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STUDY OF THE PERFORMANCE OF DIFFERENT ALIGNMENT METHODS IN PATTERN RECOGNITION

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ABSTRACT

In this paper the alignment of noisy signals using different methods is studied. The methods studied in this paper are the Maximum Position method, the Cross-correlation method and the Zero Phase method. In order to evaluate the performance of the alignment methods, a database of high range resolution radar profiles containing patterns belonging to six different targets has been used, and the classification error using the k-Nearest Neighbor method is calculated. Results show the best performance of the Maximum Position method, in terms of error rate.

1. INTRODUCTION

Most of the classification algorithms (k-Nearest Neighbor (kNN), Multilayer Perceptrons, Radial Basis Function networks, Support Vector Machines, etc.) highly depend on shifts over the input signal. So, the alignment of each signal previously to any classification technique or preprocessing stage is very important. In this paper we study the accuracy of various methods for the alignment of High Range Resolution (HRR) radar signals, a kind of noisy signals used in Automatic Target Recognition tasks, evaluating the error rate using a kNN classifier.

Automatic classification of HRR radar targets is a difficult task. This kind of radar uses broad-band linear frequency modulation or step frequency waveforms to measure range profiles (signatures) of targets [1]. HRR radar profiles are essentially one-dimensional images of radar targets, and their values depends on the considered target and on the values of azimuth and elevation. A range profile is defined as the absolute magnitude of the coherent complex radar returns, and all phase information is usually discarded. If a range profile is measured with sufficient resolution, the parts of the aircraft that strongly reflect the radar energy, are resolved. Therefore, range profiles provide information about the geometry and structure of the aircraft, and so they are suitable features for automatic aircraft classification.

The inner characteristics of the HRR signals makes that small variations in the distance to the target cause circular shifts of the received signal. This fact makes very important the design of the alignment stage, in order to implement efficient classifiers.

The aligning algorithms are divided in terms of methodology in three groups:

- *Absolute alignment methods*. In these methods each profile is aligned independently. So, a measure over each profile must be obtained in order to estimate the shift for aligning the signal. One of these methods uses the position of the maximum of the signal (Maximum Position method). Another alternative consists in using the position of the maximum of the cross-correlation of the profile with a pattern signal (Naive Cross-correlation method). Another possibility consists in using the shift property of the Fourier transform to align the pattern (Zero Phase method).
- *Relative alignment methods*. They try to align the data set using the existing relationships among the patterns. An example of this kind of methods is the Complete Cross-Correlation method, which uses the cross-correlation of all patterns in the data set to obtain the shifts. Due to the need for a priori knowledge of the relationships (they are only valid for aligning data sets), the implementation of this kind of methods in classifications problems is not easy.
- *Integrated alignment methods*. The last group of alignment methods are not alignment methods themselves. They are included in the classification process. They are based on the design of translation invariant classifiers. The main example of this group of methods is known as Sliding Euclidean Distance, which consists in selecting the nearest pattern taking into account any possible slide of the signal. These classifiers use to have associated a very high computational cost, which make them unpractical in actual implementations.

The alignment of signals has been studied many times in the literature. In [2] methods based on the centroid and the cross-correlation are reviewed. Centroid methods are very sensitive to noise, and obtain worse results than the cross-correlation based methods. In [3] and [4] cross-correlation methods applied to ultrasonic signals are studied, trying to find theoretical expressions for the shift estimation error over simple signals. The method for aligning the signals described in [5] is a variant of the cross-correlation method applied to HRR radar profiles alignment. In [6] a new HRR aligning method is proposed mixing the relative cross-correlation and the zero phase method. At last, in [7] the maximum position method, the cross-correlation method and the zero phase method are studied, taking into account the mean error on the estimation of the shift of the methods, using a HRR database. The paper demonstrates the superior performance of the Zero Phase method, in terms of average error on the estimation of the shift.

This paper deals with the study of the classification error with patterns aligned with the zero phase method, with cross correlation based methods and with the maximum position

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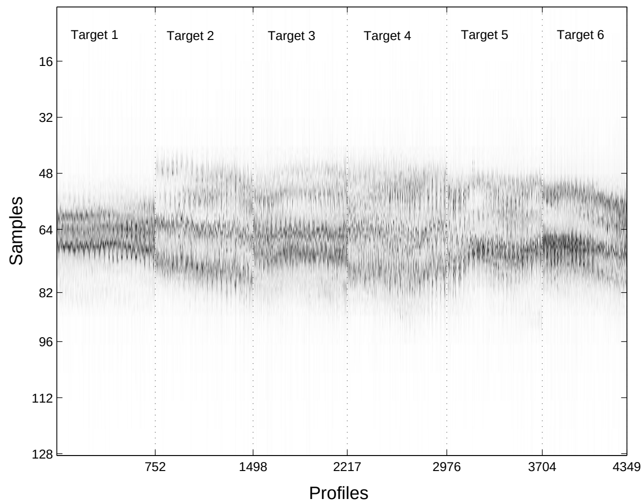

Figure 1: Radar profiles of the database. Each column of the image represents each one of the 4349 profiles. The black color is the higher value, and the white color is the lower value

method. In order to evaluate the classification error, the kNN method is used as classifier. In the literature, this method represents a standard in classification, due to the simplicity of its implementation, and, therefore, it is used to evaluate the performance of the alignment methods.

2. MATERIALS AND METHODS

In this section a short description of the database used in the experiments is included. A short review of the kNN method is also included.

2.1 Database Characteristics

In order to evaluate the performance of the different methods for aligning HRR profiles, a database containing HRR radar profiles of six types of aircrafts has been used. The assumed target position is head-on with an azimuth range of 25^o and elevations of -20^o to 0^o. The database contains 4349 profiles, and the length of each profile is 128.

Figure 1 represents the available signals. It is clearly visible that the significant information is concentrated in the middle of each time segment, whereas towards the limit of the radar profiles we encounter nondiscriminatory values.

Each profile of the database has been randomly shifted, in order to study the capabilities of the alignment methods. Therefore, the original data has been shifted using an uniform random integer variable from 0 to 127, which represents a complete misalignment of the profiles.

For each experiment, three subsets are used: a training set, composed of M profiles ($M/6$ per class), randomly selected from the original data set (the poses could be different for different targets), a validation set composed of other $M_{va}l$ profiles ($M_{val}/6$ per class) and a test set, composed of M_test profiles ($M_{test}/6$ per class). To study the influence the size of the training set has on the performance of the developed classifiers, we used training sets of different sizes. Table 1 resumes the sizes of the sets considered in the experiments. The validation set is used to select the necessary classifiers'

Table 1: Sizes of the different sets considered in the paper

M_{train}	1920	960	480	240	120	60
M_{val}	240	180	138	102	78	60
M_{test}	1710	1710	1710	1710	1710	1710

parameters. The test set is used to assess the classifier's quality after training. The test set remains unaltered for all the experiments described in this paper.

The performance of the kNN classifier can be specified as the *probability of correct classification* (P_{cc}), the *probability of misclassification* (P_{mc}) or the *error rate*. The probability of correct classification is the probability of a given target being classified correctly. The probability of misclassification is the probability that a given target is wrongly classified ($P_{mc} = 1 - P_{cc}$). Finally, the error rate expresses the percentage of overall classification errors, and these probabilities are estimated using the Monte Carlo simulation [8].

The Signal to Noise Ratio (SNR) has been a parameter of the study. Due to the temporal localization of the signal, the SNR has been defined using the peak energy of the signal (1). In this paper, the SNR varies from 5 dB to 50 dB in steps of 5 dB.

$$SNR = 10\log\left(\frac{\max\{x[n]\}^2}{\sigma_n^2}\right)dB \qquad (1)$$

2.2 k-nearest neighbor classifier

The k-Nearest Neighbor method is frequently used in ATR. As early as 1975, a target identification scheme based on multi-frequency measurements of the Radar Cross Section (RCS), using this technique for classification, was proposed [9].

This technique assumes that the data sets contain N_i points of class C_i and N points in total, so that $\sum_i N_i = N$. Then a hypersphere around the observation point $\mathbf{x}$ is taken, which encompasses k points irrespective of their class label. Suppose this sphere, of volume V, contains k_i points of class C_i, then applying Bayes' theorem, we obtain (2) [10].

$$P(C_i \mid \mathbf{x}) \simeq \frac{p(\mathbf{x} \mid C_i)P(C_i)}{p(\mathbf{x})} = \frac{k_i}{k} \qquad (2)$$

Thus, to minimize the probability of misclassifying a vector $\mathbf{x}$, it should be assigned to the class C_i for which the ratio k_i/k is highest.

The value of k must be previously selected, in order to implement the method. In this paper its value has been selected using the validation set. So, for each SNR and each size of the training set, the validation classification error has been measured, and best value of k has been selected.

3. RESULTS

This section describes the results obtained with the kNN method over the HRR data, using the different methods studied in this paper: the Maximum Position method, the Zero Phase method, and Cross-correlation based methods.

Table 2: Classification error (%) using the kNN method with patterns aligned using the Maximum Position method for different SNR values

M_{train}	1920	960	480	240	120	60
5 dB	83.51	84.09	82.46	83.80	84.27	81.99
10 dB	80.53	80.41	79.47	78.83	80.29	82.69
15 dB	47.13	44.62	54.15	55.32	59.36	66.78
20 dB	18.25	20.53	25.61	32.81	38.83	45.44
25 dB	9.53	12.69	16.73	21.40	26.78	32.81
30 dB	8.19	10.53	14.33	19.18	25.67	30.99
35 dB	6.55	8.25	11.52	17.66	23.80	28.95
40 dB	7.19	8.13	12.81	17.37	24.44	28.01
45 dB	5.61	7.72	11.52	15.32	21.52	26.37
50 dB	5.03	7.43	11.05	15.26	21.81	28.95

Table 3: Classification error (%) using the kNN method with patterns aligned using the Zero Phase method for different SNR values

M_{train}	1920	960	480	240	120	60
5 dB	82.05	83.74	83.98	83.16	83.74	83.68
10 dB	83.86	82.46	83.45	82.98	83.22	83.45
15 dB	58.60	64.50	69.82	74.50	75.85	77.66
20 dB	25.03	32.22	38.65	45.61	60.99	62.11
25 dB	13.16	19.65	24.33	29.82	37.31	50.53
30 dB	10.35	12.81	18.42	26.73	34.39	40.29
35 dB	8.19	11.99	18.07	23.39	30.94	39.42
40 dB	8.42	14.91	23.86	22.11	33.98	48.89
45 dB	8.71	12.92	16.37	23.57	36.78	44.97
50 dB	9.12	11.64	17.02	22.16	33.39	47.19

3.1 Aligning signals using the Maximum position method

The Maximum Position method proposes to align the signals using the position of the maximum value. So, this method is only useful when the signals show a clear global maximum value. This is the simplest method, both in terms of performance and computational cost. Therefore, it has been used many times in the literature.

In HRR radar signal alignment, this method is based on the modeling of the signal with scatterers. In [11] a preprocessing method based on the extraction of the position of the main scatterer is proposed. Using the position of the main scatterer as reference, a new set of aligned profiles is obtained. The main disadvantage of this method is the high sensibility of the performance to the presence of noise.

Table 2 shows the classification error obtained by the kNN method, aligning the HRR signals using the position of the maximum value. This table shows the relationship of the classification error measured with the test set, and the SNR, for different training set sizes. Parameter k of the method has been selected using the validation set.

3.2 Aligning signals using the Zero Phase method

The Zero Phase method has been previously used for alignment of panoramic images [12]. The basis of this method is the shift property of the Fourier transform. For any function $x(t)$ with Fourier transform $X(\omega)$, the Fourier transform of $x(t-\triangle t)$ is given by (3):

$$\mathscr{F}\{x(t-\triangle t)\} = X(\omega)e^{-j\omega\triangle t} \tag{3}$$

In the discrete time case, where we are dealing with sampled functions $x[n]$ $(n = 0,...,N-1)$, a similar property holds for circularly shifted versions of $x[n]$:

$$DFT\{x[((n-m))_N]\} = X(n)e^{-j2\pi nm/N} \tag{4}$$

So, for a discrete shift m, the phase ϕ of the n-th component of the Discrete Fourier Transform (DFT) $(n = 0,...,N-1)$ will be shifted by $-2\pi nm/N$. These phase shifts can be used to obtain an estimation of the discrete shift m.

A phase shift $\phi + 2n\pi$ generates uncertainty, because it is taken like a phase shift ϕ. This fact makes the measure of differences between two consecutive phase shifts necessary. So, if the phase shift for the component n_0 is $\phi(X(n_0)) = -2\pi n_0 m/N$ and the phase shift for the component n_0+1 is $\phi(X(n_0+1)) = -2\pi(n_0+1)m/N$, then the difference between both phases is given by (5), and this value is annotated between 0 and 2π, solving the uncertainty.

$$\phi(X(n_0)) - \phi(X(n_0+1)) = \frac{2\pi m}{N} \tag{5}$$

It is usual the use of $n_0 = 0$. In order to study the shift of the signal, it is necessary to study the differences in phase of two consecutive DFT samples. Using $n_0 = 0$, it is only necessary to study one phase, because the phase of the DC component $(n = 0)$ is always zero. So, using this value of n_0, the value of the shift m can be obtained with expression (6).

$$m = -\frac{N}{2\pi}(\phi(X(1))) \tag{6}$$

Finally, classification errors of the kNN method with patterns aligned using the Zero Phase method are also included in table 3. The value of parameter k has been selected using the validation set.

3.3 Aligning signals using cross correlation based methods

The Cross-correlation method is rather more complex than the first one. It is based on the value of the cross-correlation of the profiles. If the true signal $x[n]$ were known, the relative shifts of a set of M_{train} signals can be determined by an optimal matched filtering. In this approach, cross-correlation analysis of each signal with respect to a template signal (equal to $x[n]$) would yield shift estimates. Since $x[n]$ is unknown, an optimal matched filtering approach is not feasible. However, relative shifts can be estimated by a suboptimal matched filtering [13, 14], in which the true signal is approximated by the other signals of the available database.

In a first approach, it is necessary to calculate the circular cross-correlation of the signal with a template. The position of the maximum value of the cross-correlation of the signal with the template indicates the shift necessary to align the pattern. The Naive Cross-Correlation method uses the j-th signal of the database as template, and each signal is aligned with respect to this profile. So, in order to align a profile, it is necessary to calculate the cross-correlation of the profile only with the first signal. So, the computational cost is relatively low and independent of the database size.

Table 4: Classification error (%) using the kNN method with patterns aligned using the naive cross-correlation method for differen SNR values

M_{train}	1920	960	480	240	120	60
5 dB	80.99	82.34	84.44	82.34	82.63	82.57
10 dB	80.23	81.64	82.81	83.51	81.99	81.87
15 dB	49.01	53.86	60.18	62.16	66.78	71.93
20 dB	20.47	23.33	26.78	31.99	38.30	45.15
25 dB	10.47	14.04	18.13	23.10	29.36	35.32
30 dB	7.78	10.47	15.38	19.65	26.02	31.87
35 dB	7.43	9.82	13.16	17.89	24.09	31.17
40 dB	7.49	9.42	13.74	18.30	24.09	29.36
45 dB	8.01	9.18	12.98	17.43	22.51	28.89
50 dB	7.84	9.06	12.69	17.72	22.98	28.60

Table 5: Classification error (%) using the kNN method with patterns aligned using the complete cross-correlation method for different SNR values

M_{train}	1920	960	480	240	120	60
5 dB	82.51	81.23	84.21	83.33	84.04	83.63
10 dB	81.46	82.87	82.92	84.27	82.63	82.22
15 dB	50.58	55.73	64.09	67.66	71.93	75.85
20 dB	23.10	27.89	31.40	37.25	46.26	53.57
25 dB	13.10	17.19	21.93	25.79	35.56	44.68
30 dB	11.05	14.33	17.84	23.04	32.11	43.68
35 dB	8.77	12.11	17.43	23.80	29.47	42.63
40 dB	7.54	10.99	16.14	21.35	29.82	39.82
45 dB	7.02	11.05	16.02	23.22	31.75	39.65
50 dB	7.25	12.51	15.73	20.64	30.47	39.47

Table 4 shows the results obtained using the kNN method with the patterns aligned using the complete cross-correlation method.

More complex approaches use the cross-correlation of the signal with all the patterns in the available database. Considering $\triangle_{mn}$ the position of the maximum of the cross-correlation of signal m with n, the objective is to find the shifts d_k of each signal k that minimice the next expression.

$$C(d_1, d_2, ..., d_M) = min\{\sum_{i=1}^{M}\sum_{j=1}^{M}(\triangle_{ij} + d_i - d_j)^2\} \quad (7)$$

Equaling to zero all partial derivatives of equation (7), and fixing the position of the j-th, the estimated shift d_k of signal k is given by (8)[2].

$$d_k = \begin{cases} 0 & if \quad k = j \\ \sum_{i=1}^{M}\triangle_{ik} - \triangle_{ij} & if \quad k \neq j \end{cases} \quad (8)$$

This implementation is called the Complete Cross-correlation method, and the associated computational cost depends on the data set size (the number of cross-correlations needed are $(M_{train}+1)M_{train}/2$, being necessaries $2N^2$ simple operations to calculate a circular cross-correlation of two signals of length N). This fact makes this strategy unpractical in real-time applications.

Equation (8) can be simplified by adding a term $\sum_{i=1}^{M}\triangle_{ij}$. This new term does not increment the cost function (7), and therefore the obtained solution is a valid minimization of the cost function

$$d_k = \sum_{i=1}^{M}\triangle_{ik} \quad (9)$$

The cost function to minimize does not take into account the circular characteristic of the data, so in practical implementations it is important to previously align the patterns in the middle of the vector using an absolute method, so any value of $\triangle_{ij}$ is lower than $N/4$, where N is the length of the pattern.

On the other hand, the complete correlation method is a relative alignment method that can be used to align the training set, but it is realistic the use of the method to align the patterns of the test set. Test patterns must be aligned independently with an absolute alignment method, because in real implementations you mus decide without knowledge about the others test patterns. The validation set must be aligned in the same way as the test set, in order to establish a reliable estimation of the performance of the classifier.

So, in order to align the validation set and the test set, the values of the shifts must be calculated using only the cross correlations of each pattern t with the k-th pattern of the training set. Defining $\hat{\triangle}_{tk}$ as the position of the maximum value of the cross-correlation of test pattern t and each pre-aligned training pattern k, the shift d_t necessary to align the test pattern t must be calculated by minimizing the next cost function.

$$C(d_t) = \sum_{k=1}^{M}(\hat{\triangle}_{kt} - d_t)^2 \quad (10)$$

Calculation the derivatives with respect to d_t and equaling to zero, the value of d_t is obtained.

$$d_t = \frac{1}{N}\sum_{k=1}^{M}\hat{\triangle}_{kt} \quad (11)$$

So, circularly shifting d_t vector t, a new vector is obtained, which is aligned with respect to the training set.

The main disadvantage of this approach is the high computational cost associated to the calculus of all needed cross-correlation, being necessaries M_{train} cross correlations in order to shift each signal. This fact makes this method to be not suitable in many real time implementations.

Table 5 shows the results obtained using the kNN method with the patterns aligned using the complete cross-correlation method.

4. CONCLUSIONS

In this paper we study the performance of different methods to align noisy one-dimensional signals shifted in time. The methods studied in this paper are the Maximum Position method, two Cross-correlation based methods and the Zero Phase method. To study the performance, the classification error using the kNN method is measured for different values of SNR. Results show the best performance of the Maximum Position method, independently of the SNR, followed by the naive cross-correlation method. Previous works had demonstrated that, taking only into account the sensitivity of the alignment methods with the SNR, the Maximum Position obtained the worst results. So, the improvement of the classification error using this method may be explained by taking into account the posterior classification of the signals, and the alignment of the signals using the mean peak.

Results presented in this paper demonstrates that, in order to implement a classifier, the alignment of the main peaks of the signals is more important than the sensitivity of the alignment methods to the noise. In future works, the development lines of alignment algorithms for automatic target recognition using HRR radars must be oriented to increase the robustness of the Maximum Position method with respect to the SNR, by combining it with other less sensible methods, like the Zero Phase method or cross-correlation based methods.

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