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Improvements in HMM Based Spectral Frequency Line Estimation

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***Abstract*— This paper considers the application of Hidden Markov Models to the problem of tracking frequency lines in spectrograms of strongly non-stationary signals such as encountered in aero-acoustics and sonar where tracking difficulties arise from low SNR and large variances associated with spectral estimates. In the proposed method, we introduce a novel method to determine the observation (measurement) likelihoods by interpolation between local maxima. We also show that use of low variance AutoRegressiveMultiTaper (ARMT) spectral estimates results in improved tracking. The frequency line is tracked using the Forward-Backward and Viterbi algorithms.**

I. INTRODUCTION

ONE of the limiting factors restricting aircraft landings at major airports is the minimum spacing requirements due to vortex wake avoidance. If it can be shown that the separation requirements are too conservative, then it may be possible to increase the rate of landings on a given runway. During August/September 2003, NASA and the USDOT sponsored a wake acoustics test at the Denver International Airport. The central instrument of the test was a large microphone phased array. Different types of aircrafts were recorded during landing and the acoustic data obtained was stored. From acoustic data the spectrograms were generated using the technique of autoregressive (AR) spectral estimation from multitaper autocorrelation estimates [2]. A sample spectrogram obtained using this technique is shown in Fig. 1.

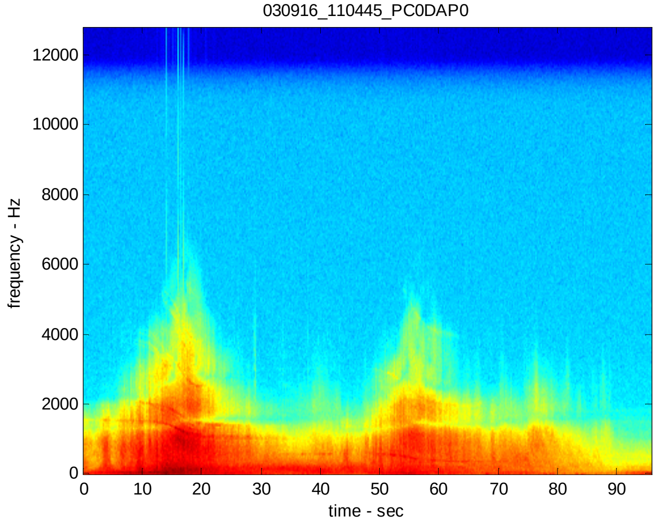

Fig. 1. Example of a spectrogram

The lines in the spectrogram bear crucial information about the nature of the vortex such as frequency, power and duration. Hence tracking of lines gives us the possibility to compare vortices generated by different flights of one type of aircraft and thus make a generalization about its characteristics. For this reason we have developed an efficient statistical method to track the frequency lines due to vortex in the lack of knowledge of their distribution and SNR.

For our purpose of tracking the frequency lines we use the first order Hidden Markov Models. There are many applications of HMMs in different areas such as econometrics, neural networks, bayesian networks, pattern recognition, control systems and DNA sequences.

This paper is organized as follows. We outline the steps in ARMT method and justify the reason why we choose it obtain the spectral estimate in section II. In section III the background of Hidden Markov Models is introduced. In section IV expressions for the quantities used in HMMs are developed and an algorithm to estimate the state sequence is introduced. Then we test the efficiency of our method using one real and one simulated signal.

II. ARMT SPECTRAL ESTIMATION

It is known that the covariance estimates have a strong effect on the performance of the AR spectral estimation. In our method the covariance estimates are obtained from the non-parametric spectral estimates [2]. The inverse DTFT of MTSE (Multitaper Spectrum Estimate) is shown to yield low bias and consistent autocorrelation estimates. Furthermore the AR spectrum obtained from the MT autocorrelation estimates is a smoothed and denoised version of the MTSE.

The smoothness property of the ARMT spectral estimate allows us to estimate the frequency track more efficiently by eliminating the redundant local maxima and thus reducing the set of nominees for the hidden state. This property is illustrated by comparing ARMT with another spectral estimation method (MTWT) which is also described in [2].

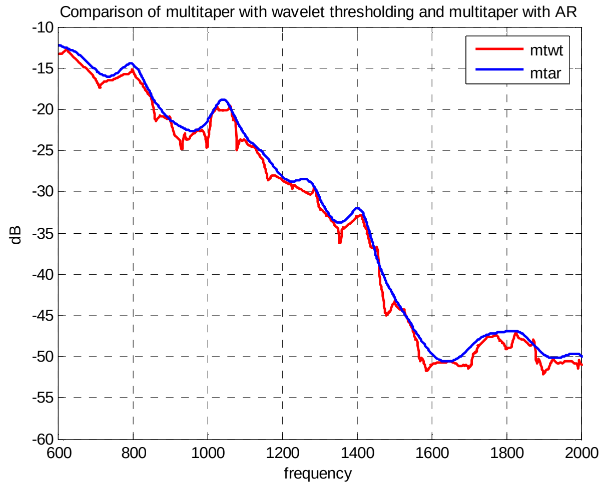

Fig. 2. Illustration of smoothness of ARMT spectral estimate

III. BACKGROUND OF HIDDEN MARKOV MODELS

The elements of the HMM theory are described in [5] and [6]. To summarize a (first order) Markov process is a stochastic model having discrete states in which the probability of being in any state at any time depends only on the state at the previous time. Let x_k denote the state at time k. Then we have

$$P\left(x_k \middle| x_{k-1}, x_{k-2}, \ldots, x_1\right) = P\left(x_k \middle| x_{k-1}\right) \quad (3.1)$$

A hidden Markov model (HMM) is a finite set of states, each of which is associated with a probability distribution. Transitions among the states are governed by a set of probabilities called transition probabilities. In a particular state an outcome or observation can be generated, according to the associated probability distribution. It is only the outcome, not the state visible to an external observer and therefore states are hidden to the outside; hence the name Hidden Markov Model.

In order to define an HMM completely, the following elements are needed

M : The number of states of the model

N : The number of observation symbols in the alphabet

A set of state transition probabilities $A = \{a_{ji}\}$

$$a_{ji} = P\left(x_k = i \middle| x_{k-1} = j\right), \quad 1 \le i, j \le M \quad (3.2)$$

A probability distribution in each of the states, $B = \{b_i(z_k)\}$

$$b_i(z_k) = P\left(z_k \middle| x_k = i\right), \quad 1 \le i \le M \quad (3.3)$$

where z_k is defined as the observation vector at time k.

The initial state distribution $\pi = \{\pi_i\}$, where

$$\pi_i = P\left(x_1 = i\right), \quad 1 \le i \le M \quad (3.4)$$

There are two important assumptions on the HMMs.

The transition probabilities are independent of the time at which the transitions take place. Mathematically this can be expressed as:

$$P\left(x_{k_1} = i \middle| x_{k_1-1} = j\right) = P\left(x_{k_2} = i \middle| x_{k_2-1} = j\right) \quad (3.5)$$

for any k_1 and k_2.

The second assumption which is known as the output independence assumption states that the current observation is statistically independent of the previous observations. Consider the sequence of observations $Z_K = \left(z_1, \ldots, z_K\right)$, then by the assumption we have:

$$P\left(Z_K \middle| x_1, x_2, \ldots, x_K\right) = \prod_{k=1}^{K} P\left(z_k \middle| x_k\right) \quad (3.6)$$

IV. APPLICATION OF HMM TO FREQUENCY LINE TRACKING

Consider a smaller portion of the spectrogram in Fig. 1 that contains a frequency line. The new image is given in Fig. 4.

The colors represent the variation of the spectral power with time and frequency. Spectral power is given in dB and the color scale for spectral powers is given on the right of the figure.

Our purpose is to track the frequency line which has the highest consistency and probability of observation given a suitable definition of the HMM parameters. That is, we are trying to estimate the sequence of unknown frequencies $X_K = \{x_1, \ldots, x_K\}$. Next we derive meaningful expressions for the elements defined in section II.

We denote the state at time instant k with i where it can take values in the range $1, \ldots, M$. First we choose a suitable model to describe the state transitions. It is meaningful to describe the change of line frequency with time as a random walk. By this assumption the difference between two consecutive frequencies obeys a normal distribution, i.e., $x_k - x_{k-1} \sim N\left(0, \sigma^2\right)$. The transition probabilities can be formulated as follows:

$$a_{ji} = P\left(x_k = i \middle| x_{k-1} = j\right) = \frac{c_j}{\sqrt{2\pi\sigma^2}} e^{\frac{(i-j)^2}{2\sigma^2}}, \quad 1 \le i, j \le M \quad (4.1)$$

where c_j is a scaling constant such that

$$\sum_{i=1}^{M} P\left(x_k = i \middle| x_{k-1} = j\right) = 1 \quad (4.2)$$

which is a result of the total probability theorem.

The transition probabilities are stored in the matrix A which is given by

$$A \triangleq \begin{bmatrix} \vdots & \cdots & \vdots \\ a_{j1} & \cdots & a_{jM} \\ \vdots & \cdots & \vdots \end{bmatrix} \quad (4.3)$$

Note that the rows of A add up to one.

The most crucial step is defining the probability distribution of the states or in other words the observation likelihoods. Let us denote the dB power of the spectrogram at time k and frequency i by $S(k,i)$. In [7] and [8] the authors derive the observation likelihoods by taking into account the power of all the frequencies. In our case there is a high amount of noise in the lower frequencies and therefore using the methods where all the frequencies are involved will lead to undesired results.

In our method we offer a different scheme where the local maxima are involved along with interpolation techniques. The procedure to construct the observation likelihood matrix B is as follows.

1. Find the local maxima of $S(k,i)$ for each k. Denote the number of local maxima located at time k by n_k and let $L_k(m)$ be the set of local maxima frequencies at time k where $1 \le m \le n_k$.

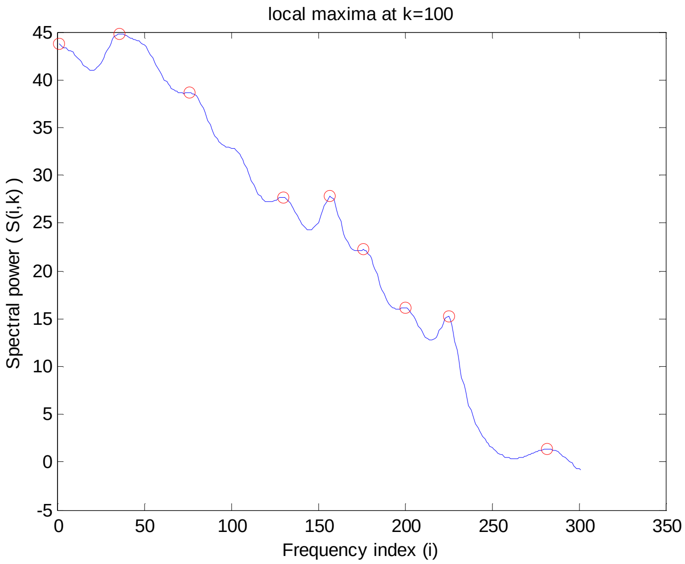

Fig. 3. $S(k,i)$ for $k=100$ and its local maxima

2. Interpolate the local maxima: Start from $k=1$ and for each k do the following:
For each m check if the set $\{L_k(m)-\lfloor\sigma\rfloor,\ldots,L_k(m)+\lfloor\sigma\rfloor\}$ contains an element of the set L_{k+1} where $\lfloor\ \rfloor$ means "largest integer less than". If not check if the same set contains an element of the set L_{k+2}. If it contains, find an element, say $L_{k+2}(m')$ that is in the interval then add a frequency to the set L_{k+1} which is calculated by rounding $(L_{k+2}(m')+L_k(m))/2$ to the nearest integer. At the end we will have a new set of frequencies for each k. Let us denote this new set as L'_k.

3. Calculate the observation likelihoods as follows:

$$b_i(z_k) = P(z_k|x_k=i) = \begin{cases} \dfrac{S(i,k)}{S(k)}, & \text{if } i \in L'_k \\ 0, & \text{otherwise} \end{cases} \tag{4.4}$$

where $S(k)$ is a scaling constant such that

$$\sum_{i=1}^{M} P(z_k|x_k=i) = 1 \tag{4.5}$$

The matrix B is constructed using observation likelihoods calculated above.

$$B = \begin{bmatrix} \cdots & b_1(z_k) & \cdots \\ \vdots & \vdots & \vdots \\ \cdots & b_M(z_k) & \cdots \end{bmatrix} \tag{4.6}$$

The initial state distribution is assumed to be uniform since we don't have any prior information regarding the initial states.

$$\pi_i = P(x_1=i) = \frac{1}{M}, \quad 1 \le i \le M \tag{4.7}$$

Now that we have all the elements that we need we can calculate the state estimates. The probabilities

$$\gamma_k(i) = P(x_k=i|Z_K) \tag{4.8}$$

can be used to compute the estimate of x_k defined as follows

$$\hat{x}_k = \arg\max_{i=1,\ldots,M}\{P(x_k=i|Z_K)\} \tag{4.9}$$

In order to calculate $\gamma_k(i)$ we use the Forward-Backward algorithm. We define the forward and backward probabilities α_k and β_k as follows:

$$\begin{aligned} \alpha_k &= P(x_k=i, Z_k) \\ \beta_k &= P(z_{k+1},\ldots,z_K|x_k=i) \end{aligned} \tag{4.10}$$

Using the following recursions

$$\begin{aligned} \alpha_k(i) &= b_i(z_k)\sum_{j=1}^{M} a_{ji}\alpha_{k-1}(j), \quad k=2,\ldots,K \\ \beta_k(i) &= \sum_{j=1}^{M} b_j(z_{k+1})a_{ij}\beta_{k+1}(j), \quad k=K-1,\ldots,1 \end{aligned} \tag{4.11}$$

Then $\gamma_k(i)$ can be calculated using

$$\gamma_k(i) = \frac{\alpha_k(i)\beta_k(i)}{\sum_{j=1}^{M}\alpha_k(j)\beta_k(j)} \tag{4.12}$$

The forward and backward probabilities are initialized as follows:

$$\begin{aligned} \beta_K(i) &= 1, \quad i=1,\ldots,M \\ \alpha_1(i) &= b_i(z_1)\pi(i), \quad i=1,\ldots,M \end{aligned} \tag{4.13}$$

A second algorithm that can be used to compute the estimate of x_k is the Viterbi algorithm. We define the quantity

$$\begin{aligned} \delta_k(i) &= \max_{X_{k-1}}\{P(X_{k-1},x_k=i,Z_k)\}, \\ & i=0,\ldots,M-1, \quad k=2,\ldots,K \end{aligned} \tag{4.14}$$

which can be recursively computed by

$$i=0,\ldots,M-1, \quad k=2,\ldots,K$$

$$\begin{cases} \delta_k(i) = \max_{j\in I}\{\delta_{k-1}(j)a_{ji}\}b_i(z_k) \\ \phi_k(i) = \arg\max_{j\in I}\{\delta_{k-1}(j)a_{ji}\} \end{cases} \tag{4.15}$$

We need to initialize the variables $\delta_1(i)$ and $\phi_1(i)$ to start the recursion. The proper initialization of these variables is given as

$$i = 0,\ldots,M-1, \quad \begin{cases} \delta_1(i) = \pi(i) \\ \phi_1(i) = 0 \end{cases} \tag{4.16}$$

Then we can obtain the estimated sequence $\hat{X}_K = \{\hat{x}_1,\ldots,\hat{x}_K\}$ using the backward recursion

$$\hat{x}_k = \phi_{k+1}(\hat{x}_{k+1}), \quad k = K-1,\ldots,1 \tag{4.17}$$

Now we have two methods to compute the estimate $\hat{X}_K$. In general these two methods should give the same results, however because of the nature of our data we can have different estimates from two algorithms. In this case we need to choose the best of the estimates to continue further processing. Consider, for example the estimates for spectrogram in Fig. 4 found using both algorithms. The estimates are given in Fig. 5.

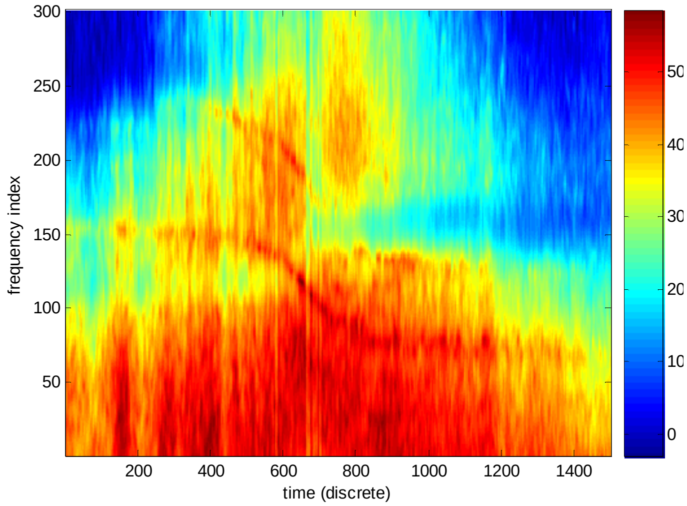

Fig. 4. Smaller spectrogram image obtained from spectrogram in Fig. 1 containing a perceivable frequency line.

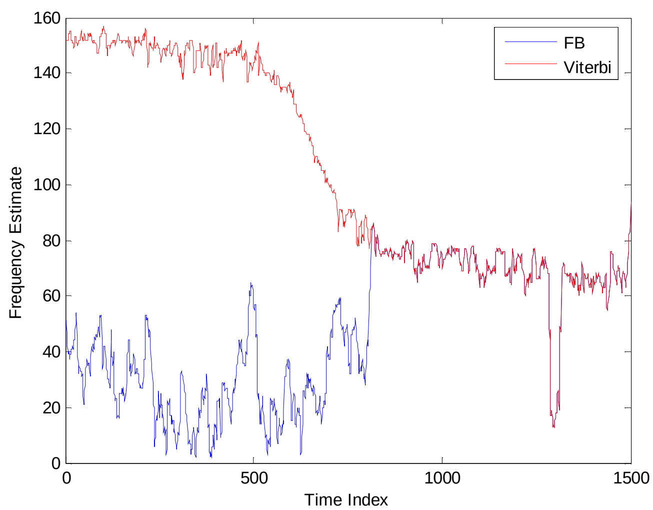

Fig. 5. Frequency line estimates for the spectrogram in Fig. 4. using both FB and Viterbi algorithms

As we see in the figure, for this case, the Viterbi algorithm is clearly the one that gives a better result, since its line is "smoother" compared to the FB estimate. In order to make it clear which estimate is better we need to define a general measure for the smoothness of a line. Here's the method we developed in order to measure the smoothness of a line.

First of all we decompose the estimate using L level wavelet transformation using one of the standard wavelet filters (we used db8 filter in our calculations) and calculate its approximation and detail coefficients. Then in order to obtain an approximation for our estimate we set the detail coefficients to zero and leave the approximation coefficients untouched. Applying wavelet reconstruction to the new set of coefficients will give us an approximation of our line. The approximates thus obtained for the estimates in Fig. 5 are given as black lines in Fig. 6.

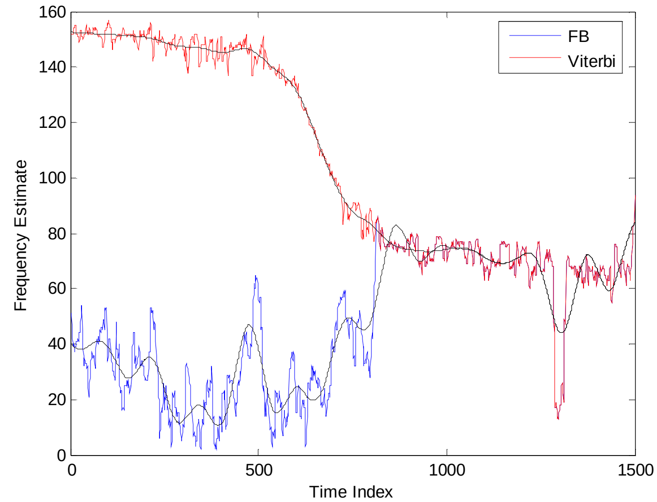

Fig. 6. Approximations for the estimates in Fig. 5.

In order to determine which estimate is smoother, we simply calculate the variances of the differences between the estimates and their approximations. In this example the variance of the difference is less for the Viterbi algorithm, and hence we decide to continue further processing with the estimate found using the Viterbi algorithm.

Using the aforementioned methods we estimate the frequency path in time, the plots of two spectrograms and their respective frequency line estimates are shown in the following figures. Fig. 4 is a spectrogram of a real signal whereas Fig. 8 is the spectrogram of a logarithmic chirp signal corrupted with high amount of Gaussian noise (-12.3 dB SNR).

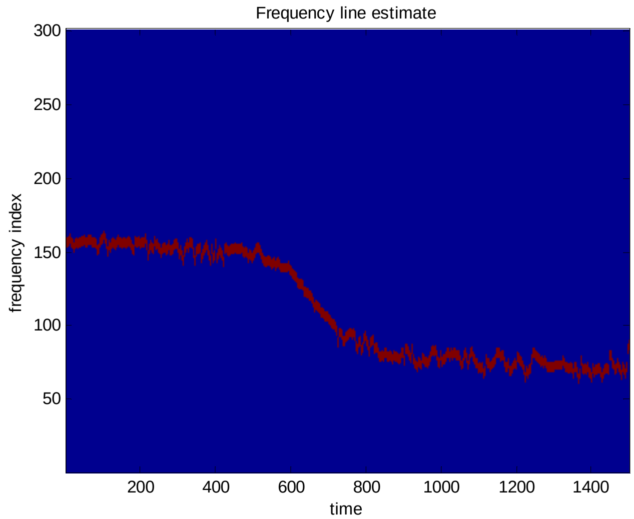

Fig. 7. Estimated frequency line for the spectrogram in Fig. 4.

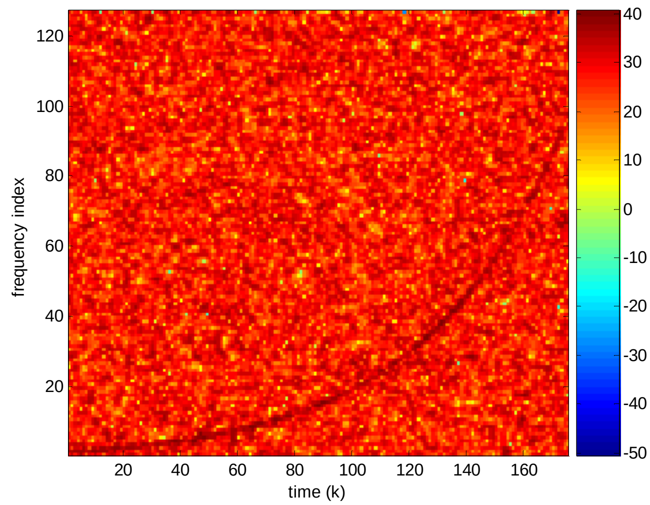

Fig. 8. Test signal generated using chirp with additive Gaussian noise (SNR -12.3 dB).

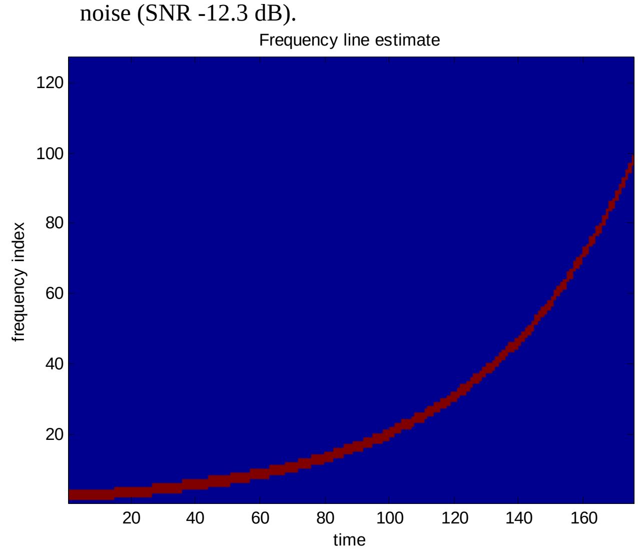

Fig. 9. Estimated frequency line for the spectrogram in Fig. 8.

V. CONCLUSION

We addressed the problem of HMM based line extraction from spectrograms. The algorithms we developed provide us with satisfying results both for real and simulated data. The challenge of having no prior information about the SNR the data is efficiently overcome by our method. The problems of multiple line tracking and estimation of birth and death of tracks were under investigation at the time this paper was submitted.

ACKNOWLEDGMENT

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