d(k-1) \\ \vdots \\ d(k-M) \end{bmatrix} \begin{bmatrix} y(k) \\ \vdots \\ y(k-M) \end{bmatrix}^H \right\}, \quad (23)$$

and p is defined as

$$p = E \left\{ \begin{bmatrix} y(k) \\ \vdots \\ y(k-M) \end{bmatrix} d(k)^* \right\}. \quad (24)$$

The feedback weights are then $\mathbf{f}_{LP+DFE} = -Q \mathbf{w}_{LP+DFE}$.

The convergence of the main tap for both the DFE after LP and the DFE-only ($L = 0, M = 6$) can be seen in Fig. 7. For the two-stage structure, the length of the predictor is set to $L = 4$, and the number of equalizer taps (M) is varied. It is clearly seen that the convergence time of the LP+DFE has been reduced more than on order of magnitude, when compared to the system employing only the DFE.

7. BER PERFORMANCE RESULTS

The number of taps for the LP and the DFE governs both steady-state performance and convergence time. The convergence time was studied above and here we will examine how varying filter orders affects performance of the overall system. To this end, the total number of filter taps ($L + 2M + 1$) in the system will be set to 13. The bit error rate (BER) is calculated after the weights have converged.

The BER results for both the LP and DFE are shown in Fig. 8. The first three curves labeled $L = 4, M = 0; L = 6, M = 0; L = 8, M = 0$ illustrate that the performance of the LP improves as the predictor order is increased. The

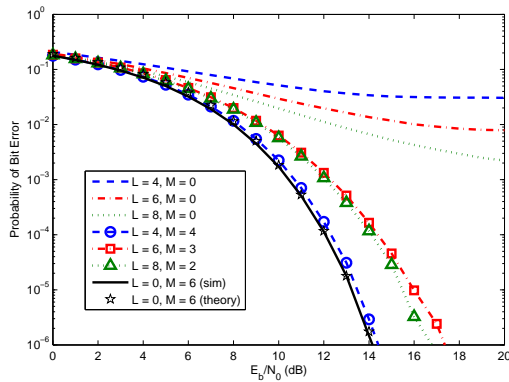


Figure 8: BER performance for LP, LP+DFE, and DFE for $SIR = -20$ dB.

two-stage systems are evaluated for the cases of $L = 4, M = 4$; $L = 6, M = 3$; $L = 8, M = 2$. The last case is the scenario ($L = 0, M = 6$) where no taps are assigned to the predictor and all the taps are given to the equalizer. It is apparent that the steady-state BER performance for the two-stage system is bounded by that of a system using the DFE only.

In order to evaluate the trade-offs in convergence time and BER performance, we will define the total convergence time necessary for the two-stage system as the sum of the convergence times for each stage individually. The results are shown in Table 1 for $E_b/N_0 = 10$ dB and indicate that there is a trade-off between convergence time and steady-state probability of error. The DFE-only case possesses the longest convergence time and the best probability of error. Using different parameters for the two-stage structure, the convergence times can be reduced, at the expense of steady-state probability of error. Notice that when comparing a two-stage system to the DFE-only case, there is more than an order of magnitude improvement in convergence time with only a small degradation in the probability of error.

L	M	t_L	t_M	t_{total}	P_e
0	6	-	60,000	60,000	1.84×10^{-3}
4	4	140	3,892	4,032	2.24×10^{-3}
6	3	100	2,378	2,478	6.33×10^{-3}
8	2	80	3,876	3,956	5.72×10^{-3}

Table 1: DFE and LP+DFE results, $\mu_{LP} = 0.0001$, $\mu_{DFE} = 0.005$, $SIR = -20$ dB, and P_e is for $E_b/N_0 = 10$ dB.

8. CONCLUSIONS

We investigated the response of the LMS DFE in the presence of severe narrowband interference. Due to the absence of a reference for the interference, the convergence time for this equalizer is unacceptably slow for use in some realistic systems. An adaptive linear predictor was introduced as a pre-filter in an effort to provide a direct reference for the interference. It is shown that this two-stage filtering approach reduces the time necessary for convergence by more than an order of magnitude. The achievable steady-state BER performance of this two-stage structure is reduced slightly due to the signal distortion introduced by the predictor. It is shown that a suitable selection of the parameters for the two-stage

filter allows effective BER performance to be achieved in a substantially reduced time interval.

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