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A NEW IMPLEMENTATION OF THE ICP ALGORITHM FOR 3D SURFACE REGISTRATION USING A COMPREHENSIVE LOOK UP MATRIX

A. Almhdie[1], C. Léger[1], M. Deriche[2] and R. Lédée[1]

[1] Laboratory of Electronics, Signals and Images (LESI), Polytech'Orléans, University of Orléans, France
12 rue de Blois, 45067, Orléans, France
[2] Electrical Engineering Department, King Fahd University, Dhahran, Saudi Arabia
phone: + (33) 238494563, fax: + (33) 238417377,
emails: {*ahmad.almhdie, christophe.leger, roger.ledee*}*@univ-orleans.fr* and *mderiche@kfupm.edu.sa*
web: *http://www.univ-orleans.fr/lesi*

ABSTRACT

The iterative closest point algorithm is one of the most efficient algorithms for robust rigid registration of 3D data. It consists in finding the closest points between two sets of data which are then used to estimate the parameters of the global rigid transformation to register the two data sets. All the steps of the algorithm are highly dependent upon the accuracy with which correspondence pairs of points are found. In this paper, a new enhanced implementation of the ICP algorithm proposes to use a look up matrix for finding the best correspondence pairs. It results in reducing the minimum mean square error between the two data sets after registration, compared to existing implementations. The algorithm was implemented and tested to evaluate its convergence properties and robustness to noise. Performance improvements are obtained. The new algorithm has successfully been applied to register 3D medical data.

1. INTRODUCTION

The registration of 3D data sets is an important task in both Computer Vision and Photogrammetry, especially in the medical field. A detailed overview of image registration techniques can be found in [1], and different methods proposed for medical image registration have been discussed in [2, 3].

The iterative closest point (ICP) algorithm, originally proposed by Besl and McKey [4], is one of the most popular methods used for estimating the rigid transformation of roughly aligned 3D data sets. It is widely used for the registration of free-form surfaces where dense data is assumed, and a good initial estimate is available or can easily be obtained. Additionally, all selected scene points from the scene surface are assumed to have correspondences in the reference surface. The first step of the ICP algorithm consists in choosing corresponding (closest) points in the two 3D data sets. Since the accuracy of the search for correspondence points highly affects the estimation of the transformation parameters, the output of the first step has a major impact over the following stages and strongly affects the overall performance of the algorithm. This step strongly depends upon both the selection of the points of the two surfaces, and the method used for finding the correspondence of the selected points. The Original ICP algorithm [4], denoted OICP in this paper, searches for the closest point in the reference surface for each point in the scene surface without any restrictions.

Widespread interest in 3D surface registration using the OICP algorithm has motivated the scientific community to propose other techniques to improve the different steps of the original algorithm. Many variants have been developed for speeding up the convergence and/or improving the performance of the different phases of the algorithm [5]. A good review of these variants can be found in [6]. There has been significant interest regarding the selection of points used for the estimation of transformation parameters. In [7], the Trimmed ICP selects only a predefined number of estimated matched pairs for the calculation of the 'optimal motion'. The projection of the scene points onto the reference surface can also be used for the correspondence search [8]. The Picky ICP (PICP) [9] rejects all points previously estimated to correspond to one reference point, except the one with the smallest distance. This approach reduces convergence problems that may arise using the original ICP algorithm, when a common reference point is assigned to multiple points in the scene surface. However, this affects negatively the performance of the algorithm in noisy situations, since many points are discarded in the estimation step.

This paper focuses on the correspondence search of 3D surface points. The aim is to enhance the performance of the correspondence search step of the OICP algorithm. The use of a new comprehensive look up matrix is investigated and evaluated. The proposed CICP (C for comprehensive) algorithm ensures *unique* matches of correspondence pairs.

The rest of the paper is organized as follows. First, the original OICP algorithm is summarized. Next, the new CICP algorithm is described and performance analyses details are given. Results of comparisons based on both synthetic and medical data are then presented to show the performance improvement of the CICP algorithm. Finally some concluding remarks are given.

2. OVERVIEW OF THE OICP ALGORITHM

Assume that the given two surfaces to be registered can be described as point sets; the scene data points, **P**, with N_p

points, $\{p_i,\ i=1, \ldots, N_p\}$, and the reference data points, **M**, with N_m points, $\{m_i,\ i=1, \ldots, N_m\}$. Depending on the accuracy of the constructed surfaces, N_p is not necessarily equal to N_m. Furthermore, the point p_i of the scene surface does not necessarily represent 3D correspondence to the point m_i of the reference surface. The search space, however, is determined by the size of the scene data set; i.e., N_p. The OICP algorithm can be summarized as follows:

A. Initialization:

1) Let the initial scene surface $\mathbf{P}_0$, be equal to **P**.
2) Define the maximum number of iterations $\mathbf{k}_{max}$.
3) Initialize the translation vector and the rotation matrix as follows:

$$\mathbf{T} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } \mathbf{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (1)$$

This corresponds to zero translation and no rotation.

B. Iterations:

1) For each point p_i (i=1, ... , N_p) of the scene **P**, compute the closest point $y_i \in \mathbf{M}$ from the model using the Euclidian distance. Let y_i be the point on **M** corresponding to the minimum distance p_i.
2) Using the selected correspondence pairs, compute the transformation, rotation (**R**) and translation (**T**), that minimises the mean square error (MSE) of these pairs:

$$\text{MSE} = \frac{1}{\mathbf{N_p}} \sum_{\mathbf{i}=1}^{\mathbf{N_p}} \left\| y_{\mathbf{i}} - \mathbf{R}(p_{\mathbf{i}}) - \mathbf{T} \right\|^2 . \quad (2)$$

The resulting transformation from the minimization of the above equation will be denoted $\mathbf{R}_k$ and $\mathbf{T}_k$. This step also provides the minimum distances which correspond to the matched pairs.
3) Transform **P** according to (**R**, **T**) and restart a new iteration if the change in the MSE is above a predefined threshold ζ, and if the maximum number of iterations $\mathbf{k}_{max}$ is not reached. If not, stop the iterations and exit.

3. THE PROPOSED CICP ALGORITHM

In previous variants of the OICP algorithm, the search procedures of corresponding pairs of points are based on a line-by-line (vector) search within a P-M distance matrix described in Table 1, where $d_{i,j}$ is the distance between p_i and m_j. Duplicate matches may hence occur, since multiple m_i (columns) can be assigned to different p_i (lines). The PICP variant ensures unique matches by rejecting all duplicate pairs, except the one with the smallest distance. This can be described as a line-by-line followed by a column-by-column search within the P-M distance matrix. Unfortunately, this may lead to the exclusion of good markers from the estimation procedure. To overcome this drawback, a more comprehensive search is needed.

A novel effective evaluation metric is introduced for correspondence search, called comprehensive lookup matrix measure. This measure ensures that ***every*** selected point on the scene surface has a ***unique*** match in the reference surface.

	m_1	m_2	…	m_{Nm}
p_1	$d_{1,1}$	$d_{1,2}$		$d_{1,Nm}$
p_2	$d_{2,1}$	$d_{2,2}$		$d_{2,Nm}$
:				
p_{Np}	$d_{Np,1}$	$d_{Np,2}$		$d_{Np,Nm}$

Table 1 – the P-M distance matrix.

The CICP is different in that it sorts the $d_{i,j}$, distances in ascending order within the entire P-M distance matrix. Moreover, the point m_j is not considered to be a correspondence to p_i if either m_j or p_i has been previously assigned a correspondence. This ensures that each point in the scene surface will have a different association in the reference surface.

The CICP is the only ICP algorithm that makes use of all scene points in the search procedure to find the best and unique correspondence pairs. In other words, the P-M distance matrix is introduced to comply with the fact that a rotation is a bijective (one to one) function. Previous ICP implementations are based on vector not matrix analysis of the assignment problem which can yield surjection correspondences, incompatible with correct estimations of rotation parameters. In this case, some elements in **M** may be mapped by more than one element in **P**. When the number of points in the two sets to be registered is not the same, the CICP algorithm considers the one with a smaller number of points as a scene data set to ensure bijectivity of the resulting correspondence pairs. To reduce computation time introduced by matrix search procedure, fast assignment algorithms can be used. The CICP algorithm can be summarized as follows:

1) For each point $p_i \in \mathbf{P}$, (i=1, ..., N_p), the algorithm computes the Euclidian distance to each point $m_j \in \mathbf{M}$, (j=1, ..., N_m). Then, for N_p times, the algorithm:
   a. looks for the location (i,j) that corresponds to the minimum distance in the current look up matrix,
   b. assigns p_i to m_j as a correspondence pair,
   c. removes this correspondence pair from future consideration by eliminating the i^{th} row and j^{th} column.
2) Using the estimated correspondence pairs, the algorithm computes the transformation parameters (**R, T**), and transforms **P** accordingly.
3) The algorithm computes the MSE between the reference and transformed scene data sets. If the change in the MSE is above a predefined threshold and $\mathbf{k}_{max}$ is not reached, the algorithm starts a new iteration. If not, the iterative procedure is stopped.

Instead of leaving the decision of rejecting worse pairs till the end of every iteration as in [9], the CICP algorithm makes such a decision at the end of every selection of pair correspondence. Such an approach improves the accuracy and the convergence of the ICP algorithms, as shown in the following sections.

4. PERFORMANCE ANALYSIS

In this paper, the new CICP algorithm will be compared to the OICP and PICP as benchmarks. The robustness of the CICP algorithm will be studied under the presence of noise, with both synthetic and real medical data.

4.1 Noise generation

Real data are usually corrupted by noise caused by a wide range of sources, e.g. detector variations, environmental variations, transmission or quantization errors, etc.
Here, the performance of the selected ICP algorithms is investigated under the effect of Gaussian and impulsive noise.

4.1.1 Gaussian noise

Gaussian noise is added to the original data according to the following method:

1) Transform all points of the data set from Cartesian to spherical coordinates:
$$p(x_i, y_i, z_i) \rightarrow p(\theta_i, \varphi_i, \rho_i)\ \forall\ i = \{1, \ldots, \mathrm{N_m}\}, \quad (3)$$
2) Add noise to each resulting $\rho_i\ \forall\ i = \{1, \ldots, \mathrm{N_m}\}$:
$$\rho_i = \rho_i + |\rho_i - \mu_\rho| \times 10^{-\mathbf{SNR\_dB}/20} \times rand(.), \quad (4)$$
where $\mu_\rho = \sum_i^{\mathbf{N}_m} \rho_i$, $rand(.)$ is a random Gaussian number generator with mean zero and variance one, and **SNR_dB** is the required signal to noise ratio in dB.
3) Transform points back to Cartesian coordinates.

4.1.2 Impulsive noise

Impulsive noise is commonly referred to as outliers. In this case, a set of *h*% of the data points is assumed to be corrupted by impulsive noise. To generate outliers, replace step 2 of Gaussian noise generation by:

2a) Randomly select *h*% of the total data points.
2b) Modify each selected point: $\rho_j\ \forall\ j = \{1,..., h \times \mathrm{N_m}/100\}$
$$\rho_j = \rho_j \times (1+\beta) \quad (5)$$
where β represents the distribution of the outliers relative to the points of the original data set.

4.2 Performance parameters

To compare the performance characteristics of the CICP algorithm to OICP and PICP algorithms, two parameters are taken into consideration: the percentage of correct matches and the mean square error (MSE) between the registered data sets.

4.2.1 Percentage of correct matches

Correct matches analysis can be carried out when exact orientation of the data sets is known. Under such a hypothesis, the percentage of correct associations of corresponding points from the scene and reference surfaces is counted at each step of the algorithm. This measurement is a straightforward indicator of the performance of the correspondence pair search method.

4.2.2 Mean Square Error

The convergence property of the algorithm can be estimated by computing the mean square error (MSE) between the reference and the registered data sets, at each iteration of the algorithm. When orientation is known for both registered surfaces, MSE is given by computing the mean square distances between correspondence pairs of points. Otherwise, 3D to 2D mesh interpolation can be integrated, and MSE can be calculated with the corresponding non-zero elements of the resulting 2D meshes

5. SIMULATED SCENE SURFACE RESULTS

The CICP algorithm was tested under a noise-free situation as well as with Gaussian noise (with SNR of 5 dB, 10 dB, 15 dB and 20 dB) and outliers (with percentage of outliers of 5 %, 10%, 15 % or 20 % of the data set) conditions. The convergence properties and the accuracy of the proposed CICP algorithm have been evaluated using two types of point sets: synthetic and real medical data sets.

5.1 Registration of synthetic data

In the first experiment, the 'Rabbit' synthetic data set[1] was used as a reference, as shown in Figure 1a. Figure 1b shows an example of noise-free scene data, rotated -29°, -4° and 8° around the x-axis, y-axis and z-axis, respectively. The number of reference and scene data points is the same, i.e., 1000 points. The OICP, PICP and CICP algorithms were tested to register the reference surface (Figure 1a) with the scene surface (Figure 1b), corrupted with Gaussian and outlier noise.

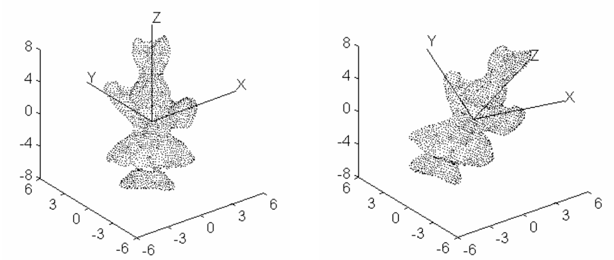


Figure 1 – Bunny data. a) Reference surface. b) Scene surface (rotation, noise free).

5.1.1 Correct matches evaluation

Figure 2 and Figure 3 show the percentage of correct matches at each iteration of the OICP, PICP and CICP algorithms. It can be seen that the CICP gives the highest number of correct matches, in fewer iterations, compared to both OICP and PICP algorithms, for most noise situation tests.

5.1.2 MSE comparison

Figure 4 and Figure 5 show the evolution of the mean square error computed at each step of the OICP, PICP and CICP algorithms. Under 5dB Gaussian noise data corruption, the CICP algorithm reaches minimum MSE faster (4 iterations) than the OICP and PICP algorithms (respectively 17 and 37 iterations). Similar results are obtained for other amounts of Gaussian noise, as well as for noise-free scene data.
In the case of outliers, the CICP algorithm reaches minimum MSE faster than for the OICP and PICP algorithms. Moreover, the change in outlier ratio slightly affects the number of iterations of the CICP, compared to OICP and PICP algorithms.
To evaluate the repeatability of the results presented above with particular scene data rotation, the same experiments were carried out on more than twenty randomly selected rotations, within 30° around each x, y and z axis. In all cases, the CICP algorithm showed improvements compared to the OCIP and PICP algorithms.

[1] Special thanks go to Mr A. Ajmal, at *The University of Western Australia*, for providing us with the data.

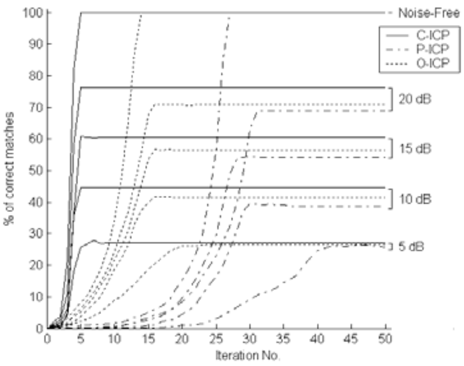


Figure 2 – Accuracy performance in the case of Gaussian noise.

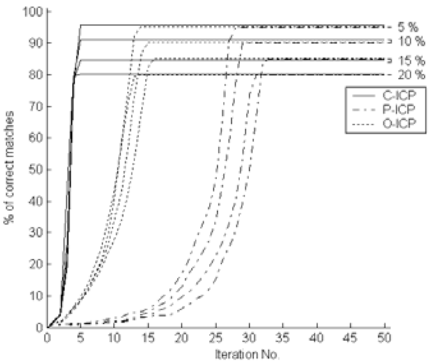


Figure 3 – Accuracy performance in the case of outliers.

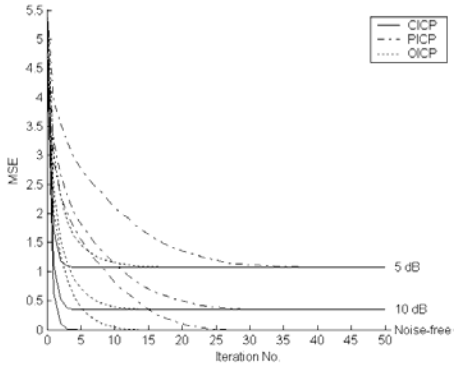


Figure 4 – Convergence comparison in the case of Gaussian noise.

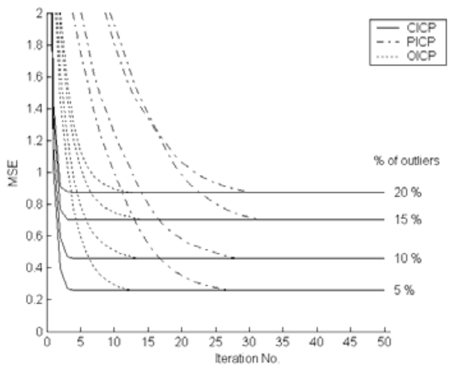
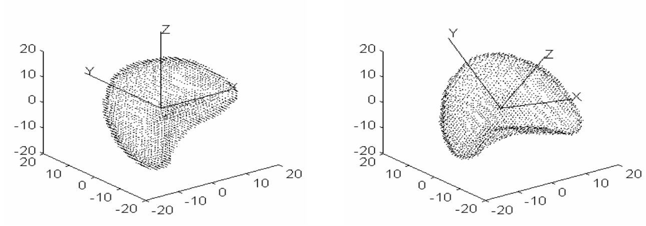


Figure 5 – Convergence comparison in the presence of outliers.

5.2 Registration of medical data

The second experiment considers a set of 922 points of real data of human Lung[2] as a reference surface (Figure 6a). The scene data set is simulated by rotating the reference scene data of -29°, -4° and 8° around the x-axis, y-axis and z-axis, respectively (Figure 6b). In the case presented, the number of points in the reference and scene data sets is equal.



(a) (b)

Figure 6: Lung data. a) Reference lung data. b) Scene lung data (rotation, noise-free).

The MSE between the reference and registered surfaces is measured at each iteration of the OICP, PICP and CICP algorithms, considering scene surface degraded with Gaussian noise (Figures 7) and outliers (Figure 8).

Figure 7 confirms the performance enhancement of the CICP algorithm, which reaches the minimum MSE faster than the OICP and PICP algorithms. Figure 8 shows similar results with outliers.

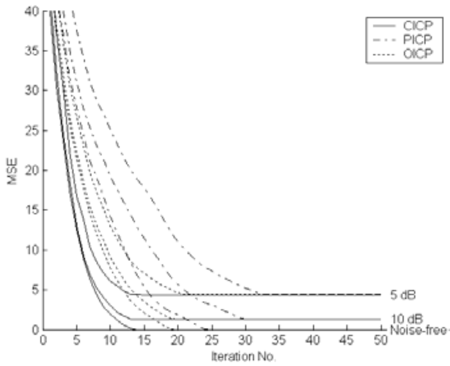


Figure 7: Convergence comparison in the case of Gaussian noise.

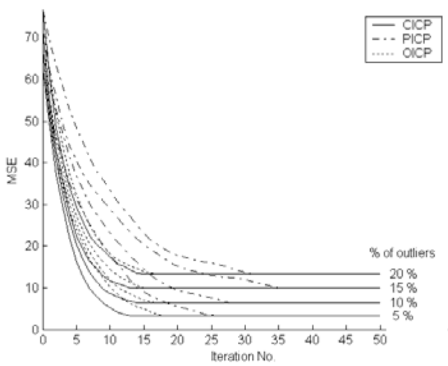


Figure 8: Convergence comparison in the case of outliers.

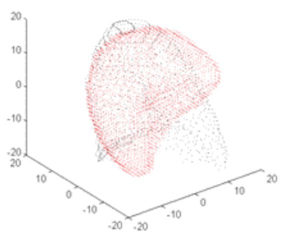
[2] Special thanks go to Dr Fabienne Thérain, Chef du service de *Médecine Nucléaire, Centre Hospitalier Régional d'Orléans, France.*

6. MEDICAL APPLICATION

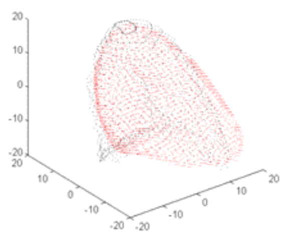
In this experiment, we consider the registration of the real lung data of Figure 6a, to a lung atlas presented in Figure 9a. The atlas data set consists of 1150 points, whereas a set of 922 points were uniformly selected from the scene lung data. The motion parameters that “best” align the lung data set with the atlas are estimated using the three OICP, PICP and CICP algorithms. The corresponding registered data are displayed in Figure 9.

For the experiment carried out, the MSE threshold ζ between the reference surface and the registered scene surface is set to a low value (10^{-3}), and the maximum number of iterations ($k_{\max}$) is not limited. This ensures that the estimation of the motion parameters is correct. The algorithm stops when the estimated motion parameters are constant within several iterations, and MSE between the reference and registered surfaces is computed. With lung atlas and lung data, the CICP, PICP and OICP algorithms achieve convergence in 31, 135 and 84 iterations, respectively.

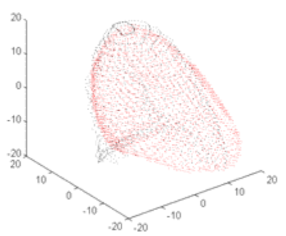
In Figure 9, when stability of motion parameters is reached, the CICP algorithm (Figure 9d) seems to give a better registration of the two data surfaces (Figure 9a), compared to the OICP (Figure 9b) and PICP (Figure 9c) algorithms. This qualitative result was confirmed by an expert in the field of medical imaging. Further experiments will be conducted to quantify precisely these preliminary results.



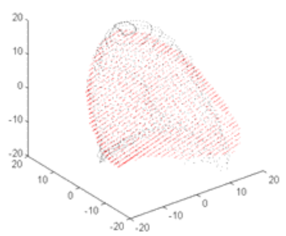
(a)



(b)



(c)



(d)

Figure 9: a) Initial views: lung atlas (reference surface, black) and lung data (scene surface, red). Registered data using (b) OICP, (c) PICP, (d) CICP algorithms.

7. CONCLUSION

In this work, a novel enhanced implementation of the ICP algorithm is presented. The use of the complete look-up distance matrix during the point association procedure guarantees that unique matches are obtained for all points from the scene data. The substitution of a vector by a matrix based search of correspondence pairs ensures correct rigid registration, in agreement with the bijective property of the rotation. Computing time expansion due to the switch from vector to matrix search can be limited by using many valuable techniques used to solve assignment problems. Compared to other ICP implementations, the proposed CICP algorithm provides: a faster convergence, in terms of number of iterations, a more precise estimation of pair of points correspondence, and a better resilience to additive Gaussian noise and outliers. The accuracy of the proposed CICP has been investigated and very promising results have been shown for 3D real medical data registration.

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