14th European Signal Processing Conference (EUSIPCO 2006), Florence, Italy, September 4-8, 2006, copyright by EURASIP

RADAR DETECTION AND CLASSIFICATION OF JAMMING SIGNALS *

Maria Greco[1], Fulvio Gini[1], Alfonso Farina[2] and Valentina Ravenni[3]

[1] Dip. di Ingegneria dell'Informazione, University of Pisa, via G. Caruso, 14 - 56122 Pisa, Italia

Tel: +39-050-2217620, Fax: +39-050-2217522, e-mail: m.greco@ing.unipi.it , f.gini@ing.unipi.it

[2] SELEX Sistemi Integrati (formerly AMS SpA), via Tiburtina Km. 12.400 - 00131 Rome, Italy

Tel: +39-06-4150-2279, fax: +39-06-4150-3755, e-mail: afarina@selex-si.com

[3] GEM Elettronica S.r.l., via A. Vespucci, 9 – 63039 San Benedetto del Tronto (AP), Italy

Tel +39-0735-59051, Fax +39-0735-590528, E-mail: ravenni@gemrad.com

ABSTRACT

This paper considers the problem of detecting and classifying a radar target against jamming signals emitted by electronic countermeasure (ECM) systems. The detection-classification algorithm proposed here exploits the presence in the jamming spectrum of spurious terms due to phase quantization performed by the radio frequency digital memory (DRFM) device.

1. INTRODUCTION

Electronic countermeasure (ECM) techniques against tracking radar are often enhanced by using radio frequency digital memory. The DRFM is a device in which high-speed sampling digital memory is used for storage and recreation of radio frequency signals in order to deceive hostile radar systems. In a DRFM system, the input RF signal is generally first down-shifted in frequency and then sampled with a high-speed analog-to-digital converter (ADC). The samples are stored in memory and they can be manipulated in amplitude, frequency and phase to generate a wide range of jamming signals. The stored samples are later recalled, processed by the digital-to-analog converter (DAC), up converted and transmitted back to the victim radar [1].

In this paper, we approach the problem of detecting and classifying a radar target against jamming signals in presence of thermal noise. The model of the jamming signal emitted by a phase-quantizer DRFM, very common in jamming systems, is introduced in Section 2. In Section 3 we define the jamming signal error angle (JSEA) as a measure of the errors introduced by the DRFM on the false target. In Section 4 we propose a detection/classification algorithm that exploits the JSEA to distinguish between true and false target. Finally, some results of our Monte Carlo simulations are described in Section 5.

*This work has been funded by SELEX Sistemi Integrati.

2. JAMMING SIGNAL MODEL

A pulse Doppler radar transmits a coherent signal that is produced by modulating a complex sinusoid with a pulsed waveform. For ease of treatment, we assume here that the signal is infinite in time. Using the infinite time assumption, the pulsed waveform can be written as

$$p(t) = \sum_{n=-\infty}^{+\infty} p_{T_i}(t - nT_r) = p_{T_i}(t) \otimes \sum_{n=-\infty}^{+\infty} \delta(t - nT_r), \qquad (1)$$

where $p_{T_i}(t) = rect(t/T_i)$. Then, the target signal received by the radar is proportional to $y_P(t) = p(t)\exp[j\phi(t)]$, where $\phi(t) = 2\pi F_0 t$ and the frequency F_0 takes into account the down-conversion performed by the DRFM and the Doppler frequency of the target.

Generally, the information content of an intercepted radar signal is mainly carried in the phase of the signal, then the amplitude information is discarded and only the phase is quantized, using $N=2^M$ levels. M is the number of bits of the DRFM.The detailed mathematical derivation and analysis of the jamming signal has been performed in [2]. Here we summarize the main results.

The jamming signal can we written as

$$x(t) = e^{j\hat{\phi}(t)} = \sum_{m=-\infty}^{+\infty} \text{sinc}\left(m + \frac{1}{N}\right) e^{j(Nm+1)\phi(t)} \qquad (2)$$

The spectrum of the quantized signal, that is the Fourier transform of signal $x(t)$, is

$$\hat{X}(f) = \sum_{m=-\infty}^{+\infty} \text{sinc}\left(m + \frac{1}{N}\right) \delta\left[f - (Nm+1)F_0\right] \qquad (3)$$

where $\delta(f - f_X)$ is the Dirac delta function, centered at $f = f_X$. This result demonstrates that the spectrum of the jamming signal consists of the superposition of several different terms. The contribution for m=0 can be called the "primary image" term [3] and it is located at the frequency F_0. In addition to this primary term, there are contributions from $m \neq 0$ terms. The strength of the mth term is attenuated by the factor $\mathrm{sinc}(m + 1/N)$. Since m is always an integer and since the value of the *sinc* function for integer arguments is zero, the strength of the spurious terms decreases as the number N of quantization levels increases. These spurious signals occur at the frequencies $f_s = (Nm+1)F_0$. For a 2-bit quantization, as instance, the strongest spurious harmonic is obtained for m=-3 and its "amplitude" ratio with the primary term is about -9.6 dB.

3. THE JAMMING SIGNAL ERROR ANGLE

The jamming signal and the true signal are received by the radar. Both signals are down-converted at a low frequency or in baseband and filtered by an anti-aliasing low-pass filter with impulsive response $h(t)$, centered at null frequency, before sampling and processing [4]. It is important to remember that the frequency F_0 takes into account the down-conversion performed by the DRFM and the Doppler frequency of the target, that is $F_0 = F_{I_{DRFM}} + F_D$, where $F_{I_{DRFM}}$ is the intermediate frequency of the DRFM and F_D is the Doppler frequency of the target. After the baseband conversion performed by the radar receiver, both jammer and true target spectrum are centered on F_D.

The true and jamming signal vectors can be obtained by sampling at the frequency F_c the filtered signals $x(t) = g(t) \otimes h(t)$ and $y(t) = y(t) \otimes h(t)$. We assume that the anti-aliasing filter has a bandwidth large enough to leave the true signal unmodified and to suppress the components of the jamming signal external to the bandwidth of the filter, such that: $-F_c/2 \leq (Nm+1)F_0 - F_{I_{DRFM}} \leq F_c/2$.

It results that

$$x(n) = x\left(\frac{n}{F_c}\right) = \sum_{m=M_{\min}}^{M_{\max}} \mathrm{sinc}\left(m + \frac{1}{N}\right) e^{j(Nm+1)2\pi f_0 n} e^{-j2\pi f_{IDRFM} n} \qquad (4)$$

and

$$y(n) = y\left(\frac{n}{F_c}\right) = e^{j2\pi f_D n}, \qquad (5)$$

where:

$$f_0 = F_0/F_c\,,\ f_{I_{DRFM}} = F_{I_{DRFM}}/F_c\,,\ f_D = F_D/F_c\,,$$

$$n = 0,1,\ldots,K-1\,,\ M_{\min} = -\left\lceil \frac{1}{2Nf_0} \right\rceil \text{ and } M_{\max} = \left\lfloor \frac{1}{2Nf_0} \right\rfloor.$$

To measure the errors introduced by the DRFM on the jamming signal, we introduce a new measure: the jamming signal error angle (JSEA) μ. For its definition, we used a generalized cosine:

$$\cos(\mathbf{x}, \mathbf{y}) = \frac{\left|\mathbf{x}^H \mathbf{y}\right|}{\|\mathbf{x}\| \cdot \|\mathbf{y}\|} \qquad (6)$$

to define the angle between two complex vectors **x** and **y** [5, 6]. The angle μ depends on the number of quantization levels N=2^M, the normalized frequency f_0 and the number of samples K. Some result has been reported in Fig. 1 for M=2 and K=32 as a function of f_0.

In Fig. 2 we show the spectra of the true and jamming signals for f_0=0.097, M=2 and K=32. The differences between the two spectra increase with decreasing number of bits and decreasing value of $f_{I_{DRFM}}$.

Based on our results we can conclude that, when the number of quantization bits is low, the K-dimensional directions of the steering vectors of true and jamming signals differ by the angle μ. This information can be used in the radar detector to recognize the jamming signal and avoid false detection.

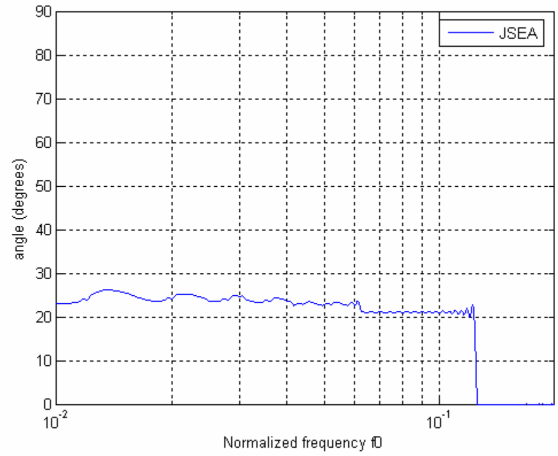

Fig. 1 – Jamming signal error angle (abs) as a function of f_0

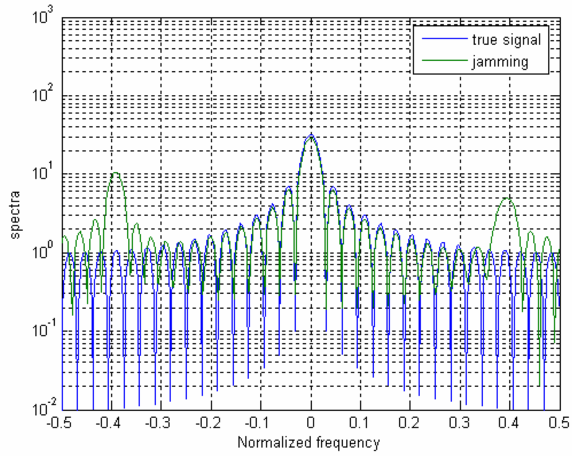

Fig. 2 – Jamming and true signal spectra signal centered of f_0

4. THE DETECTION/CLASSIFICATION SCHEME

The detection problem and the classification between true target and jammer is formulated here as a multiple hypotheses testing (MHT) problem on the complex measured vector $\mathbf{z}=[z(0)\ z(1)\ \cdots\ z(K-1)]^T$ recorded by the radar during the time-on-target (*ToT*), where *K* is the number of samples collected by the radar during a single pulse.

Under the null hypothesis (H_0) it is assumed that the data consist of disturbance **d** (clutter plus thermal noise). Under the alternative hypotheses it is instead assumed that the data consist of the sum of disturbance and signal backscattered by the true target (H_1) or the jammer (H_2) Therefore, we formulate our MHT problem as follows [7]:

$$\begin{cases} H_0 : \mathbf{z} = \mathbf{d} \\ H_k : \mathbf{z} = \mathbf{d} + \mathbf{s}_k, \qquad k = 1, 2. \end{cases} \tag{7}$$

Denote by $P_k = \Pr\{H_k\}$ the *a priori* probability of hypothesis H_k. We assume that $\{P_k\}_{k=0}^{2}$ are unknown, but those pertaining to the two target classes are the same,[1] i.e., $P_1 = P_2 = (1-P_0)/2$, where P_0 is the probability of the hypothesis H_0.

The detection/classification scheme should detect the presence of a target and correctly classify it. In this work we suppose the true target $\mathbf{s}_1 = \alpha\mathbf{p}$, where α is a complex Gaussian random variable with zero mean and variance σ_α^2, in short notation $\alpha \sim CN(0,\ \sigma_\alpha^2)$. The steering vector **p** is assumed a priori known, $p(n) = e^{j2\pi f_D n}$, where f_D is the Doppler frequency of the true target normalized to the pulse repetition frequency (PRF) of the radar. The jamming signal vector is $\mathbf{s}_2 = \beta\mathbf{p}_j$, where β is a complex Gaussian random variable with zero mean and variance σ_β^2, in short notation $\beta \sim CN(0,\ \sigma_\beta^2)$. The vector $\mathbf{p}_j$ is unknown. The disturbance **d** is modeled as complex Gaussian distributed random vector with zero-mean and correlation matrix $\mathbf{M}_d = E\{\mathbf{z}\mathbf{z}^H\} = \sigma_d^2\mathbf{M}$, where $(\cdot)^H$ is the conjugate-transpose operator, σ_d^2 is the power of each disturbance component, and **M** is the normalized covariance matrix, i.e. $[\mathbf{M}]_{i,i} = 1$ for $i = 1,\ 2,\ \cdots,\ N$. We assume here that **M** is full-rank. In shorthand notation $\mathbf{d} \sim CN(\mathbf{0},\ \sigma_d^2\mathbf{M})$.

If P_0, P_1, P_2 and $\mathbf{p}_j$ were known we could use the maximum a posteriori (MAP) criterion and decide in favor of one of the three hypotheses according to the following rule:

$$\hat{H}_{\bar{k}} :\ \bar{k} = \arg\max_k \Pr\{H_k|\mathbf{z}\} = \arg\max_k f_{\mathbf{z}|H_k}(\mathbf{z}|H_k)P_k \tag{8}$$

where $k = 0, 1, \cdots, 2$, $\Pr\{H_k|\mathbf{z}\}$ is the *a posteriori* probability of H_k and $f_{\mathbf{z}|H_k}(\mathbf{z}|H_k)$ is the data probability density function (pdf) conditioned to hypothesis H_k.

In this work we propose a different approach that exploits the presence in the jamming spectrum of spurious terms due to the phase quantization. Based on the spectral shape of true target and jamming signal, it is clear that a filter matched to the true target $\mathbf{s}_1 = \alpha\mathbf{p}$ does not allow the radar system to distinguish between target and jammer, because both signals present a spectral peak at the frequency f_D. The difference is in the spurious spectral lines centered at $f_s = (Nm+1)f_0$, with $m \neq 0$, that would be cancelled by a narrowband matched filter centered on f_D.

As already said, the classifier should exploit the information contained in the JSEA for classification purposes and measure it from the observation vector. A similar operation is performed by the ACE (Adaptive Coherence Estimator) detector [8,9]:

$$l_{ACE}(\mathbf{z}) = \frac{\left|\mathbf{p}^H\mathbf{S}^{-1}\mathbf{z}\right|^2}{\left(\mathbf{p}^H\mathbf{S}^{-1}\mathbf{p}\right)\left(\mathbf{z}^H\mathbf{S}^{-1}\mathbf{z}\right)}. \tag{9}$$

It is easy to observe that, if the noise is negligible and **S**=**I**, where **I** is the *N*x*N* identity matrix, $l_{ACE} = \cos^2\mu$, where μ is the JSEA. Then, l_{ACE} uses the measure of the angle between known target steering vector and observation vector for detection purposes and originally, it was thought in the adaptive form, that is, it estimates the covariance matrix of the disturbance using secondary vectors.

In this work we assume a priori knowledge of the disturbance covariance matrix, i.e., **S**=**M** (otherwise it can be estimated as described in [10]), and we use l_{ACE} as a classifier. Then the detection/classification scheme is composed of two blocks as shown in Fig. 3.

The first one is a whitening matched filter [11]. It acts as a detector, then it tests hypothesis H_0 against H_1+H_2,

$$l_{GLRT}(\mathbf{z}) = \frac{\left|\mathbf{p}^H\mathbf{M}^{-1}\mathbf{z}\right|^2}{\mathbf{p}^H\mathbf{M}^{-1}\mathbf{p}} \underset{H_0}{\overset{H_1+H_2}{\gtrless}} \eta \tag{10}$$

If the threshold η is exceeded the target is declared present without any other information, if not, the target is declared absent. In the first case, in the second block, the system tries to classify the signal as true target or jamming [12]:

$$l_{ACE}(\mathbf{z}) = \frac{\left|\mathbf{p}^H\mathbf{M}^{-1}\mathbf{z}\right|^2}{\left(\mathbf{p}^H\mathbf{M}^{-1}\mathbf{p}\right)\left(\mathbf{z}^H\mathbf{M}^{-1}\mathbf{z}\right)} = \frac{l_{GLRT}(\mathbf{z})}{\left(\mathbf{z}^H\mathbf{M}^{-1}\mathbf{z}\right)} \underset{H_2}{\overset{H_1}{\gtrless}} \lambda \tag{11}$$

If the threshold λ is exceeded, that is, if the $\cos^2$ of the estimated JSEA $\hat{\mu}$ is greater than λ, the target is classified

[1] This assumption is somewhat artificial, but necessary to come up with a practical decision strategy. However, if some a priori information is available, it can be easily taken into account in the selection of the decision thresholds.

as true, if not it is classified as jammer. Being $l_{ACE} = \cos^2 \hat{\mu}$, the threshold λ spans in the interval [0,1].

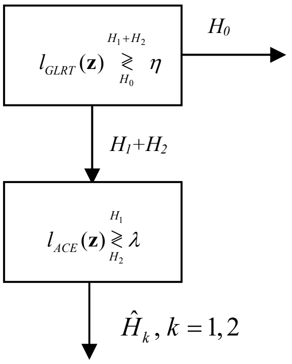

Fig. 3 – Detection/classification scheme

To evaluate the performance of the proposed scheme, we calculated the probability of false alarm (P_{FA}), the probability of detection (P_D), and the probability of correct classification (P_C).

The **probability of false alarm** is given by the probability of deciding that a target is present, belonging to any of the possible classes, when the null hypothesis is the correct one. Therefore, $P_{FA} = \Pr\{l_{GLRT}(\mathbf{x}) > \eta | H_0\}$.

The **probability of correct classification** is defined as the probability of deciding for the correct hypothesis when a target is present, that is $P_C = P_1 P_{C|H_1} + P_2 P_{C|H_2}$ where we defined $P_{C|H_k} = \Pr\{\hat{H}_k | H_k\}$, for k=1,2, then

$$P_{C|H_1} = \Pr\{l_{GLRT}(\mathbf{x}) > \eta, l_{ACE}(\mathbf{x}) > \lambda | H_1\} \tag{12}$$

and

$$P_{C|H_2} = \Pr\{l_{GLRT}(\mathbf{x}) > \eta, l_{ACE}(\mathbf{x}) < \lambda | H_2\} \tag{13}$$

The **probability of detection** is defined as the probability of declaring a target to be present, whichever the class the target belongs to, so we have $P_D = P_1 P_{D|H_1} + P_2 P_{D|H_2}$ where:

$$P_{D|H_1} = \Pr\{\hat{H}_{12} | H_1\} = \Pr\{l_{GLRT}(\mathbf{x}) > \eta | H_1\} \tag{14}$$

and

$$P_{D|H_2} = \Pr\{\hat{H}_{12} | H_2\} = \Pr\{l_{GLRT}(\mathbf{x}) > \eta | H_2\} \tag{15}$$

It is also useful to calculate the **probability of error** conditioned to hypotheses H_1 and H_2. We define

$$P_{E|H_1} = \Pr\{\hat{H}_2 | H_1\} = \Pr\{l_{GLRT}(\mathbf{x}) > \eta, l_{ACE}(\mathbf{x}) < \lambda | H_1\} \tag{16}$$

as the probability of label the target as true when a jamming signal is present and

$$P_{E|H_2} = \Pr\{\hat{H}_1 | H_2\} = \Pr\{l_{GLRT}(\mathbf{x}) > \eta, l_{ACE}(\mathbf{x}) > \lambda | H_2\} \tag{17}$$

as the probability of label the received signal as jamming when a true target is present.

5. NUMERICAL RESULTS

In this section we report some numerical results of our performance analysis. In Figs. 4 and 5 we report the probabilities of classification $P_{C|H_1}$ and $P_{C|H_2}$ for M=2, K=32 and $P_{FA}=10^{-4}$. The color relates to the value of the probability of classification, the x-axis represents the threshold of the ACE detector and the y-axis represents the target signal-to-noise power ratio $SNR_t = \sigma_\alpha^2 / \sigma_d^2$ in Fig. 4, and the jamming-to-noise power ratio $SNR_j = \sigma_\beta^2 / \sigma_d^2$ in Fig. 5.

In the Monte Carlo simulations the jamming steering vector $\mathbf{p}_j$ has been normalized to have same norm as $\mathbf{p}$. We set the target Doppler frequency $F_D = 100\,KHz$, the intermediate frequency of the DRFM $F_{I_{DRFM}} = 100\,MHz$ and the sampling frequency $F_C = 1.024\,GHz$.

In the SNR_t and SNR_j ranges of values we analyzed, the detection probability P_D is unitary. With high values of the threshold and of the signal-to-noise ratios the probability of correct classification, conditioned to each hypothesis is almost unitary.

This results is confirmed by Fig. 6, where we report the overall P_C. In this figure we set $SNR_t = SNR_j = SNR$ and again M=2, K=32, and $P_{FA}=10^{-4}$.

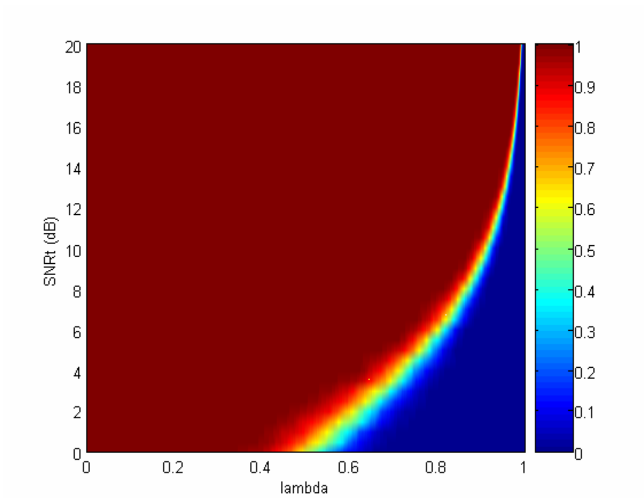

Fig. 4 – Probability of correct classification of the true target

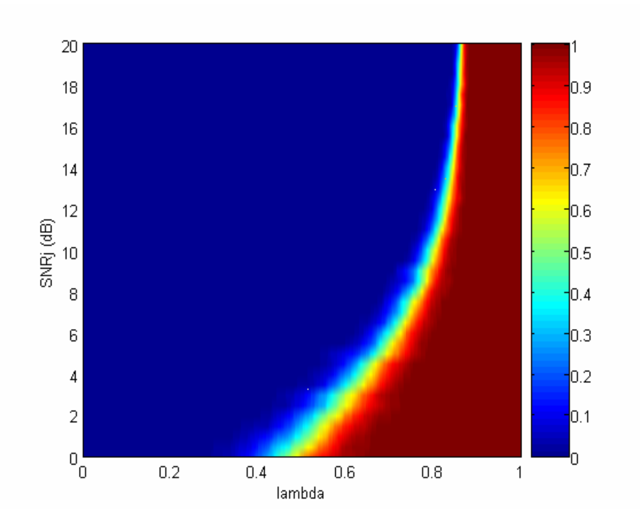

Fig. 5 – Probability of correct classification of the jamming signal

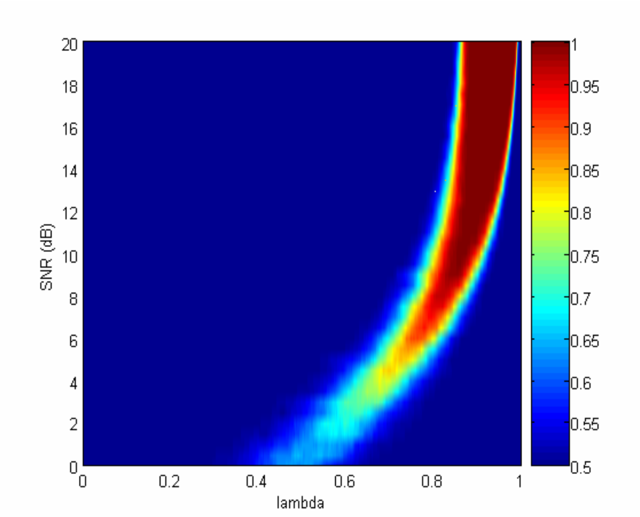

Fig. 6 – Probability of correct classification

Based on our first encouraging results, we can conclude that the proposed two-step detection/classification algorithm is effective and easy to implement.

A different approach to this multi-hypothesis detection problem could make use of the results in [13] concerning the problem of robust adaptive radar detection in the presence of steering vector mismatches. This is the subject of current research.

REFERENCES

[1] S. J. Roome, "Digital Radio Frequency Memory", *Electronic & Communication Engineering Journal*, August 1990.

[2] M. Greco, F. Gini, A. Farina, "Combined effect of phase and RGPO delay quantization on jamming signal spectrum," *IEEE International Radar Conference*, May 2005, Arlington, VA, USA.

[3] J. W. Goodman, M. Silvestri, "Some Effects of Fourier- Domain Phase Quantization", *IBM J. Res. Develop*., September 1970.

[4] S. D. Berger., "Digital Radio Frequency Memory Linear Range Gate Stealer Spectrum", *IEEE Trans. on Aerospace and Electronic System*, Vol. 29, April 2003, pp. 725-735.

[5] M. Knaak, S. Araki, S. Makino, "Geometrically Constraint ICA for Convolutive Mixtures of Sound", *Proceedings of ICASSP '03 Conference*, Vol.2, April 2003, pp. 725-8.

[6] S. Ramphashad, T. W. Parks, R. Shenoy, "Signal Modeling and Detection Using Cone Classes", *IEEE Trans on Signal Processing*, Vol. 46, No. 2 February 1996, pp. 329-338.

[7] F. Gini, A. Farina, M. Greco, "Radar Detection and Pre-Classification Based on Multiple Hypotheses Testing," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. 40, No. 3, pp. 1046-1059, July 2004.

[8] F. Gini, M. Greco, M. Diani, and L. Verrazzani, "Performance Analysis of Adaptive Radar Detectors Against Non-Gaussian Real Sea Clutter Data," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 36, No. 4, pp. 1429-1439, October 2000.

[9] S. Kraut, L.L. Sharf, R. W. Buther, "The Adaptive Coherence Estimator: A Uniformly most-Powerful-Invariant Adaptive Detection Statistic," *IEEE Trans on Signal Processing*, Vol. 53, No. 2, pp. 427-438, February 2005.

[10] F. Gini and J. H. Michels, "Performance Analysis of Two Covariance Matrix Estimators in Compound-Gaussian Clutter," *IEE Proceedings Part-F*, vol. 146, No. 3, pp. 133-140, June 1999.

[11] F. Gini, A. Farina, "Vector Subspace Detection in Compound-Gaussian Clutter. Part I: Survey and New Results," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. 38, No. 4, pp. 1295-1311, October 2002.

[12] N. B. Pulsone, C. M. Rader, "Adaptive Beamformer Orthogonal Rejection Test, *IEEE Trans on Signal Processing*, Vol. 49, No. 3, March 2001, pp. 521-529.

[13] A. De Maio, "Robust adaptive radar detection in the presence of steering vector mismatches," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. 41, No. 4, pp. 1322 - 1337, October 2005.