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GROUP-WISE BLIND OFDM ML DETECTION FOR COMPLEXITY REDUCTION

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ABSTRACT

This paper presents a low-complexity blind Maximum-Likelihood (ML) detector for Orthogonal Frequency Division Multiplexing (OFDM) systems in block fading channels. The receiver complexity is reduced by subcarrier grouping (SG) for which the OFDM block is partitioned into smaller groups, and then the data are detected on a group-by-group basis. An identifiability analysis is also provided. We show that the data in each group can be identified under a more relaxed condition than that in [1], therefore enabling us to use smaller group size for implementation efficiency. Our simulation results show that the proposed detector can provide good symbol error performance even when the group size is much smaller than the discrete Fourier transform size.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has been recognized as a promising modulation scheme for wired and wireless communications due to its robustness against frequency selective fading and low receiver complexity. The OFDM modulator/demodulator transforms a frequency selective fading channel into a multitude of flat fading subchannels in the frequency domain, so that only one-tap equalizers are needed to detect the data provided that channel state information (CSI) is available.

To estimate the CSI, either non-blind or blind methods can be considered. For non-blind methods, periodically transmitted training blocks or pilot tones can be used to estimate the CSI [2, 3]. However, these known data incur spectral efficiency loss, especially for fast time-varying channels and at low signal-to-noise ratios (SNRs) [4]. For the second-order statistics (SOS) based blind channel estimation methods [5], many OFDM blocks are often required to estimate the SOS. In these methods the channel is assumed to be static for a long period of time, which may be violated in *block fading channels*; i.e., when the channel coefficients remain constant only for one OFDM time block. Differentially encoded OFDM [6] only requires the channel to be static over two consecutive OFDM blocks, but it incurs a 3 dB performance penalty in SNR. The blind channel estimator in [10] estimates the channel in one OFDM block using multi-receiver diversity, but its performance is sensitive to noise effects and the channel length. In this paper, we focus on a Maximum-Likelihood (ML) method for joint channel estimation and data detection in one OFDM block, which is suitable for block fading channels. In [7], a blind OFDM ML data detector/channel estimator was presented and realized by Sphere Decoding (SD) algorithm [8] and V-BLAST algorithm [9]. The channel identifiability of this blind ML detection/estimation method is proven in [1].

If we consider an OFDM system for which the number of subcarriers is large (e.g., 1024 subcarriers), the blind ML detector becomes computationally infeasible because it is a large scale optimization problem under such circumstances. In the paper, we propose a low-complexity implementation alternative, called *subcarrier grouping* (SG). The idea is to partition the OFDM block into smaller groups, and then apply the blind ML detection method to each group. We provide a data/channel identifiability analysis for the SG, and show that for Gaussian distributed fading channels, the transmitted data and channel can be identified up to a phase shift in probability one sense. Unlike the condition in [1] which depends on the signal constellation size, the presented condition is independent of the signal constellation size, thereby allowing us to have SG with smaller group size. Our simulation results show that the proposed ML detector has promising performance and outperforms those based on the pilot-assisted least-squares (LS) channel estimator in [3] and the blind channel estimator in [10], even when the group size is much smaller than the total number of subcarriers.

2. OFDM SIGNAL MODEL AND BLIND ML DETECTION

2.1 Signal Model

Consider an OFDM system in which the standard procedures of cyclic prefix insertion and guard interval removal are applied. Suppose that there are N_R antennas at the receiver, and let N denote the discrete Fourier transform (DFT) size (or the OFDM block size). The received signal for the kth subcarrier at antenna r is given by

$$y_r(k) = H_r(k)s(k) + w_r(k),$$

where $k = 1, 2, ..., N$, $r = 1, 2, ..., N_R$,

- $s(k) \in \mathbb{Q}$: transmitted symbol for subcarrier k where $\mathbb{Q}$ is the signal constellation with size $|\mathbb{Q}|$
- $H_r(k)$: channel frequency response for subcarrier k
- $w_r(k)$: spatially uncorrelated additive white Gaussian noise with variance σ_w^2 for all r.

Let $\mathbf{h}_r \in \mathbb{C}^{L\times 1}$ be the vector containing the channel impulse response coefficients for the rth antenna, for $r = 1, 2, ..., N_R$.

Let $\boldsymbol{\Xi} \in \mathbb{C}^{N\times N}$ be the DFT matrix with the kth row given by

$$(\boldsymbol{\Xi})_{k,\bullet} = \frac{1}{\sqrt{N}}[1, e^{-j\frac{2\pi}{N}(k-1)}, \ldots, e^{-j\frac{2\pi}{N}(k-1)(N-1)}],$$

where $(\cdot)_{k,\bullet}$ denotes the kth row of a matrix. Then $H_r(k)$ are given by

$$\begin{bmatrix} H_r(1) \\ H_r(2) \\ \vdots \\ H_r(N) \end{bmatrix} = \boldsymbol{\Xi} \begin{bmatrix} \mathbf{h}_r \\ \mathbf{0} \end{bmatrix} = [\mathbf{F}, \widetilde{\mathbf{F}}] \begin{bmatrix} \mathbf{h}_r \\ \mathbf{0} \end{bmatrix}$$
$$= \mathbf{F}\mathbf{h}_r, \quad r = 1,\ 2, \ldots,\ N_R,$$

where $\mathbf{F} \in \mathbb{C}^{N\times L}$ and $\widetilde{\mathbf{F}} \in \mathbb{C}^{N\times(N-L)}$. Therefore, the received OFDM block can be expressed as

$$\begin{aligned} \mathbf{y}_r &= [y_r(1), y_r(2), \ldots, y_r(N)]^T \\ &= \mathbf{D}(\mathbf{s})\mathbf{F}\mathbf{h}_r + \mathbf{w}_r, \ \ r = 1,\ 2, \ldots,\ N_R, \end{aligned}$$

where

$$\begin{aligned} \mathbf{s} &= [s(1), s(2), \ldots, s(N)]^T, \\ \mathbf{w}_r &= [w_r(1), w_r(2), \ldots, w_r(N)]^T, \end{aligned}$$

and $\mathbf{D}(\mathbf{s}) \in \mathbb{C}^{N\times N}$ which is a diagonal matrix with $\mathbf{s}$ on its main diagonal. In this paper, we consider block channel fading where the channel coefficients remain static only for one OFDM block, and we investigate methods for joint estimation of $\mathbf{h}_r$ and detection of $\mathbf{s}$ over one OFDM block.

2.2 Blind ML Detection

A blind ML detector [1, 7] for the aforementioned OFDM system is briefly reviewed as follows. The blind ML detector can be derived according to the deterministic ML criterion:

$$\{\hat{\mathbf{s}}, \hat{\mathbf{H}}\} = \arg \min_{\mathbf{s}\in\mathbb{Q}^N, \mathbf{H}\in\mathbb{C}^{L\times N_R}} ||\mathbf{Y} - \mathbf{D}(\mathbf{s})\mathbf{F}\mathbf{H}||_{\mathrm{F}}^2, \qquad (1)$$

where $\|\cdot\|_{\mathrm{F}}$ denotes the Frobenius norm, and

$$\begin{aligned} \mathbf{Y} &= [\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_{N_R}], \\ \mathbf{H} &= [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_{N_R}]. \end{aligned}$$

Assume that the signal constellation $\mathbb{Q}$ is constant modulus. By exploiting the semi-unitary property of $\mathbf{F}$ (i.e., $\mathbf{F}^H\mathbf{F} = \mathbf{I}_L$ where $\mathbf{I}_L$ is the $L\times L$ identity matrix) and following the same procedure as in [7], one can show that (1) can be reformulated as a quadratic minimization problem

$$\hat{\mathbf{s}} = \arg \min_{\mathbf{s}\in\mathbb{Q}^N} \mathbf{s}^T\mathbf{G}\mathbf{s}^*, \qquad (2)$$

where

$$\mathbf{G} = \sum_{r=1}^{N_R} \mathbf{D}(\mathbf{y}_r^*)\left(\mathbf{I}_N - \mathbf{F}\mathbf{F}^H\right)\mathbf{D}(\mathbf{y}_r).$$

Equ. (2) can be solved by using the optimal SD algorithm or the suboptimal V-BLAST algorithm [7]. Moreover, the semidefinite relaxation (SDR) algorithm [11–13] is an effective suboptimal alternative for solving (2), which exhibits near-optimal performance and has a polynomial-time worst-case complexity of $O(N^{3.5})$. (Note that the SD algorithm does not have a polynomial-time worst-case complexity.)

2.3 Identification Condition

An important theoretical basis for the blind detection problem in (1) is data/channel identifiability. A sufficient condition for unique channel identification has been presented in [1]. Consider the signal constellation $\mathbb{Q}$ which is PSK and satisfies two properties: i) if $s \in \mathbb{Q}$, then $s^* \in \mathbb{Q}$, ii) if $s_1, s_2 \in \mathbb{Q}$, then $s_1 \times s_2 \in \mathbb{Q}$. We reinterpret the identifiability condition in [1] as follows:

***Lemma 1* [1]:** *Assume that there is no noise; i.e., $\mathbf{w}_r = \mathbf{0}$ for all r. For the blind OFDM ML detector in (1), the channel $\mathbf{H}$ can be identified up to a phase shift if $N \geq |\mathbb{Q}|L$.*

Though the channel can be properly identified under the premises in Lemma 1, it is not necessarily true for data identification. The reason is that if for some k we have $H_r(k) = 0$ for all r = 1, 2, ..., N_R, then $s(k)$ can never be correctly detected even when perfect CSI is available. Let us consider the following channel fading assumption:

A1) The channel vectors $\mathbf{h}_r$, r = 1, 2, ..., N_R, are Gaussian distributed, and at least one of them has a positive definite covariance matrix.

One can easily show that under **A1)**, the probability of the event $\{H_r(k) = 0,\ r = 1, 2, ..., N_R\}$ is of measure zero for any k = 1, 2, ..., N. Thus the following lemma is obtained from Lemma 1:

***Lemma 2*:** *Under the premises of Lemma 1 and under* **A1**), *the data vector* $\mathbf{s}$ *can be identified up to a phase shift with probability one.*

3. LOW-COMPLEXITY IMPLEMENTATION VIA SUBCARRIER GROUPING

The direct application of the aforementioned blind ML detector would be computationally too complex for large DFT size. For example, for a typical DFT size of $N = 256$, the complexity of using SDR to implement the ML detector is of the order of $256^{3.5}$ which is obviously unaffordable in practice. To tackle this complexity issue, a subcarrier grouping (SG) method is proposed in Section 3.1. Then, an identifiability condition for SG is developed in Section 3.2.

3.1 Subcarrier Grouping

As mentioned earlier, the idea of SG is to partition the OFDM block into smaller groups, and then deal with each group independently. Let us partition the OFDM block into P groups with equal size M; i.e., $N = PM$. Let $S_p = \{i_{1,p}, i_{2,p}, \ldots, i_{M,p}\}$ be the subcarrier index set associated with group p. These index sets satisfy $S_1 \cup S_2 \cup \cdots \cup S_P = \{1, 2, \ldots,\ N\}$, and $S_p \cap S_q = \emptyset$ for all $p \neq q$ (i.e., nonoverlapping subcarrier indices). The received signal associated with the pth group can be represented by

$$\mathbf{Y}_p = \mathbf{D}(\mathbf{s}_p)\mathbf{F}_p\mathbf{H} + \mathbf{W}_p, \qquad (3)$$

where $\mathbf{s}_p = [s_{i_{1,p}}, s_{i_{2,p}}, \ldots, s_{i_{M,p}}]^T$, and

$$\mathbf{Y}_p = \begin{bmatrix} (\mathbf{Y})_{i_{1,p},\bullet} \\ \vdots \\ (\mathbf{Y})_{i_{M,p},\bullet} \end{bmatrix}, \mathbf{F}_p = \begin{bmatrix} (\mathbf{F})_{i_{1,p},\bullet} \\ \vdots \\ (\mathbf{F})_{i_{M,p},\bullet} \end{bmatrix}, \mathbf{W}_p = \begin{bmatrix} (\mathbf{W})_{i_{1,p},\bullet} \\ \vdots \\ (\mathbf{W})_{i_{M,p},\bullet} \end{bmatrix}. \qquad (4)$$

If the subcarrier index sets S_p are chosen such that $\mathbf{F}_p$ is semi-unitary (i.e., $\mathbf{F}_p^H \mathbf{F}_p = (M/N)\mathbf{I}_L$) for all p, the blind OFDM ML detector in (2) then can be directly applied to each individual group in (3), thereby reducing the receiver complexity. It has been shown [14] that if the subcarrier index sets S_p are chosen to be

$$S_p = \left\{ p, p+\frac{N}{M}, p+\frac{N}{M}\cdot 2, \ldots, p+\frac{N}{M}\cdot(M-1) \right\}, \quad (5)$$

[i.e., $i_{k,p} = p + \frac{N}{M}(k-1)$] where M divides N and $M > L$, then the associated $\mathbf{F}_p$ in (4) are semi-unitary for all p.

Suppose that SDR algorithm is used to handle the blind ML problem associated with SG. Then the receiver complexity is $O(PM^{3.5}) = O(NM^{2.5})$, which, for the choice of $M \ll N$, is a significant reduction compared to the complexity $O(N^{3.5})$ required by the full blind ML detector. In the simulation results shown in the next section, it will be illustrated that SG can provide good performance for $M \ll N$.

3.2 Data Identifiability of SG

For SG, one can show, following the proof in [1], that the identifiability conditions in Lemmas 1 and 2 are also true; that is, each SG data vector $\mathbf{s}_p$ and the channel $\mathbf{H}$ can be identified up to a phase shift with probability one if $\mathbf{H}$ satisfies the fading assumption **A1)** and $M \geq |\mathbb{Q}|L$. In the following, we provide another data identifiability condition which is more relaxed than the above mentioned:

***Theorem 1*:** *Assume that there is no noise, and that* **A1)** *holds. For the blind OFDM ML detector using the SG as given in (3)-(5), the data vector* $\mathbf{s}_p$ *for every* $p = 1, 2, ..., P$ *and the channel* $\mathbf{H}$ *can be identified up to a phase shift with probability one if* $M > L$.

The proof of Theorem 1 is given in the Appendix.

4. SIMULATION RESULTS

In the following simulation examples, we show the efficacy of the proposed blind OFDM ML detector using SG. The coefficients of frequency selective fading channels are zero-mean i.i.d. complex Gaussian distributed, and change from block to block. We define the SNR as

$$\text{SNR} = \frac{\text{E}\{\|\mathbf{H}\|_F^2\}}{N\sigma_w^2}.$$

If not mentioned specifically, the signal constellation was BPSK. The blind OFDM ML detector was implemented by the SDR algorithm [11]. The performance (in terms of symbol error rate (SER)) of the proposed blind OFDM ML detector was compared to its coherent counterpart, and the detectors based on the blind channel estimator in [10] and the pilot-based LS channel estimator in [3]. The number of pilots used for the pilot-based LS channel estimator was always L (the minimum number of pilots to uniquely estimate the channel). The phase ambiguities of the proposed detector and the blind channel estimator in [10] were solved by assuming that one symbol is known. The number of trials of the simulation was 5×10^4. We should mention that the blind OFDM ML detector without SG [7] is computationally too intensive to use in the following examples because the considered DFT size is large.

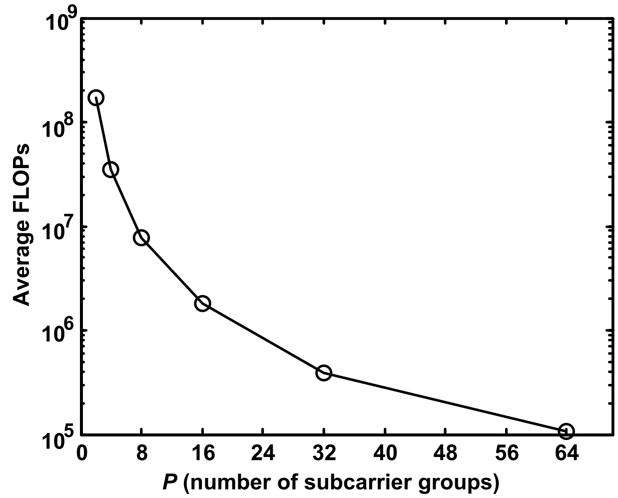

Fig. 1. Computational complexity (Average FLOPs vs. P) of the proposed blind ML detector using SG for $N = 256$, $N_R = 4$, $L = 3$, $|\mathbb{Q}| = 2$ and SNR= 20 dB.

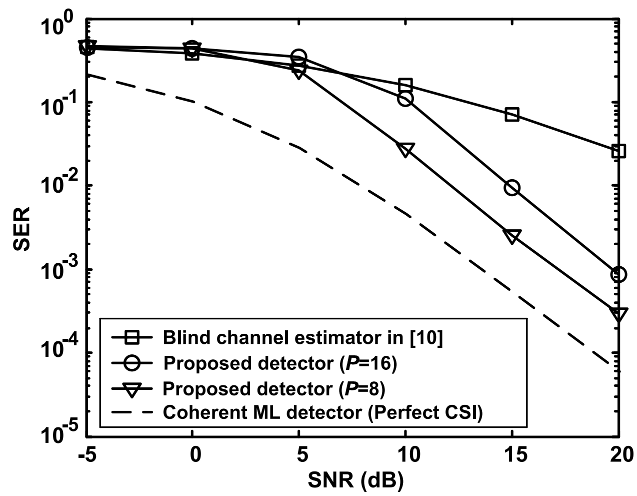

Fig. 2. Performance (SER vs. SNR) of the proposed blind OFDM ML detector using SG for $N = 256$, $N_R = 2$, $L = 10$ and $|\mathbb{Q}| = 2$.

Figure 1 shows the average computational complexity performance (in terms of floating point operations (FLOPs) [13]) of the proposed detector using the SDR algorithm for $N = 256$, $N_R = 4$, $L = 3$, and SNR= 20 dB. One can see that the receiver complexity is reduced when P increases.

Figure 2 shows the results of the proposed blind OFDM ML detector using SG for $N = 256$, $N_R = 2$, and $L = 10$. It can be seen that the proposed detector significantly outperforms the method in [10]. One can also see that for the proposed detector with $P = 16$ ($M = 16$), the data can be identified under the condition of $M > L$ in spite of $M < |\mathbb{Q}|L$ ($|\mathbb{Q}| = 2$ for BPSK). Moreover, we observe that there is a performance gain when P reduces from 16 to 8 (or M increases from 16 to 32). This verifies the expectation that increasing the group size improves performance.

Figure 3 shows the results for $N = 256$, $N_R = 4$, $L = 16$, and $P = 4$ (thus M=64). One can see from the figure that the proposed detector outperforms the pilot-assisted LS channel estimation method in [3]. Note that the LS channel estimator requires at least $L = 16$ pilots, whereas the proposed ML detector requires only one pilot. Moreover, it can be seen from the figure that the proposed detector using $P = 4$ has performance loss less than 3 dB compared to its coherent counterpart as SER=10^{-6}. This demonstrates that the performance loss compared to that without SG [7] is small while the complexity reduction of the proposed detector is apparent (see Fig. 1). Similar results can also be observed in Fig. 4 for $N = 64$, $N_R = 4$, $L = 8$, $P = 2$ (thus M=32) and the QPSK constellation.

5. CONCLUSION

We have presented a blind ML receiver for OFDM in block fading channels. The proposed detector, using subcarrier grouping (SG), is a group-wise ML method aiming to gain computational efficiency, a benefit that is inherently not possible when applying the full ML detector [7] to a large scale OFDM system. An identifiability condition for the proposed detector is also derived. In particular, our identifiability condition is more relaxed than that in [1], in that the former enables the possibility of using smaller group size for computational efficiency. Simulation results show that SG exhibits affordable complexity while maintaining promising performance.

6. APPENDIX PROOF OF THEOREM 1

Let us rewrite the deterministic ML criterion in (1) for the received signal in (3) as

$$\{\hat{\mathbf{s}}_p, \hat{\mathbf{H}}\} = \arg \min_{\mathbf{s}_p \in \mathbb{Q}^M} \left\{ \min_{\mathbf{H} \in \mathbb{C}^{L \times N_R}} ||\mathbf{Y}_p - \mathbf{D}(\mathbf{s}_p)\mathbf{F}_p\mathbf{H}||_F^2 \right\}, \quad \text{(A.1)}$$

where the subscript 'p' is the index of the subcarrier group, and $\mathbf{F}_p$ is semi-unitary (i.e., $\mathbf{F}_p^H\mathbf{F}_p = (M/N)\mathbf{I}_L$). The inner minimization term of (A.1) is actually a least-squares problem given $\hat{\mathbf{s}}_p$, and has a closed-form solution

$$\hat{\mathbf{H}} = \left(\frac{N}{M}\right)\mathbf{F}_p^H\mathbf{D}^H(\hat{\mathbf{s}}_p)\mathbf{Y}_p. \quad \text{(A.2)}$$

Let $\mathbf{s}'_p \in \mathbb{Q}^M$ be the transmitted data vector for group p. For the noise-free case, the received signal in (3) is given by

$$\mathbf{Y}_p = \mathbf{D}(\mathbf{s}'_p)\mathbf{F}_p\mathbf{H}. \quad \text{(A.3)}$$

It is easy to verify that $\mathbf{s}'_p$ and $\mathbf{H}$ are a solution set to (A.1) for the noise-free case. If $\mathbf{s}''_p \in \mathbb{Q}^M$ and $\mathbf{H}_2$ are also a solution set to (A.1) (for the noise-free case), then we have

$$\mathbf{D}(\mathbf{s}'_p)\mathbf{F}_p\mathbf{H} = \mathbf{D}(\mathbf{s}''_p)\mathbf{F}_p\mathbf{H}_2. \quad \text{(A.4)}$$

According to (A.2) and (A.3), we can express $\mathbf{H}_2$ as

$$\mathbf{H}_2 = \left(\frac{N}{M}\right)\mathbf{F}_p^H\mathbf{D}((\mathbf{s}''_p)^* \odot \mathbf{s}'_p)\mathbf{F}_p\mathbf{H}, \quad \text{(A.5)}$$

where $\odot$ is the Hadamard product. Substituting (A.5) into (A.4) and through some manipulations, one can reexpress (A.4) as

$$\left(\left(\frac{M}{N}\right)^2\mathbf{I}_L - \mathbf{F}_p^H\mathbf{D}^H(\mathbf{s}'_p \odot (\mathbf{s}''_p)^*)\mathbf{F}_p\mathbf{F}_p^H\mathbf{D}(\mathbf{s}'_p \odot (\mathbf{s}''_p)^*)\mathbf{F}_p\right) \cdot \mathbf{H} = \mathbf{0}, \quad \text{(A.6)}$$

or alternatively as

$$\left(\mathbf{F}_p^H\mathbf{D}^H(\mathbf{s}'_p \odot (\mathbf{s}''_p)^*)\left(\left(\frac{M}{N}\right)\mathbf{I}_M - \mathbf{F}_p\mathbf{F}_p^H\right)\mathbf{D}(\mathbf{s}'_p \odot (\mathbf{s}''_p)^*)\mathbf{F}_p\right) \cdot \mathbf{H} = \mathbf{0}. \quad \text{(A.7)}$$

Let $\overline{\mathbf{F}}_p = \sqrt{\frac{N}{M}}[\mathbf{F}_p, \widetilde{\mathbf{F}}_p] \in \mathbb{C}^{M \times M}$ which is a unitary matrix where $\widetilde{\mathbf{F}}_p \in \mathbb{C}^{M \times (M-L)}$. Then we have

$$\left(\frac{M}{N}\right)\overline{\mathbf{F}}_p\overline{\mathbf{F}}_p^H = \mathbf{F}_p\mathbf{F}_p^H + \widetilde{\mathbf{F}}_p\widetilde{\mathbf{F}}_p^H = \left(\frac{M}{N}\right)\mathbf{I}_M. \quad \text{(A.8)}$$

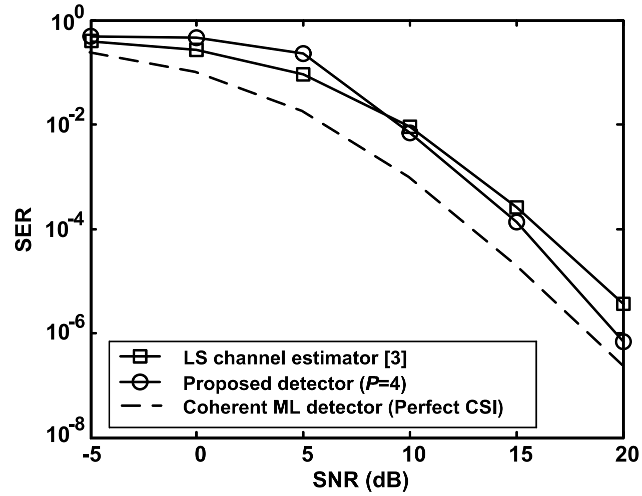

Fig. 3. Performance (SER vs. SNR) of the proposed blind OFDM ML detector using SG for $N = 256$, $N_R = 4$, $L = 16$, $P = 4$ and $|\mathbb{Q}| = 2$.

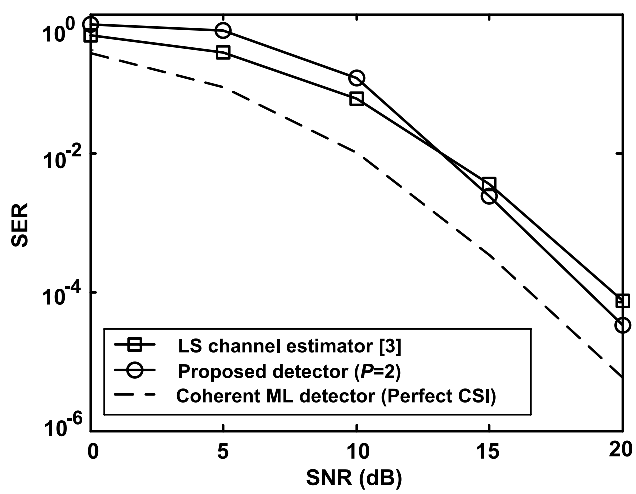

Fig. 4. Performance (SER vs. SNR) of the proposed blind OFDM ML detector using SG for $N = 64$, $N_R = 4$, $L = 8$, $P = 2$ and $|\mathbb{Q}| = 4$.

Substituting (A.8) into (A.7) gives rise to

$$\mathbf{U}_p\mathbf{H} = \mathbf{0}, \tag{A.9}$$

where $\mathbf{U}_p = \widetilde{\mathbf{F}}_p^H \mathbf{D}(\mathbf{s}'_p \odot (\mathbf{s}''_p)^*)\mathbf{F}_p$. Because the probability

$$\mathrm{P_r}\left(\mathbf{U}_p\mathbf{H} = \mathbf{0}\right) = \mathrm{P_r}\left(\bigcap_{r=1}^{N_R} \mathbf{U}_p\mathbf{h}_r = \mathbf{0}\right) \leq \mathrm{P_r}\left(\mathbf{U}_p\mathbf{h}_r = \mathbf{0}\right)$$

for any $r = 1, 2, ..., N_R$. By **A1)**, let the covariance matrix of $\mathbf{h}_r$ be positive definite. It can be shown that the probability of the event $\mathbf{U}_p\mathbf{h}_r = \mathbf{0}$ is of measure zero unless $\mathbf{U}_p = \mathbf{0}$. Therefore, (A.9) implies

$$\widetilde{\mathbf{F}}_p^H \mathbf{D}(\mathbf{s}'_p \odot (\mathbf{s}''_p)^*)\mathbf{F}_p = \mathbf{0} \tag{A.10}$$

with probability one. Now we show that (A.10) holds only if $\mathbf{s}''_p = e^{j\theta}\mathbf{s}'_p$ where $e^{j\theta} \in \mathbb{Q}$. According to (4) and (5), one can show that $\widetilde{\mathbf{F}}_p$ can have the kth row given by

$$(\widetilde{\mathbf{F}}_p)_{k,\bullet} = \frac{1}{\sqrt{N}}[e^{-j\frac{2\pi}{N}i_{k,p}L}, e^{-j\frac{2\pi}{N}i_{k,p}(L+1)}, \ldots, e^{-j\frac{2\pi}{N}i_{k,p}(M-1)}], \tag{A.11}$$

where $i_{k,p} = p + \frac{N}{M}(k-1)$. Let

$$\mathbf{D}(\mathbf{s}'_p \odot (\mathbf{s}''_p)^*) = \mathrm{diag}\left\{e^{-j\theta_1}, e^{-j\theta_2}, \ldots, e^{-j\theta_M}\right\}, \tag{A.12}$$

where $e^{-j\theta_i} \in \mathbb{Q}$ for all i. According to (4), (5), (A.11) and (A.12), the (m,n)th element in (A.10) can be shown to be

$$\frac{1}{N}\sum_{i=0}^{M-1} e^{-j\theta_{i+1}} e^{j\frac{2\pi}{N}(p+\frac{N}{M}i)(L+m-n)} = 0 \tag{A.13}$$

for $m = 1, 2, ..., M-L$, and $n = 1, 2, ..., L$. Let $\ell = L+m-n$. Then (A.13) becomes

$$\frac{1}{N}\sum_{i=0}^{M-1} e^{-j\theta_{i+1}} e^{j\frac{2\pi}{M}i\ell} = 0 \ \text{ for } \ell = 1, 2, \ \ldots, \ M-1,$$

which implies $\theta = \theta_1 = \cdots = \theta_M$, i.e., $\mathbf{s}''_p = e^{j\theta}\mathbf{s}'_p$, by the DFT property [15]. Hence the data vector $\mathbf{s}'_p$ can be identified up to a phase shift with probability one. Substituting $\mathbf{s}''_p = e^{j\theta}\mathbf{s}'_p$ into (A.5) gives rise to

$$\mathbf{H}_2 = e^{-j\theta}\mathbf{H},$$

i.e., the channel $\mathbf{H}$ can also be identified up to a phase shift with probability one. Thus we have completed the proof.

7. ACKNOWLEDGEMENT

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