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A COMPARISON OF SOFT AND HARD DECISION-DIRECTED FEEDFORWARD PHASE ESTIMATORS

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ABSTRACT

This paper is devoted to the comparison of hard and soft decision directed feedforward phase estimators based on the maximum likelihood principle. The particular structure of these estimators is taken into account to derive a new lower bound on the estimation variance for a constant phase error. Equivalent noise bandwidths for decision directed estimators are also studied. They allow us to characterize the estimator behaviour for the tracking of a time varying phase error. Simulation results are finally proposed, comparing the root mean square errors of the different estimators, for a constant phase error and for a time varying phase error respectively.

1. INTRODUCTION

Feedforward carrier recovery is usually preferred to feedback techniques for burst transmission systems. Indeed, contrary to the classical phase locked loop, a feedforward structure provides phase estimates without any acquisition delay. Besides, several phase estimation strategies are discussed in the literature when the transmitted data are totally unknown by the receiver [1]. These strategies are commonly divided into non data aided (NDA) and decision directed (DD) techniques. The recent success of solutions where channel decoding and phase estimation are embedded [2] highlights the interest of DD approaches. In this case, the phase estimation relies on the symbol detection, possibly more reliable because of the decoding process. This study focuses on feedforward DD phase recovery algorithms.

The DD phase estimation can either involve *hard* or *soft* decisions from the receiver. We propose here to compare both approaches it absence of channel decoding. The relevance of the decisions relies on a preliminary phase precorrection step. To ensure this pre-correction, a feedback component is introduced into the estimation structure. As a consequence, the DD estimators cannot be studied as the classical non data aided feedforward estimators.

This paper derives a specific lower bound for the variances of the hard- and soft-decision directed estimators in presence of constant carrier phase error. Moreover, equivalent noise bandwidths are defined and evaluated for the proposed schemes, allowing a better understanding of the estimators behaviour when the phase error is not constant. The performances of the hard and soft DD estimators are finally compared for a constant and a time varying carrier phase error. All simulation results presented in this paper have been obtained with QPSK modulation.

The paper is organized as follows: the proposed phase recovery schemes are described in Section 2. Section 3 derives the modified Cramer Rao lower bound for the DD phase estimation variance. Section 4 characterizes the estimators in terms of equivalent noise bandwidths. The estimator performances are finally compared through simulation results in Section 5. Conclusions are reported in Section 6.

2. PROBLEM FORMULATION

2.1 Decision directed phase estimation

This paper addresses the problem of feedforward phase recovery. A random sequence of N complex QPSK symbols $d = (d_0, d_1, \dots, d_{N-1}) \in \{\pm 1 \pm j\}^N$ is sent through a Gaussian channel. The transmitted signal is affected by a phase error φ. The possible phase variations are assumed to be slow: φ is assumed to be constant on duration NT_s, where T_s denotes the symbol duration. Assuming perfect timing recovery in a classical baseband model - including Nyquist pulse shaping and matched filtering - the received samples $y = (y_0, \dots, y_{N-1})$ can be expressed as:

$$y_k = d_k e^{j\varphi} + n_k, \tag{1}$$

where the complex noise samples $n = (n_k)_{k=0,\dots,N-1}$ can be decomposed into independent Gaussian real and imaginary components with zero means and variances σ^2.

The log-likelihood function for the joint phase and symbol estimation problem can be expressed as follows (up to an additive constant)[1]:

$$\Lambda(y|d,\varphi) = \frac{-1}{2\sigma^2} \sum_{k=0}^{N-1} \left| y_k - d_k e^{-j\varphi} \right|^2. \tag{2}$$

In a DD approach, (hard or soft) decisions $\hat{d}_k$ are taken from the received samples y_k in order to obtain a more tractable phase log-likelihood expression:

$$\Lambda_{DD}\left(y|\hat{d},\varphi\right) = \frac{-1}{2\sigma^2} \sum_{k=0}^{N-1} \left| y_k - \hat{d}_k e^{-j\varphi} \right|^2. \tag{3}$$

The maximization of the log-likelihood function (3) leads to the following feedforward estimator [1]:

$$\hat{\varphi}_{DD} = \arg\left(\sum_{k=0}^{N-1} y_k \hat{d}_k^{\,*} \right), \tag{4}$$

where $.^*$ denotes complex conjugation.

Remark: when the decisions $\hat{d}_k$ are taken directly from the received samples, their relevance depends on the phase

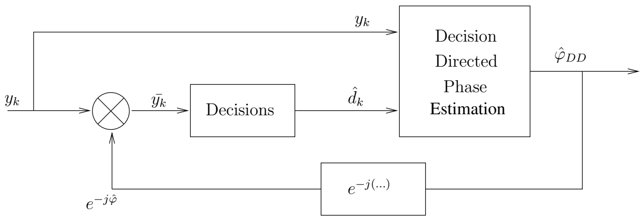

Figure 1: DD phase estimation with data pre-correction.

error φ, which has to be avoided. It is therefore preferable to pre-correct the signal phase before making the decisions. The phase estimated from a block of data is classically used to pre-correct the samples of the next block [3, 4], as shown on Fig. 1. The initial pre-correction step is based on some non data aided algorithm: the Viterbi & Viterbi estimator [5] is considered in our simulations. The pre-corrected samples are denoted as $\bar{y_k} \triangleq y_k e^{-j\bar{\varphi}}$, where $\bar{\varphi}$ is the phase estimate obtained from the previous block of samples.

2.2 DD algorithms

We propose to compare the following algorithms for the estimation of a constant phase error:

1. The Hard Decision Directed (HDD) estimator $\hat{\varphi}_{HDD}$ [3] given by (4) where
$$\hat{d}_k = \text{sign}\left(\Re\left(\bar{y_k}\right)\right) + j\text{sign}\left(\Im\left(\bar{y_k}\right)\right), \quad (5)$$
and $\Re\left(\bar{y_k}\right)$, $\Im\left(\bar{y_k}\right)$ denote the real and imaginary parts of $\bar{y_k}$, respectively,
2. The Soft Decision Directed (SDD) estimator $\hat{\varphi}_{SDD}$ given by the same expression, except that the hard decisions $\hat{d}_k$ are replaced by soft decisions δ_k:
$$\delta_k \triangleq \sum_{s=\pm1\pm j} P\left[d_k = s|\bar{y_k}\right] s. \quad (6)$$
The probabilities $P\left[d_k = s|\bar{y_k}\right]$ are classically computed as [1]:
$$P\left[d_k = s|\bar{y_k}\right] = \lambda \exp\left(-\frac{|s-\bar{y_k}|^2}{2\sigma^2}\right), \quad (7)$$
with
$$\lambda = \frac{1}{\sum_{s=\pm1\pm j}\exp\left(-\frac{|s-\bar{y_k}|^2}{2\sigma^2}\right)}. \quad (8)$$

3. LOWER BOUND ON THE ESTIMATION VARIANCE

Section 3.1 shows that the phase pre-correction step (mentioned in 2.1) provides *a priori* information about the unknown parameter φ. Based on this remark, Section 3.2 derives a lower bound on the variance of the SDD/HDD phase estimators when the phase error is constant.

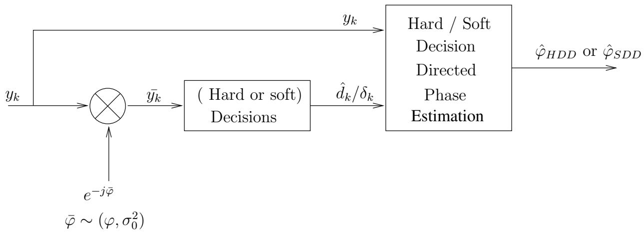

Figure 2: Phase estimation model including a priori information .

3.1 Phase estimation with a priori information

The feedback pre-correction step illustrated on Fig. 1 introduces additional information about the phase to be estimated. Indeed, the phase estimate from the previous block (denoted as $\bar{\varphi}$) provides additional information to the current data samples. We assume that the distribution of $\bar{\varphi}$ is Gaussian with mean φ and variance σ_0^2:
$$\bar{\varphi} \sim \mathcal{N}\left(\varphi, \sigma_0^2\right). \quad (9)$$
Under this assumption, we propose the equivalent model depicted on Fig. 2: each phase estimate $\hat{\varphi}_{SDD}$ or $\hat{\varphi}_{HDD}$ is obtained from N received samples $\left(y_0, \ldots, y_{N-1}\right)$ and $\bar{\varphi}$.

Remarks:

- The Gaussian assumption for $\bar{\varphi}$ has been motivated by many simulations for both HDD and SDD schemes,
- The expectation of $\bar{\varphi}$ is equal to φ, reflecting the unbiasness of HDD and SDD estimators,
- The looped structures proposed in this paper ensure small variations of the variance σ_0^2 in consecutive blocks (in steady state regime), inducing $\sigma_0^2 \simeq \sigma_\varphi^2$, where σ_φ^2 is the phase jitter,
- It is interesting to note that $\bar{\varphi}$ does not depend on y (since this estimate is issued from a former data block). As a consequence, the joint log-likelihood of $(y, \bar{\varphi})$ can be written:
$$\Lambda\left(y, \bar{\varphi}|d, \varphi\right) = \sum_{k=0}^{N-1} \ln p\left[y_k|d_k, \varphi\right] + \ln p\left(\bar{\varphi}\right). \quad (10)$$

3.2 Modified Cramer-Rao lower bound

Determining a lower bound on the variance of a phase estimator is a classical problem in synchronization [1]. The modified Cramer Rao lower bound (MCRB) is usually preferred to the true Cramer Rao bound (CRB) for its simplicity [6]. In absence of prior information regarding φ, the MCRB is defined as:
$$MCRB\left(\varphi\right) \triangleq \frac{-1}{E\left[\frac{\partial^2}{\partial\varphi^2}\left(\Lambda\left(y|d,\varphi\right)\right)\right]}, \quad (11)$$
yielding the well known result [6]:
$$MCRB\left(\varphi\right) = \frac{1}{2N\left(E_s/N_0\right)}, \quad (12)$$

where E_s/N_0 denotes the signal to noise ratio (SNR) on the transmission channel.

This section takes advantage of the additional information $p(\bar{\varphi})$ defined in (9) (coming from a previous block of samples) to derive a new MCRB on the variance of the DD phase estimators:

$$\overline{MCRB}(\varphi) \triangleq \frac{-1}{E\left[\frac{\partial^2}{\partial\varphi^2}\left(\Lambda(y,\bar{\varphi}|d,\varphi)\right)\right]}. \quad (13)$$

Combining (10) and (13), straightforward computations yield:

$$\overline{MCRB}(\varphi) = \left(\frac{1}{MCRB(\varphi)} + \frac{1}{\sigma_0^2}\right)^{-1}, \quad (14)$$

Remarks:

- Eq. (14) clearly shows that the proposed bound is smaller than the classical MCRB. This means that the knowledge of an *a priori* information $\bar{\varphi}$ improves the phase estimation,
- When σ_0^2 tends to zero, $\overline{MCRB}(\varphi)$ tends to zero as well. This makes sense, since $\bar{\varphi}$ is itself an estimator of φ, with variance σ_0^2,
- When $\sigma_0^2 \gg MCRB(\varphi)$, the benefit brought by $\bar{\varphi}$ is negligible and we recover the classical bound:

$$\overline{MCRB}(\varphi) \simeq MCRB(\varphi), \quad (15)$$

- It is important to note that $MCRB(\varphi)$ defined in (12) is no longer a bound for the phase estimation variance in presence of feedback pre-correction.

4. EQUIVALENT NOISE BANDWIDTHS

Section 3 derived a lower bound for the variance of phase estimation errors. This bound is applicable when the carrier phase is constant. When the phase to estimate is not constant, we propose to resort to the equivalent noise bandwidth parameter in order to characterize the estimator behaviour.

It should indeed be noted that the consecutive phase estimates are correlated, due to:

- The filtering operation (4), and
- The phase pre-correction step discussed in Sect. 2.1.

Considering the problem in the frequency domain, it is interesting to compare the power spectral density (PSD) of the phase estimates for both HDD and SDD schemes. For instance, Fig. 3 shows the normalized PSD of the estimates (obtained by simulations with a constant phase error) when $E_s/N_0 = 0\,dB$ (SNR) and $N = 32$ (number of samples per block): the PSD of the HDD estimates appears to be larger than the PSD of the SDD estimates.

As for classical feedback algorithms, we propose to compare the HDD/SDD estimation schemes in terms of equivalent noise bandwidth [7]. This parameter, denoted as B_l, is defined as the bandwidth of an equivalent rectangular PSD function with the same maximum value A_0, such that $A_0 B_l$ equals the average power of the phase estimates, as shown on Fig. 4.

A theoretical expression for the HDD estimator's equivalent noise bandwidth was analytically derived in [3]. A similar result is much more difficult to obtain when soft decisions

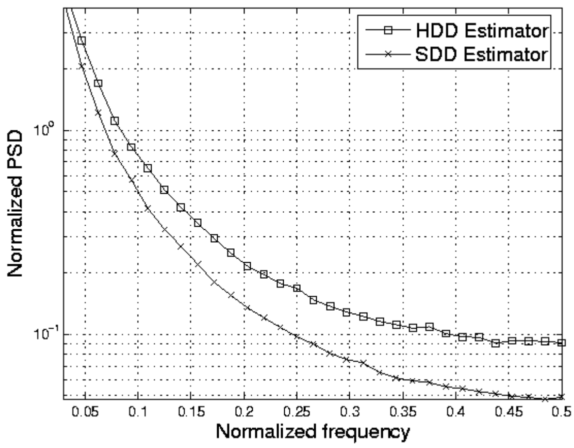

Figure 3: PSD of phase estimates for $E_s/N_0 = 0\,dB$ and $N = 32$.

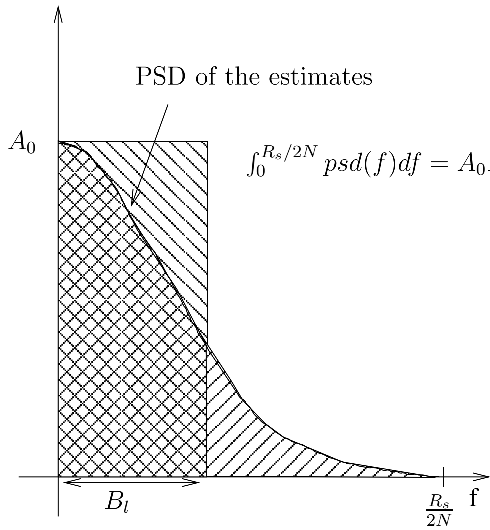

Figure 4: Definition of the equivalent noise bandwidth.

are used in the decision process. Thus, we resort to simulations in order to evaluate the noise bandwidth of the SDD estimator. More precisely, the values of B_l are deduced from the PSD of the estimates when the phase error is constant and normalized by the symbol rate $\frac{1}{T_s}$. They appear to be inversely proportional to the estimation block size N. Fig. 5 shows the product $B_l T_s N$ for both HDD and SDD estimation structures, as a function of the channel SNR E_s/N_0. It is important to mention the following points:

- It is noticeable that $B_l T_s$ tends to $\frac{1}{2N}$ for high SNRs, as for classical data aided estimators. This was predictable, since at high SNRs, the (soft or hard) decisions are likely to be reliable. Consequently, both HDD and SDD algorithms behave like data aided estimators.
- When the SNR decreases and for a given value of N, the SDD noise bandwidth decreases faster than that of the HDD scheme.

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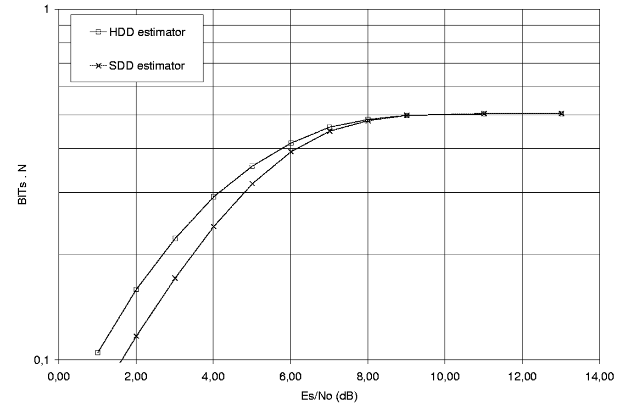

Figure 5: Equivalent noise bandwidths for the SDD and HDD estimators (obtained by simulation).

Finally, we recall that a small equivalent noise bandwidth is favorable to the mitigation of the Gaussian noise effects in the phase estimation. On the other hand, a large noise bandwidth is more suited to the tracking of a time varying phase error. A trade-off on this parameter is thus necessary, in order to optimize the estimation performance.

5. SIMULATION RESULTS

In this section, the HDD and SDD estimator performance are compared in terms of root mean square error (RMSE). Simulations with a constant phase error are analysed in section 5.1. A time-varying phase error is then considered in section 5.2, including a DVB-S2 compliant phase noise model.

5.1 Constant phase error

Fig. 6 shows the root mean square errors (RMSEs) for the phase estimates versus the signal to noise ratio E_s/N_0. The circle (resp. square) curves corresponds to the the SDD (resp. HDD) estimates. The parameters for this example are $N = 32$ and $\varphi = 0°$ (without loss of generality). The MCRBs are computed from (14), where σ_0^2 has been estimated from Monte Carlo runs:

$$\sigma_0^2 \simeq \frac{1}{K}\sum_{k=0}^{K-1}\left(\hat{\varphi}_k - \varphi\right)^2, \qquad (16)$$

and $\hat{\varphi}_k$ are consecutive phase estimates.

For a given estimation block size N, the SDD estimator outperforms the HDD estimator, especially at low SNRs. For instance, in order to guaranty a $RMSE < 8°$ (with $N = 32$ QPSK symbols per block), the SDD estimator requires $E_s/N_0 > -3\,dB$, whereas the HDD estimator requires $E_s/N_0 > 1\,dB$. A gain of $4\,dB$ is thus provided at these SNR levels.

We recall that at low SNR and for a fixed value of N, the SDD estimator noise bandwidth is smaller than the HDD estimator noise bandwidth (cf. Fig. 5), which is more favorable when the phase error is constant. However, in most applications, the receiver has to estimate and correct a time varying phase error. The next section studies the performance of HDD and SDD estimators in the presence of phase noise.

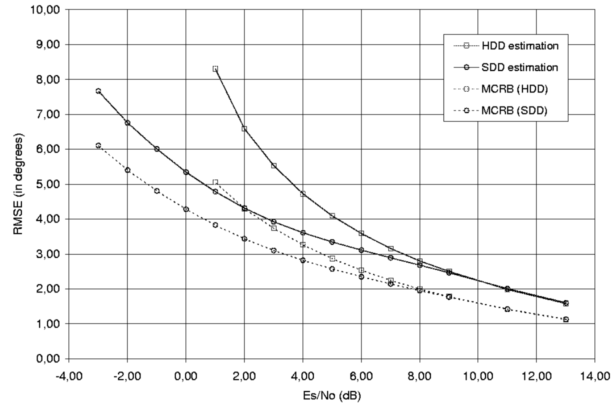

Figure 6: RMSEs and MCRBs for HDD and SDD algorithms (Constant phase error, N=32 , QPSK modulation).

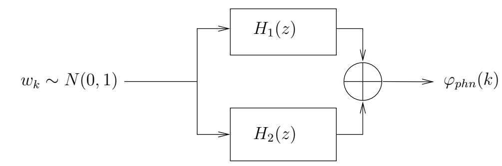

Figure 7: Phase noise model .

5.2 Time varying phase error

We propose to evaluate the HDD and SDD estimator performance in presence of phase noise. The proposed model for phase noise is decribed in Sect. 5.2.1. The estimator performances are presented in Sect. 5.2.2.

5.2.1 Phase noise model

The proposed phase noise model is based on a white Gaussian sequence, sent through two parallel recursive filters H_1 and H_2, as depicted on Fig. 7. With the transfer functions $H_1(z)$ and $H_2(z)$ given in [8], this model complies with the aggregate mask suggested in [9] for the evaluation of carrier recovery in DVB-S2 receivers, as shown on Fig. 8.

With the notation defined of Fig. 7, the phase error $\varphi(k)$ affecting the k-th transmitted sample can be expressed as:

$$\varphi(k) = \varphi_0 + \varphi_{phn}(k), \qquad (17)$$

where φ_0 is the phase error affecting the sample y_0.

5.2.2 Estimation performance

A trade-off on the noise bandwidth allows to minimize the phase jitter, as explained in section 4. The RMSE is plotted on Fig. 9 for HDD and SDD estimators, when the estimation block size (which determines the noise bandwidth) is set to its optimal value. We observe that the gain over the SNR provided by the soft decisions approaches 1 dB for $-2\,dB < E_s/N_0 < 1\,dB$. As a conclusion, the SDD estimator remains more performant than the HDD estimator for the tracking of phase noise.

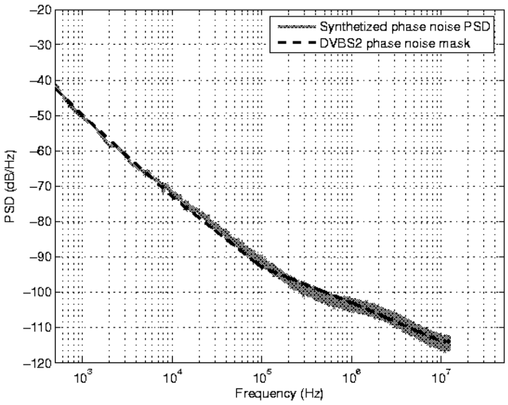

Figure 8: PSD of the synthetized phase noise and target mask .

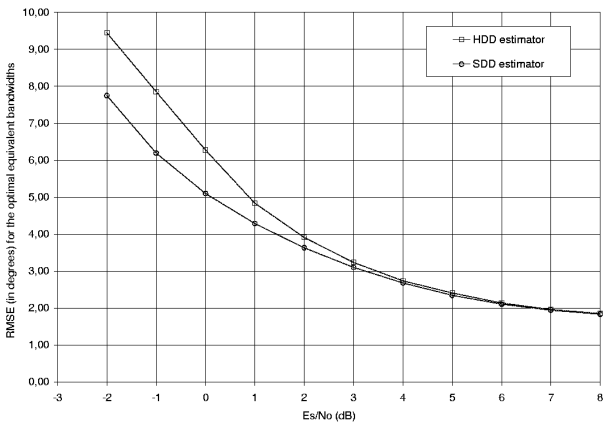

Figure 9: RMSE in presence of phase noise, with the HDD and SDD estimators and for the optimal estimation block sizes (QPSK modulation).

Remark:

At low signal to noise ratio, the optimal block sizes (allowing to minimize the MMSEs in presence of phase noise) are greater than the reference block size $N = 32$ (considered in sect. 5.1). This explains why in such case the estimators seem to perform better in presence of phase noise (Fig. 6) than with a constant phase error (Fig. 9).

6. CONCLUSION

The decision directed feedforward estimators require a signal phase pre-correction step. To ensure this pre-correction, the phase estimate obtained from a block of samples is used to correct the samples of the next block. In other words, a feedback component is introduced into the classical feedforward estimation structure.

This paper derived the modified Cramer-Rao lower bound for these estimators, assuming that the carrier phase error is constant. Besides, equivalent noise bandwidths were evaluated for both HDD and SDD schemes by means of simulations, for different SNR levels. The estimator performances obtained for constant and time-varying phase errors were also presented. The main conclusion was that the SDD estimator outperformed the HDD estimator in both situations.

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