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BLIND RECOVERY OF MIMO QAM SIGNALS: A CRITERION WITH ITS CONVERGENCE ANALYSYS

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ABSTRACT

In this paper, The problem of blind recovery of QAM signals for Multiple-Input Multiple-Output (MIMO) communication systems is investigated. We propose a new criterion based on the real (or equivalently the imaginary) part of the equalizer outputs, with a cross correlation constraint. A performance analysis reveals the absence of any undesirable local stationary points, which ensures perfect recovery of all transmitted signals and global convergence of the algorithm. From the proposed criterion an adaptive algorithm is derived. It is shown that the proposed algorithm presents a lower computational complexity compared to the constant modulus algorithm (CMA) and multimodulus algorithm (MMA). The effectiveness of the proposed algorithm is illustrated by some numerical results.

1. INTRODUCTION

The use of multiple antennas at both sides of communication link increases significantly the spectral efficiency [1], especially when blind techniques, without the use of training sequences, are considered. A large number of techniques have been proposed so far in the literature that offer different trade-offs of complexity and performance. The well known and more studied is the constant modulus algorithm (CMA) [2], [3]. It is very robust in practice and can be applied to non-constant modulus communication signals. Other algorithms have been proposed: Multi-modulus algorithms (MMA) [4], Constant norm algorithms (CNA) [5], Multiuser kurtosis (MUK) [6], and others [7]. In contrast to the large number of algorithms that have been proposed is the low number of algorithms whose convergence has been studied.

In this paper, we present a new criterion for blind recovery of QAM signals for MIMO communication systems, with its convergence analysis. The algorithm combines HOS criterion with a deflation convergence procedure. The criterion consists in penalizing the deviation of the real (the imaginary) part of the equalizer output from a constant calculated from a statistics of input signals. The use of only the real (the imaginary) part results in low computational complexity compared to CMA [2], [3] and MMA [4] that use both real and imaginary parts. The analysis of the criterion shows that the algorithm is free of any stable undesired local stationary points for any number of source signals; hence, it is globally convergent to a setting that recovers them all. From the proposed criterion we derive an adaptive algorithm. At each iteration, the algorithm combines a stochastic gradient algorithm (SGA) update and a Gram Schmidt orthogonalization procedure.

The rest of the paper is organized as follows. In section II, the problem formulation and assumptions are introduced. In section III, we present the criterion. Convergence analysis is carried out in section IV. In section V, we present an adaptive implementation of the algorithm. Finally, section VI presents some numerical results.

2. PROBLEM FORMULATION

We consider a linear data model of the form:

$$\mathrm{Y}(n) = \mathbf{H}\,\mathrm{A}(n) + \mathrm{B}(n) \qquad (1)$$

Where A is the (M×1) vector of the source signals, $\mathbf{H}$ is the (N×M) MIMO linear memoryless channel, Y(n) is the (N ×1) vector of the received signals and B is the (N×1) noise vector. M and N represent the number of antennas at the transmitter and receiver respectively.

We assume that: $\mathbf{H}$ has full column rank M, the noise is additive white Gaussian independent from the source signals, and the source signals are independent and identically distributed (i.i.d), mutually independent ($E[\mathrm{A}\mathrm{A}^H] = \sigma_a^2\,\mathrm{I}_M$) and drawn from QAM constellation.

In order to recover the source signals, the received signal Y(n) is processed by a (N × M) receiver matrix $\mathbf{W}$. Then, the receiver output can be written as :

$$\mathrm{Z}(n) = \mathbf{W}^T\,\mathrm{Y}(n) = \mathbf{W}^T\,\mathbf{H}\,\mathrm{A}(n) + \overline{\mathrm{B}}(n)$$
$$= \mathbf{G}^T\,\mathrm{A}(n) + \overline{\mathrm{B}}(n) \qquad (2)$$

Where Z(k) is the (M ×1) vector of the receiver output, $\mathbf{G} = \mathbf{H}^T\,\mathbf{W}$ is the (M × M) global system matrix and $\overline{\mathrm{B}}(n)$ is the filtered noise at the receiver output.

The matrix $\mathbf{W}$ is feasible to separate the source signals, except for a possible permutation and up to a unitary scalar rotation.

Notation: we use capital letters and capital boldface letters to denote vectors and matrices respectively. The symbols $(.)^*$ and $(.)^T$ denote the complex conjugate and transpose respectively, $(.)^H$ is the Hermitian transpose, and $\mathbf{I}_p$ is the $(p\times p)$ identity matrix.

3. THE CRITERION

The proposed cost function to be minimized is :

$$J=\sum_{i=1}^{M} E[(z_{R,i}^2-R)^2] \qquad \text{Subject to}: \mathrm{G}_i^H \mathrm{G}_j = 0,\ i\neq j, i=1\ldots M,\ j=1\ldots i-1 \tag{3}$$

Where: $z_{R,i}$ represents the real part of the receiver output z_i and $\mathrm{G}_i=[g_{1i},\ldots,g_{Mi}]^T$ is the *i*-th column vector of **G**. We can use equivalently the imaginary part of z_i thanks to the symmetry of the QAM Constellation. Throughout this paper, we consider only the real part, since the analysis is the same for the imaginary part.

The constraint in (3) prevents the extraction of the same signal at many outputs, it comes from the condition $E[z_i z_j^*]=0$, that cancels the cross correlation between equalizer outputs. The constant R is fixed by assuming a perfect equalization, and is defined as:

$$R=\frac{E[a_R^4(n)]}{E[a_R^2(n)]} \tag{4}$$

Where $a_R(n)$ is the real part of the source signal $a(n)$.

4. CONVERGENCE ANALYSIS

In order to study the stationary points, the cost function (3) is formulated in the following way [6]:

$$\underset{\mathrm{G}_i, i=1,\ldots,M}{Min}\quad J(\mathrm{G}_i)=E[(z_{R,i}^2-R)^2] \qquad \text{Subject to}: \mathrm{G}_\ell^H \mathrm{G}_i=0;\quad \ell=1,\ldots,i-1 \tag{5}$$

(5) is equivalent to (3). There are M constrained optimization criteria that are implemented in parallel, i.e. simultaneously, and the constraint must be satisfied for each $\ell=1,\ldots,i-1$. For simplicity reasons, the analysis is restricted to noise free case, i.e.:

$$\mathrm{Z}(n)=\mathbf{G}^T\,\mathrm{A}(n) \tag{6}$$

We can now use (5) in order to study the stationary points in **G** domain. From (5), we first notice that the adaptation of each G_i depends only on $\mathrm{G}_1,\ldots,\mathrm{G}_{i-1}$. Then, we can begin by the first output, because G_1 is optimized independently from all the other vectors $\mathrm{G}_2,\ldots,\mathrm{G}_M$. Hence we have:

$$\underset{\mathrm{G}_1}{Min}\quad J(\mathrm{G}_1)=E[(z_{R,1}^2-R)^2] \tag{7}$$

By developing (7), we get:

$$J(\mathrm{G}_1)=E[z_{R,1}^4]-2R\,E[z_{R,1}^2]+R^2 \tag{8}$$

After a straightforward development of the terms in (8) (see Appendix), equation (8) can be written as:

$$J(\mathrm{G}_1)=E[a_R^4]\left(\sum_{k=1}^{M}|g_{k1}|^2-1\right)^2+\beta\left[\left(\sum_{k=1}^{M}|g_{k1}|^2\right)^2-\sum_{k=1}^{M}|g_{k1}|^4\right] +2\beta\cdot\sum_{k=1}^{M}g_{R,k1}^2 g_{I,k1}^2-E[a_R^4]+R^2 \tag{9}$$

Where:

$g_{k1}=g_{R,k1}+j\,g_{I,k1}$

$\beta=3E^2[a_R^2]-E[a_R^4]=-\kappa_{a_R}>0$,because:

$\kappa_{a_R}=E[a_R^4]-3E^2[a_R^2]$ represents the kurtosis of the real parts of the symbols, it is always negative.

On the other hand, the minimum of J is given by:

$$J_{Min}=E[(a_R^2-R)^2]=E[a_R^4]-2R\,E[a_R^2]+R^2$$

From (4) we have: $R\,E[|a_R|^2]=E[|a_R|^4]$, then:

$$J_{Min}=-E[a_R^4]+R^2 \tag{10}$$

Comparing (9) and (10), we can write:

$$J(\mathrm{G}_1)=J_{Min}+\beta\left[\left(\sum_{k=1}^{M}|g_{k1}|^2\right)^2-\sum_{k=1}^{M}|g_{k1}|^4\right] +E[a_R^4]\left(\sum_{k=1}^{M}|g_{k1}|^2-1\right)^2+2\beta\cdot\sum_{k=1}^{M}g_{R,k1}^2 g_{I,k1}^2 \tag{11}$$

Since $\beta>0$ and it is known that: $\left(\sum_{k=1}^{M}|g_{k1}|^2\right)^2\geq\sum_{k=1}^{M}|g_{k1}|^4$

Then: $\left(\sum_{k=1}^{M}|g_{k1}|^2\right)^2-\sum_{k=1}^{M}|g_{k1}|^4\geq 0$, thus :

$$\beta\left[\left(\sum_{k=1}^{M}|g_{k1}|^2\right)^2-\sum_{i=1}^{M}|g_{k1}|^4\right]\geq 0 \tag{12}$$

According to (12) , $J(\mathrm{G}_1)$ is composed only of positive terms. Thus, minimizing $J(\mathrm{G}_1)$ is equivalent to finding G_1 that minimizes all terms at the same time. One way to find the minimum of (11) is by looking for a solution that cancels the gradients of each term separately. From (10) J_{Min} is a constant ($\partial J_{Min}/\partial g_{\ell 1}^*=0$), then we only deal with the reminder terms. For that, let:

$$J(\mathrm{G}_1)=J_{Min}+J_1(\mathrm{G}_1)+J_2(\mathrm{G}_1)+J_3(\mathrm{G}_1)$$

Where:

$$J_1(\mathrm{G}_1)=\beta\left[\left(\sum_{k=1}^{M}|g_{k1}|^2\right)^2-\sum_{k=1}^{M}|g_{k1}|^4\right]$$

$$J_2(\mathrm{G}_1)=E[a_R^4]\left(\sum_{k=1}^{M}|g_{k1}|^2-1\right)^2$$

$$J_3(\mathrm{G}_1)=2\beta\cdot\sum_{k=1}^{M}g_{R,k1}^2 g_{I,k1}^2$$

Computing the derivatives of $J_1(\mathrm{G}_1)$, $J_2(\mathrm{G}_1)$ and $J_3(\mathrm{G}_1)$ with respect to $g_{\ell 1}^*$:

$$\frac{\partial J_1(\mathrm{G}_1)}{\partial g_{\ell 1}^*}=2\beta\, g_{\ell 1}\left(\sum_{k=1}^{M}|g_{k1}|^2-|g_{\ell 1}|^2\right)=0$$

Then:

$$\sum_{k=1,k\neq\ell}^{M}|g_{k1}|^2=0 \tag{13}$$

$$\frac{\partial J_2(\mathrm{G}_1)}{\partial g_{\ell 1}^*}=2E[a_R^4]g_{\ell 1}\left(\sum_{k=1}^{M}|g_{k1}|^2-1\right)=0$$

Then:

$$\sum_{k=1}^{M}|g_{k1}|^2=1 \tag{14}$$

$$\frac{\partial J_3(\mathrm{G}_1)}{\partial g^*_{\ell 1}} = 2\beta(g_{R,\ell 1}\, g^2_{I,\ell 1} + j\, g^2_{R,\ell 1} g_{I,\ell 1}) = 0$$

Then:

$$g_{R,\ell 1} = 0 \ \text{ or } \ g_{I,\ell 1} = 0 \ \text{ or } \ g_{R,\ell 1} = g_{I,\ell 1} = 0 \qquad (15)$$

(13) implies that only one entry, $g_{\ell 1}$, of G_1 is nonzero and the others are zeros. Equation (14), indicates that the modulus of this entry must be equal to one ($|g_{\ell 1}|^2 = 1$). Finally, from (15) either the real part or the imaginary must be equal to zero. Based on (14), the non zero part has one as modulus, i.e. either $g^2_{R,\ell 1} = 1$ and $g^2_{I,\ell 1} = 0$ or $g^2_{R,\ell 1} = 0$ and $g^2_{I,\ell 1} = 1$. Therefore the solution $g_{\ell 1}$ is a pure-real or a pure imaginary with a modulus one, which corresponds to:

$$g_{\ell 1} = e^{jn_1\frac{\pi}{2}}, \quad n_1\text{: is an integer.} \qquad (16)$$

This solution shows that the minimization of $J(\mathrm{G}_1)$ forces the equalizer output to form a constellation that corresponds to the source constellation with a modulo $\pi/2$ phase rotation.

From (13), (14), (15) and (16), We can conclude that the only stable minima for G_1 are of the form $\mathrm{G}_1 = [0 \ldots 0\; e^{jn_1\frac{\pi}{2}}\; 0 \ldots 0]^T$, i.e. only one entry is nonzero, pure-real or pure-imaginary with modulus equal to one, and all the others are zeros. This solution corresponds to the recovery of only one source signal and cancels the others.

G_2 is updated exactly as G_1, with a constraint in order to ensure orthogonality between G_2 and G_1:

$$\begin{aligned} &\underset{\mathrm{G}_2}{Min} \quad J(\mathrm{G}_2) = E[(z^2_{R,2} - R)^2] \\ &\text{Subject to: } \ \mathrm{G}_2^H \mathrm{G}_1 = 0 \end{aligned} \qquad (17)$$

We examine the convergence of G_2 after G_1 has converged to one signal, because the adaptation of G_1 is realized independently from the other G_i. For simplicity, and without loss of generality, we consider that G_1 has converged to the first signal. Then, $\mathrm{G}_1 = [e^{jn_1\frac{\pi}{2}}\; 0 \ldots 0]^T$. Hence the orthogonal constraint, $\mathrm{G}_2^H \mathrm{G}_1 = 0$, results in $g_{12} = 0$. Then, $\mathrm{G}_2 = [0 \,|\, \overline{\mathrm{G}}_2^T]^T$. Then equation (17) can be written as:

$$\underset{\mathrm{G}_2}{Min} \quad J(\overline{\mathrm{G}}_2) = E[(z^2_{R,2} - R)^2] \qquad (18)$$

Where: $z_2 = \mathrm{G}_2^T \mathrm{A} = \overline{\mathrm{G}}_2^T \mathrm{A}$, because $g_{12} = 0$.

Equation (18) has the same form as equation (7). Then the analysis is exactly the same as described previously. Consequently, the stationary points of (18) will be of the form $\mathrm{G}_2 = [0 \,|\, 0 \ldots 0\; e^{jn_2\frac{\pi}{2}}\; 0 \ldots 0]^T$. Hence G_2 will recover perfectly a different signal than that recovered by G_1.

By application of the same analysis to each G_i, we can see that each G_i converges to a setting that has zeros at the positions of the already recovered signals. Its remaining entries, as in (18), contain only one nonzero element, which correspond to recovering a different signal. And so on until the recovery of all signals.

Based on this analysis, we can conclude that the minimization of the proposed cost function ensures perfect recovery of all source signals and that the recovered signals correspond to the source signals with a modulo $\pi/2$ phase rotation.

5. IMPLEMENTATION

Now, we present an adaptive implementation of the cost function (3) via the classical stochastic gradient algorithm (SGA). The SGA is given by:

$$\mathbf{W}(k+1) = \mathbf{W}(k) - \frac{1}{2}\mu\, \nabla_{\mathbf{W}}(J)$$

In order to use the Gram-Schmidt orthogonalization procedure, we assume that the received signal is prewhitened or the channel matrix is unitary. Hence the constraint in (3) can be written as:

$$\mathbf{G}^H\mathbf{G} = \mathbf{W}^H \underbrace{\mathbf{H}^*\mathbf{H}^T}_{\mathbf{I}_N} \mathbf{W} = \mathbf{W}^H\mathbf{W} = \frac{1}{\sigma_a^2}\; diag(\sigma_1^2 \ldots \sigma_M^2) \quad (19)$$

Where $diag(\sigma_1^2 \ldots \sigma_M^2)$ is a diagonal matrix with $\sigma_1^2 \ldots \sigma_M^2$ as its diagonal entries and $\sigma_i^2, i = 1, \ldots, M$, is the variance of the *i*-th output of the receiver.

Computing the gradient of J with respect to $\mathbf{W}$, we obtain:

$$\mathbf{W}'(k+1) = \mathbf{W}(k) - \mu\, \mathrm{Y}^*(k)\, [\Delta_1(k)\, \ldots\, \Delta_M(k)] \qquad (20)$$

Where:

$$\Delta_i(k) = (z^2_{R,i} - R)\; z_{R,i}$$

And in order to satisfy the constraint (19) we combine, at each iteration, the Gram-Schmidt orthogonalization with SGA (20) [6]. The orthogonalization is necessary because the result of SGA, $\mathbf{W}'$, is not necessarily orthogonal. The Gram-Schmidt orthogonalization on $\mathbf{W}'$ is defined as:

$$\begin{aligned} &\mathrm{W}_1(k+1) = \mathrm{W}_1'(k+1) \\ &\mathrm{W}_p(k+1) = \mathrm{W}_p'(k+1) - Q, \quad p = 2, \ldots, M \\ &\mathrm{Q} = \sum_{l=1}^{p-1} \left[\mathrm{W}_l^H(k+1)\, \mathrm{W}_p'(k+1)\right] \mathrm{W}_l(k+1) \end{aligned}$$

After convergence, the algorithm must recover all source signals up to a possible permutation and up to modulo $\pi/2$ phase rotation.

5.1 Complexity:

In order to compare the computational complexity of the proposed algorithm with that of CMA [2], We consider only the SGA (20) because the Gram Schmidt orthogonalization procedure is the same for both algorithms. We have:

$$\mathbf{W}'(k+1) = \mathbf{W}(k) - \mu\, \mathrm{Y}^*(k)\, [\Delta_1(k)\, \ldots\, \Delta_M(k)]$$

Where for:

- Our algorithm: $\Delta_i(k) = (z^2_{R,i} - R)\; z_{R,i}$
- CMA: $\Delta_i(k) = \left(|z_i|^2 - R_{CMA}\right) z_i$
- MMA: $\Delta_i(k) = (z^2_{R,i} - R)\; z_{R,i} + j(z^2_{I,i} - R)\; z_{I,i}$

Algorithm	Multiplications	Additions
Our algorithm	$(4N+3) \times \mathrm{M}$	$4M\times N$
CMA	$2(4N+3) \times \mathrm{M}$	$8M\times N$
MMA	$2(4N+3) \times \mathrm{M}$	$8M\times N$

Table 1. Comparison of complexity per weight update

According to table 1, the complexity of the proposed algorithm is two times smaller compared to CMA and MMA. Note that the number of operations is per iteration.

6. NUMERICAL RESULTS

Some numerical results are now presented in order to confirm the theoretical analysis derived in the previously sections. For that, we use signal to interference and noise ratio (SINR) defined as:

$$SINR_i(\mathrm{W}_k) = \frac{|g_{ik}|^2}{\sum_{\ell=1,\ell\neq i}^{M} |g_{\ell k}|^2 + W_k^T R_B W_k^*}$$

Where: $SINR_i(\mathrm{W}_k)$ is the signal to interference and noise ratio of the i-th source signal at the k-th output. $g_{ij} = \mathrm{H}_i^T\,\mathrm{W}_j$, where: W_j and H_i are the j-th and i-th column vector of matrices $\mathbf{W}$ and $\mathbf{H}$ respectively. $\mathbf{R}_B = E[\mathrm{BB}^H] = \sigma_{\mathrm{B}}^2\,\mathbf{I}_N$ is the noise covariance matrix.

We use the data model in (1): the system inputs are independent, uniformly distributed and drawn from 16 or 64-QAM constellations. The unitary channel matrices are chosen randomly. The variance of noise is determined according to the desired signal to noise ratio (SNR). So that the comparison be significant, we consider the same implementation for all algorithms (See section 5). Hence the results are only influenced by the used criterion and not by the implementation.

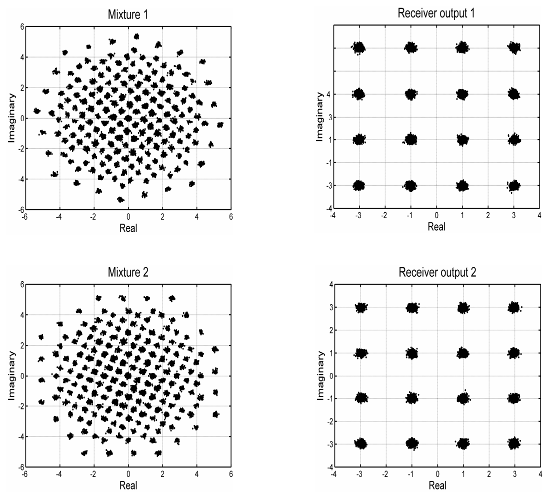

Figure 1. Left column: constellation of the received signals. Right column: constellation of the receiver outputs

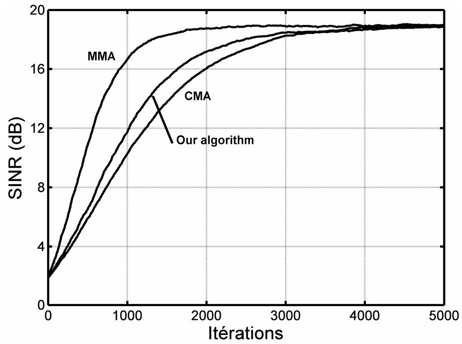

Figure 2. Performance comparison of the proposed algorithm, CMA and MMA.

Figures 1 shows the constellations of the received signals and the receiver outputs (after convergence) using the proposed algorithm. We have considered $M=2, N=2$, 16-QAM, SNR=30 dB and $\mu = 2\times10^{-3}$. It is clear that the algorithm recovers the source signals with a modulo $\pi/2$ phase rotation, which confirm the theoretical analysis.

Figure 2 shows the SINR performance plots for the proposed approach, CMA and MMA. we have considered $M=3$, $N=3$, 64-QAM, SNR=20 dB and the step sizes were chosen to have sensibly the same steady state for all algorithms. We observe that our algorithm is between MMA and CMA with a low computational complexity, it gives a good compromise between performance and complexity.

7. CONCLUSION

In this paper, we have proposed a new globally convergent algorithm for the Multiple-input multiple-output (MIMO) adaptive blind separation of QAM signals. The criterion is based on the real (the imaginary) part of the receiver output and consists in penalizing the deviation of the real (the imaginary) part from a constant. The proposed approach was shown to be globally convergent to a setting that recovers perfectly (in the absence of noise) all the source signals. Our algorithm has shown a low computational complexity compared to CMA and MMA, which make it attractive for implementation. Simulation results have shown that the proposed algorithm has, despite its low complexity, a good performance.

APPENDIX

From equation (8):

$$J(\mathrm{G}_i) = E[z_{R,i}^4] - 2R\,E[z_{R,i}^2] + R^2 \tag{21}$$

We have:

$$\mathrm{z}_i(n) = \mathrm{G}_i^T\,\mathrm{A}(n)$$

Where:

$$\mathrm{G}_i = [g_{1i},\ldots,g_{Mi}]^T\ ,\quad g_{ki} = g_{R,ki} + j\,g_{I,ki}$$

$$\mathrm{A} = [a_1,\ldots,a_M]^T\ ,\quad a_k = a_{R,k} + j\,a_{I,k}$$

Then:

$$z_{R,i} = \sum_{k=1}^{M} (g_{R,ki} a_{R,k} - g_{I,ki} a_{I,k}) \quad (22)$$

For i.i.d. and mutually independent source signals that drawn from QAM constellations, we have:

$$E[a_R] = E[a_I] = 0 \quad (23)$$

$$E[a_R^m] = E[a_I^m], \ \forall m \quad (24)$$

$$E[a_{R,k}^m a_{I,\ell}^n] = E[a_{R,k}^m] E[a_{I,\ell}^n], \quad \forall k, \ell, m, n \quad (25)$$

$$E[a_{R,k}^m a_{R,\ell}^n] = E[a_{I,k}^m a_{I,\ell}^n] = \begin{cases} E[a_R^{m+n}], & \text{if } k = \ell \\ E[a_R^m] E[a_R^n], & \text{otherwise} \end{cases} \quad (26)$$

From (22), we have:

$$\begin{aligned} E[z_{R,i}^2] &= E\left\{ \left[\sum_{k=1}^{M} (g_{R,ki} a_{R,k} - g_{I,ki} a_{I,k}) \right]^2 \right\} \\ &= E\left\{ \sum_{k=1}^{M} \sum_{\ell=1}^{M} (g_{R,ki} a_{R,k} - g_{I,ki} a_{I,k})(g_{R,\ell i} a_{R,\ell} - g_{I,\ell i} a_{I,\ell}) \right\} \\ &= \sum_{k=1}^{M} \sum_{\ell=1}^{M} \big(g_{R,ki} g_{R,\ell i} E[a_{R,k} a_{R,\ell}] - g_{R,ki} g_{I,\ell i} E[a_{R,k} a_{I,\ell}] \\ &\qquad - g_{I,ki} g_{R,\ell i} E[a_{I,k} a_{R,\ell}] + g_{I,ki} g_{I,\ell i} E[a_{I,k} a_{I,\ell}] \big) \end{aligned}$$

Using (23), (24), (25) and (26), we obtain:

$$E[z_{R,i}^2] = E[a_R^2] \sum_{k=1}^{M} |g_{ki}|^2 \quad (27)$$

Similarly :

$$\begin{aligned} E[z_{R,i}^4] &= E\left\{ \left[\sum_{k=1}^{M} (g_{R,ki} a_{R,k} - g_{I,ki} a_{I,k}) \right]^4 \right\} \\ &= \sum_{k=1}^{M} \sum_{\ell=1}^{M} \sum_{m=1}^{M} \sum_{n=1}^{M} E\big[(g_{R,ki} a_{R,k} - g_{I,ki} a_{I,k}) \\ &\quad (g_{R,\ell i} a_{R,\ell} - g_{I,\ell i} a_{I,\ell})(g_{R,mi} a_{R,m} - g_{I,mi} a_{I,m}) \\ &\quad (g_{R,ni} a_{R,n} - g_{I,ni} a_{I,n}) \big] \end{aligned} \quad (28)$$

Developing (28) and using (23), (24), (25) and (26), we have the following three cases:

- For: $k = \ell = m = n$:

$$E[|z_{R,i}|^4] = E[a_R^4] \sum_{k=1}^{M} (g_{R,ki}^4 + g_{I,ki}^4) + 6E^2[a_R^2] \sum_{k=1}^{M} (g_{R,ki}^2 g_{I,ki}^2) \quad (29)$$

- For: $k = \ell \neq m = n$ and the two other combinations:

$$E[|z_{R,i}|^4] = 3E^2[a_R^2] \sum_{k=1}^{M} \sum_{\substack{\ell=1 \\ \ell \neq k}}^{M} |g_{ki}|^2 |g_{\ell i}|^2 \quad (30)$$

- Otherwise: equation (28) equal to zero.

From (29) and (30), equation (28) becomes:

$$\begin{aligned} E[|z_{R,i}|^4] &= E[a_R^4] \sum_{k=1}^{M} (g_{R,ki}^4 + g_{I,ki}^4) + 6E^2[a_R^2] \sum_{k=1}^{M} (g_{R,ki}^2 g_{I,ki}^2) \\ &\quad + 3E^2[a_R^2] \sum_{k=1}^{M} \sum_{\substack{\ell=1 \\ \ell \neq k}}^{M} |g_{ki}|^2 |g_{\ell i}|^2 \\ &= E[a_R^4] \sum_{k=1}^{M} |g_{ki}|^4 - 2E[a_R^4] \sum_{k=1}^{M} g_{R,ki}^2 g_{I,ki}^2 + 6E^2[a_R^2] \\ &\quad \cdot \sum_{k=1}^{M} g_{R,ki}^2 g_{I,ki}^2 + 3E^2[a_R^2] \sum_{k=1}^{M} \sum_{\substack{\ell=1 \\ \ell \neq k}}^{M} |g_{ki}|^2 |g_{\ell i}|^2 \end{aligned} \quad (31)$$

On the other hand, we have:

$$\left[\left(\sum_{i=1}^{M} |g_{i1}|^2 \right)^2 - \sum_{i=1}^{M} |g_{i1}|^4 \right] = \sum_{k=1}^{M} \sum_{\substack{\ell=1 \\ \ell \neq k}}^{M} |g_{ki}|^2 |g_{\ell i}|^2 \quad (32)$$

Substituting (32) into (31):

$$\begin{aligned} E[z_{R,i}^4] &= E[a_R^4] \sum_{k=1}^{M} |g_{ki}|^4 + 3E^2[a_R^2] \left[\left(\sum_{i=1}^{M} |g_{i1}|^2 \right)^2 - \sum_{i=1}^{M} |g_{i1}|^4 \right] \\ &\quad + 2(3E^2[a_R^2] - E[a_R^4]) \sum_{k=1}^{M} g_{R,ki}^2 g_{I,ki}^2 \end{aligned} \quad (33)$$

Let $\beta = (3E^2[a_R^2] - E[a_R^4])$, and using (27) and (33) in (21), we obtain:

$$\begin{aligned} J(\mathrm{G}_i) &= E[a_R^4] \sum_{k=1}^{M} |g_{ki}|^4 + 3E^2[a_R^2] \left[\left(\sum_{i=1}^{M} |g_{i1}|^2 \right)^2 - \sum_{i=1}^{M} |g_{i1}|^4 \right] \\ &\quad + 2\beta \sum_{k=1}^{M} g_{R,ki}^2 g_{I,ki}^2 - 2E[a_R^2] \sum_{k=1}^{M} |g_{ki}|^2 + R^2 \\ &= E[a_R^4] \left(\sum_{i=1}^{M} |g_{i1}|^2 \right)^2 - E[a_R^4] \left(\sum_{i=1}^{M} |g_{i1}|^2 \right)^2 + E[a_R^4] \sum_{k=1}^{M} |g_{ki}|^4 \\ &\quad + 3E^2[a_R^2] \left[\left(\sum_{i=1}^{M} |g_{i1}|^2 \right)^2 - \sum_{i=1}^{M} |g_{i1}|^4 \right] + 2\beta \sum_{k=1}^{M} g_{R,ki}^2 g_{I,ki}^2 \\ &\quad - 2E[a_R^2] \sum_{k=1}^{M} |g_{ki}|^2 + R^2 + E[a_R^4] - E[a_R^4] \end{aligned}$$

Rearranging terms, we get:

$$\begin{aligned} J(\mathrm{G}_i) &= E[a_R^4] \left(\sum_{i=1}^{M} |g_{ki}|^2 - 1 \right)^2 + \beta \left[\left(\sum_{i=1}^{M} |g_{ki}|^2 \right)^2 - \sum_{i=1}^{M} |g_{ki}|^4 \right] \\ &\quad + 2\beta \sum_{k=1}^{M} g_{R,ki}^2 g_{I,ki}^2 - E[a_R^4] + R^2 \end{aligned}$$

Finally, we get (9).

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