LOW COMPLEXITY MIMO CHANNEL ESTIMATION FOR FEXT PRECOMPENSATION IN VECTORED xDSL SYSTEMS

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ABSTRACT

Far-end Crosstalk (FEXT) is the major limiting factor in further increase of data rate in xDSL systems. Hopefully, FEXT can be easily removed by modelling transmission channel as MIMO channel and applying vectored transmission. Having the knowledge of MIMO channel transfer function, FEXT can be completely cancelled by appropriate pre-distortion of transmitted signals. Therefore, in the paper we propose low complexity MIMO channel estimator employing Set-Membership Adaptive Recursive Techniques (SMART) with additional DFT based interpolation. As it is shown, proposed method is very accurate enabling systems with FEXT pre-compensation to approach performance of the ones operating on FEXT free channel. It is also efficient in terms of required amount of training data.

1. INTRODUCTION

Usage of existing copper wire subscriber loops originally designed for voice transmission in 300Hz-4kHz band for wide band data transmission has required solution of many challenging technical problems. At present, as demand for data rate increases the one of the most urgent problems are far-end and near-end crosstalk occurring between neighboring pairs in data cable [1].

Near-end crosstalk (NEXT) occurs when the upstream (US) signal of one modem couples into the downstream (DS) signal of another modem and vice versa. When the modems are co-located (as in central office) DS signals are much stronger than US and NEXT appears to be a very harmful form of distortion. In ADSL and VDSL NEXT can be easily avoided by frequency division duplexing (FDD). However, NEXT from alien systems like ISDN and HDSL have wide spectra that overlap with spectra used by ADSL and VDSL and can not be eliminated by FDD.

Another source of distortion in xDSL systems is far-end crosstalk (FEXT) occurring between two lines transmitting in the same direction. It is caused by electromagnetic coupling between wires. FEXT is attenuated by the channel and its level is much lower than NEXT level but it can not be avoided by FDD and that is why it presents the major limiting factor in further increase of data rate in xDSL systems.

Hopefully FEXT can be removed by vectored transmission [2] which by introduction of appropriate pre-distortions into transmitted signals results in FEXT compensation. Implementation of vectored transmission require co-location of transmitting (TX) modems at the central office (CO) as well as their full co-operation.

The paper is organized as follows. In Section 2, short overview of Discrete MultiTone (DMT) system is given. In Section 3 model of MIMO DS channel is presented. Section 4 discusses linear precoder for FEXT compensation and in Section 5 MIMO channel estimation is described. Simulation results are provided in Section 6.

2. DMT SYSTEM

For the last few years we have been observing growing popularity of multicarrier modulations (MCM). By the division of wide band transmission channel into large number of narrow band subchannel and assigning number of bits to each subchannel according to its signal-to-noise ratio (SNR) MCM approaches channel capacity as determined by Shannon's theory. The most successful examples of MCM are ADSL and VDSL [1]. They are termed Discrete MultiTone (DMT) because in all of them modulation and demodulation operations are performed by fast Fourier transforms pair (IFFT/FFT).

One of the biggest advantage of DMT is simple and efficient equalisation scheme based on convolution property of Fourier transform. Taking advantage of this property requires adding cyclic prefix (CP) to the transmitted frame which turns linear convolution actually performed by the channel into circular one. Next, simple frequency-domain equalisation can be used for compensation of channel effect.

Block diagram of DMT based transmission system is presented in Fig. 1. As long as the length of channel impulse response $h(n)$ does not exceed the length of CP (added by parallel/serial - P/S converter), linear convolution performed by the channel appears on the receiver end as a circular one. In such a case channel matrix $\mathbf{H}$ is circulant and can be written as: $\mathbf{H}=\mathbf{W}^*\Lambda\mathbf{W}$, where $\mathbf{W}$ is DFT matrix and Λ is diagonal matrix with elements composed of Fourier coefficients of the first column of $\mathbf{H}$ (i.e. the channel impulse response) [1]. Block denoted as TEQ is time-domain equaliser which is used for channel impulse response $c(n)$ shortening, which results in increase in data rate and shorter system delay [1].

Operations performed by the DMT system under conditions given above are described by equation

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$$\hat{\mathbf{X}} = \underbrace{\mathbf{\Lambda}^{-1}}_{FEQ}\mathbf{WD}\{\mathbf{h} * (\mathbf{PW}^{-1}\mathbf{X})\} + \mathbf{n}]$$

where $(\mathbf{x}*\mathbf{y})$ denotes a convolution of $\mathbf{x}$ and $\mathbf{y}$, i.e. a vector whose i-th element is $\sum_j x(j)y(i-j)$.

$$\mathbf{P} = \begin{bmatrix} \mathbf{0}_{P\times(N-P)} & \mathbf{I}_P \\ \hline \multicolumn{2}{c}{\mathbf{I}_N} \end{bmatrix} \quad \text{and} \quad \mathbf{D} = \begin{bmatrix} \mathbf{0}_{N\times P} & \mathbf{I}_N \end{bmatrix}$$

are responsible for introducing and discarding cyclic prefix respectively, $\mathbf{\Lambda}^{-1}$ is diagonal matrix of frequency-domain channel equaliser (FEQ) and $\mathbf{n}$ is a vector of noise samples.

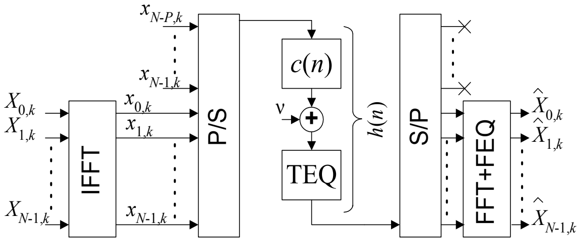

Figure 1: DMT system. P, N denotes length of CP and frame respectively, k is frame number.

3. MIMO CHANNEL MODEL

In a system with K users operating synchronously within the binder group, n-th frame received by k-th user along with FEXT originating from the remaining K-1 users is described by the equation

$$\hat{\mathbf{X}}_n^k = (\mathbf{\Lambda}^k)^{-1}\mathbf{WD}\left(\sum_{m=1}^{K-1}\left\{\mathbf{h}^{k,m} * (\mathbf{PW}^{-1}\mathbf{X}_n^m)\right\} + \mathbf{n}^k\right)$$

If the length of channel impulse response $\mathbf{h}^{k,m} = [h^{k,m}(0), \dots, h^{k,m}(L_h)]^T$ between m transmitter and k receiver (Fig. 2) does not exceed the length of CP ($L_h<P$), than transmission and crosstalk can be modeled independently on each tone, so n-th symbol received by group of K modems on i-th tone is

$$\hat{\mathbf{X}}_{i,n} = \mathbf{\Lambda}_i^{-1}\left(\mathbf{H}_i\mathbf{X}_{i,n} + \mathbf{N}_{i,n}\right) \tag{1}$$

where

$$\mathbf{X}_{i,n} = \begin{bmatrix} X_{i,n}^1 & X_{i,n}^2 & \cdots & X_{i,n}^K \end{bmatrix}^T, \quad \hat{\mathbf{X}}_{i,n} = \begin{bmatrix} \hat{X}_{i,n}^1 & \hat{X}_{i,n}^2 & \cdots & \hat{X}_{i,n}^K \end{bmatrix}^T$$

and $X_{i,n}^k$ denotes symbol transmitted by k-th user on i-th tone, $\hat{X}_{i,n}^k$ denotes symbol (corresponding to $X_{i,n}^k$) received by the k-th modem, $[\mathbf{H}_i]_{k,m} = H_i^{k,m}$ is MIMO channel transfer matrix on tone i, $[\Lambda_i^{-1}]_{kk}$ is diagonal matrix of FEQ coefficients $1/\lambda_i^k$ for i-th tone and $\mathbf{N}_{i,n}$ is vector of noise samples after FFT.

4. LINEAR FEXT PRECOMPENSATOR

In the properly designed transmission channel the crosstalk coupled signals originating from any TX can not exceed the "directly" received signal. So, channel transfer matrix on tone i

$$\left[\mathbf{H}_i\right]_{k,m} = H_i^{k,m}, \quad i = 1,\dots,M$$

is non singular and its elements obey $\forall_{k\neq m}\ |H_i^{k,k}| >> |H_i^{k,m}|$. Based on that, one can design simple and efficient linear precoder [3] for FEXT compensation

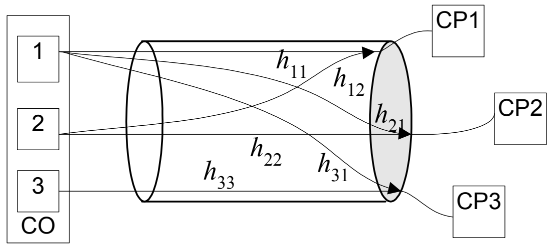

Figure 2: MIMO DS channel.

$$\mathbf{P}_i = \beta_i\mathbf{H}_i^{-1}diag(\mathbf{H}_i) \tag{2}$$

It is easy to check that precompensator defined in such a way makes channel matrix diagonal (FEXT free)

$$\mathbf{H}_i\mathbf{P}_i = \beta_i diag(\mathbf{H}_i)$$

The normalizing factor β_i should prevent increase in transmission power due to precompensation [3], i.e. $\beta_i \|[\mathbf{P}_i\mathbf{X}_i]_{\text{row } k}\|^2 \leq \|[\mathbf{X}_i]_{\text{row } k}\|^2$ which leads to condition on β_i

$$\beta_i = \min_k \left\| \left[\mathbf{H}_i^{-1}diag(\mathbf{H}_i)\right]_{\text{row } k} \right\|^{-1}$$

5. MIMO CHANNEL ESTIMATION

MIMO DS channel estimation has already been discussed in [4, 5]. The idea of impartial third-party site which collects transmitted and received signals from each modem operating on the given bundle for the predefined time span and calculates crosstalk coupling functions among twisted pairs in system has been presented in [4]. This idea although simple requires each modem to store received signals (right after AD conversion) and to transmit them back, which puts additional overhead into network and makes channel variations tracking difficult.

Because in vectored xDSL all modems operate synchronously much simple *PerTone* channel estimation is possible. In [5] such an idea was put forward with LMS gradient based implementation. In this paper we propose Set-Membership Adaptive Recursive Techniques (SMART) based estimator with additional DFT based interpolation. As it is shown, such an approach requires smaller amount of training data and offers very good estimation accuracy.

Estimation. Assumption (Section 3) that the MIMO DS channel (direct and crosstalk) is free of intersymbol and intercarrier interference requires that the length of each channel impulse response (including crosstalk) does not exceed the length of cyclic prefix

$$h^{k,m}(n) = 0 \text{ for } n \geq P$$

From this follows that channel attenuations $H_i^{k,m}$ are related to channel coefficients $h^{k,m}(n)$ as

$$H_i^{k,m} = \sum_{n=0}^{P-1} h^{k,m}(n)e^{-j\frac{2\pi}{N}in} \tag{3}$$

Therefore, knowledge of channel attenuations for P frequencies should be sufficient to determine channel attenuations at the others frequencies. So, in the first step

$H_i^{k,m}$ for F_p frequencies chosen from the range of DS frequencies $[DS_{min}, DS_{max}]$ is determined.

From (1) it follows, that the n-th symbol received by the k-th user on i-th tone is given by the equation

$$\hat{X}_{i,n}^k = (\lambda_i^k)^{-1} u_{i,n}^k$$

$$u_{i,n}^k = \sum_{m=1}^{K} H_i^{k,m} X_{i,n}^m + v_{i,n}^k$$

In order to estimate and track variation of $H_i^{k,m}$ one can define estimation error

$$e_n = u_{i,n}^k - \mathbf{h}_i^k \mathbf{X}_{i,n}$$

where

$$\mathbf{h}_i^k = \left[H_i^{k,1},\ H_i^{k,2}\ \cdots\ H_i^{k,K} \right]$$

which should be minimized. Widely used algorithms like LS, WLS, RLS try to minimize weighted 2-norm of error vector $[e_1,\ldots,e_n]$ which results in point-wise estimate for $\mathbf{h}_i^k$. Here, we apply Set-Membership Filtering (SMF) [6] whose objective is to achieve a specified bound γ on the magnitude of the estimation error e_k

$$|e_k|^2 \le |\gamma|^2 \ \text{ for } k=1,2,\ldots \tag{4}$$

It is obvious that, the solution to such defined problem is a set in the parameter space rather than a point estimate. The minimal set estimate for $\mathbf{h}^k_i$ at time n is the *membership set*

$$\psi_n = \bigcap_{k=1}^{n} \vartheta_k \tag{5}$$

where

$$\vartheta_n = \left\{ \mathbf{h}_i^k \in \mathbb{C}^N :\ |u_{i,n}^k - \mathbf{h}_i^k \mathbf{X}_{i,n}| \le \gamma \right\} \tag{6}$$

is observation set defined as the set of all $\mathbf{h}^k_i$ consistent with the specification (4) and observation $u^k_{i,n}$ and $\mathbf{X}_{i,n}$ at time instant n.

Since ψ_n (5) is an N dimensional polytope, it is not easily computed. Therefore, the idea of SMART is to find adaptively an estimate that belongs to the *feasibility set* (limiting set of ψ_n) or to one of its members. One of the approach known as SM-NLMS [6] is to compute outer approximations of the set ψ_n by minimal volume spheroids. In the N-dimensional parameter space such a spheroid, S_n is defined as

$$S_n = \left\{ \mathbf{h}_i^k \in \mathbb{C}^N :\ |\mathbf{h}_i^k - \hat{\mathbf{h}}_{i,n}^k|^2 \le \sigma_n^2 \right\} \tag{7}$$

where $\hat{\mathbf{h}}_{i,n}^k$ is the center of spheroid and σ_n is its radius.

Given S_{n-1} and observation ϑ_n (6) at time instant n updating algorithm compute a smallest spheroid that contains the intersection of S_{n-1} and ϑ_k. The algorithm works as follows

$$\hat{\mathbf{h}}_{i,n}^k = \hat{\mathbf{h}}_{i,n-1}^k + \alpha_n \frac{e_n \mathbf{X}_{i,n}^H}{\mathbf{X}_{i,n}^H \mathbf{X}_{i,n}} \tag{8}$$

$$\sigma_n^2 = \sigma_{n-1}^2 - \alpha_n^2 \frac{|e_n|^2}{\mathbf{X}_{i,n}^H \mathbf{X}_{i,n}} \tag{9}$$

where

$$\alpha_n = \begin{cases} 1-\gamma/|e_n|, & \text{if } |e_n| > \gamma \\ 0, & \text{otherwise} \end{cases}$$

Although the underlying idea of SM-NLMS and NLMS is quite different the updating formula is the same. The difference is in the step size α_n which in NLMS is a constant while in SM-NLMS is a function of current data set.

As a estimate must lie in spheroid S_n (7) formula (9) for radius updating implies that SM-NLMS guarantees non-increasing parameter error.

Interpolation. From (3) it follows, that vector of channel attenuations for frequencies F_p in matrix notation has a form

$$\mathbf{H}_P^{k,m} = \mathbf{S}_{P,red} \mathbf{W} \mathbf{S}_0^T \mathbf{h}^{k,m}$$

where

$$\mathbf{H}_P^{k,m} = [H_{F_{P-1}}^{*k,m} \ldots H_{F_1}^{*k,m}, H_{F_0}^{*k,m}, H_{F_0}^{k,m}, H_{F_1}^{k,m} \ldots H_{F_{P-1}}^{k,m}]^T,$$

$$\mathbf{S}_0 = [\mathbf{I}_P \ \vdots\ \mathbf{0}_{P\times(N-P)}]$$

$\mathbf{S}_{p,red}$ is $(2P)\times N$ selection matrix obtained by shrinking matrix $\mathbf{S}_p$ to non-zero rows. Matrix $\mathbf{S}_p$ is defined as $\mathbf{S}_p = \mathrm{diag}(s_0, s_1, \ldots, s_{N-1})$ where $s_i = 1$ for i corresponding to tone F_p and zeros elsewhere. $2P$ rows of matrix $\mathbf{S}_{p,red}$ results from that the channel in xDSL is real and therefore $H_i^{k,m}$ are complex conjugated.

Having n-th estimate of channel attenuations (8) at F_p frequencies

$$\hat{\mathbf{H}}_P^{k,m} = \mathbf{H}_P^{k,m} + \mathbf{N}_P^m$$

one can determine channel attenuations for the other frequencies using DFT based interpolation as

$$\hat{\mathbf{H}}^{k,m} = \mathbf{W}\mathbf{S}_0^T (\mathbf{S}_{P,red} \mathbf{W} \mathbf{S}_0^T)^{\dagger} (\mathbf{I} + \mathbf{R}_{NN} / \mathbf{R}_{HH})^{-1} \hat{\mathbf{H}}_P^{k,m} \tag{10}$$

where † denotes (pseudo)-inverse operator, $\mathbf{R}_{NN}$ and $\mathbf{R}_{HH}$ are autocorrelation matrices of noise and channel attenuations. Assuming that matrices $\mathbf{R}_{NN}$ and $\mathbf{R}_{HH}$ are approximately diagonal equation (10) can be simplified as

$$\hat{\mathbf{H}}^{k,m} = \mathbf{W}\mathbf{S}_0^T (\mathbf{S}_{P,red} \mathbf{W} \mathbf{S}_0^T)^{\dagger} (\mathbf{I} + \boldsymbol{\beta})^{-1} \hat{\mathbf{H}}_P^{k,m} \tag{11}$$

where $[\boldsymbol{\beta}]_{ii} = \sigma_i^2 / |H_i^{k,m}|^2$ and σ_i^2 is a noise variance in i-th subband. Values in $(\mathbf{S}_{p,red}\mathbf{W}\mathbf{S}_0^T)^\dagger$ strongly depend on positions of tones. In order to minimize noise amplification, frequencies for which estimation is performed should be placed evenly within DS frequencies range. Additional noise reduction can be obtained by incorporating additional knowledge of channel e.g. very large attenuation for DC.

6. SIMULATION RESULTS

The performance of proposed channel estimation method has been qualified during simulation of system with four 7000 feet 26AWG transmission lines. Four lines are sufficient, as most of the crosstalk power usually comes from 3 or 4 dominant sources [7]. For the simulation purpose crosstalk channel impulse responses have been calculated based on the commonly accepted FEXT model [8]

$$|H_{FEXT}(f)|^2 \approx |H_{channel}(f)|^2\, 8 \times (n/49)^{0.6} 10^{-20} f^2 l$$

where l denotes the length of the affected line in feet and f is frequency in Hz. The attenuation of direct $|H_{\mathrm{channel}}(f)|^2$ and FEXT $|H_{\mathrm{FEXT}}(f)|^2$ channels used in simulation are presented in Fig. 3. For channel shortening eigenfilter TEQ equalizer has been used. The power of transmitted signal was 23 dBm, AWGN -140 dBm/Hz and the length of cyclic prefix was set to P=32 samples.

To assess the influence of FEXT to system capacity and find out how much can be gained due to precompensation, SNR for system with FEXT free channel (SISO) and channel with crosstalk has been determined and presented on Fig 4.

In order to evaluate efficiency of proposed estimation method, SNR for system with four lines and DS transmission through channel using FEXT precompensation has been determined and compared to SNR in system operating on FEXT free (SISO) channel. Since, the precompensation matrices $\mathbf{P}_i$ (2) are computed based on estimation of channel attenuations $H_i^{k,m}$ two tests have been carried out.

In the *first test* channel attenuations $H_i^{k,m}$ have been determined using equation (8) and a set of N=100 training symbols for all DS tones. Next, matrices $\mathbf{P}_i$ (2) have been designed. Resulting mean SNR (MSNR) in system is presented in Fig. 5 by curve marked as *Direct* – due to absence of interpolation step (11). In the *second test* using equation (8) and N=100 training symbols for 32 DS frequencies only, $H_i^{k,m}$ have been estimated. Channel attenuations for the other frequencies have been determined by interpolation formula (11). Since in ADSL there are 220 DS frequencies, proposed estimation with DFT-based interpolation leads to almost 7 times reduction of required training data. Resulting mean SNR is presented in Fig. 5 by curve marked as *Interpolated* and compared to MSNR in SISO system. It can be observed that MSNR in both precompensated systems approaches MSNR in system operating on FEXT free channel.

In the next test efficiency of proposed method has been checked against number of received training symbols. The resulting MSNR is presented in Fig. 6 and compared to MSNR in SISO system. As in the previous test, estimation with additional DFT interpolation offers better accuracy although requires almost 7 times less training data. It is not surprising, interpolation usually leads to noise suppression and in this case avoids additional noise enhancement due to model rank overestimation.

Updating formula in SM-NLMS (8) is not only simple (linear computational complexity), it is executed quite rarely infrequently – depending on data. Therefore, in the additional test performance of proposed method with two estimators: low complexity SM-NLMS estimator and more computationally demanding, based on recursive realization of Least Square (RLS), estimator (quadratic computational complexity) has been compared. The resulting MSNR versus number of used (not received as in Fig. 6) training symbol is presented in Fig. 7. It can be observed, that performance of method employing low-complexity estimator very quickly approaches the one using more computationally demanding RLS.

7. CONCLUSIONS

In this paper a new method to estimate the downstream channel in a DSL environment has been proposed. Proposed method uses smaller amount of training data as compared to [5] due to DFT based interpolation. Decrease in required amount of training data is as high as 7. Also time required for estimation and tracking is limited by the same factor. Applications of low-complexity SM-NLMS estimation algorithm enables implementation of proposed method in CP modems.

The high accuracy of discussed method has been demonstrated in simulation. It has been shown that performance of vectored ADSL system with FEXT linear precompensator determined according to estimation results approach performance of ADSL system operating on FEXT free channel.

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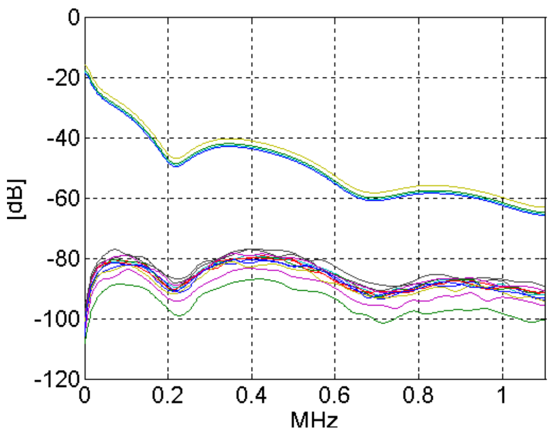

Figure 3. The attenuation of direct $|H_{channel}(f)|^2$ (upper curves) and FEXT $|H_{FEXT}(f)|^2$ (lower curves) paths of MIMO channel

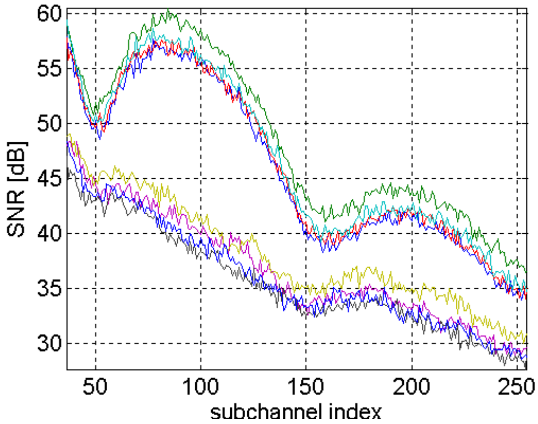

Figure 4. SNR of 4 ADSL modems, DS transmission, upper lines – with FEXT compensation, lower lines – without FEXT compensation.

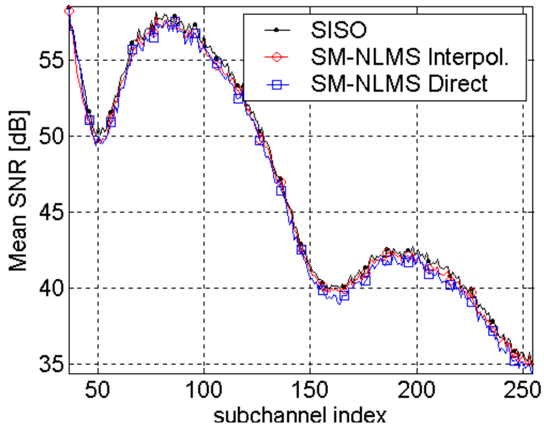

Figure 5. Mean SNR in DS transmission with FEXT precompensation achieved by group o 4 modems. Direct –estimator (8), Interpolate – estimator with additional interpolation (11).

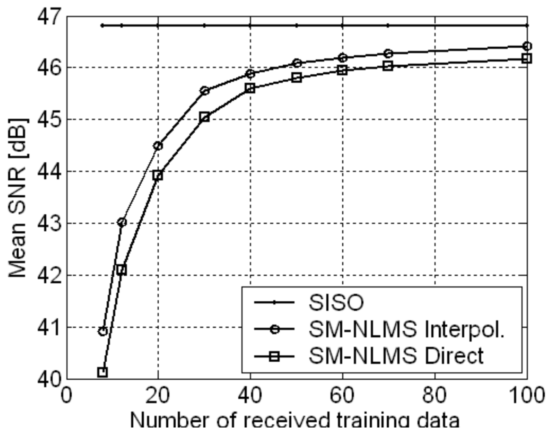

Figure 6. Mean SNR in DS transmission with FEXT precompensation achieved by group o 4 modems as a function of number of received training symbols.

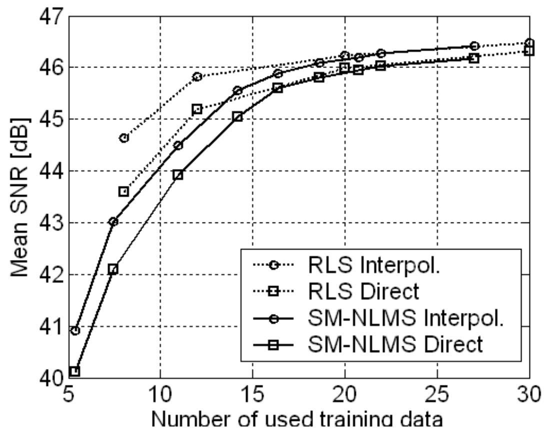

Figure 7. Mean SNR in DS transmission with FEXT precompensation achieved by group o 4 modems as a function of number of used training symbols in updating process.