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SEMI-BLIND BUSSGANG EQUALIZATION ALGORITHM

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ABSTRACT

This paper addresses the problem of semi-blind equalization following a Bussgang approach. An equalization scheme that integrates the training information into the iterative Bussgang algorithm is analyzed. The semi-blind equalization scheme allows flexible introduction of redundancy in the transmission scheme. The accuracy is assessed in the reference case of the Global System for Mobile communication (GSM), *i.e.* GMSK modulated signals received through typical mobile radio channels. Numerical simulations show that the semi-blind Bussgang equalization algorithm achieves performance comparable with the Maximum Likelihood Sequence Estimator (MLSE) implemented by Viterbi algorithm. Its flexibility allows to consider different amounts of training information as well as higher order constellations.

1. INTRODUCTION

This paper investigates a Bussgang semi-blind equalization scheme, extending the blind equalizer structure for QAM and GMSK signals derived in [1]. In fact, due to limited bandwidth resources the transmission of training information should be minimized; nevertheless, trained equalization appears necessary to cope with long, bad channels. Hence, we analyze a semi-blind Bussgang equalization scheme that integrates the training information into the iterative algorithm, allowing for flexible introduction of redundancy in the transmission scheme.

Here, we report a case study where the performance of the semi-blind Bussgang equalization algorithm is discussed for the GSM system. In fact, although the system is worldwide operating, there is still research in progress about the problem of blind and semi-blind GSM equalization, aimed at improving the bandwidth efficiency or at simplifying the receiver structure. In [2], the authors resort to a QAM approximation of the nonlinear GMSK modulation, so enabling the application of QAM blind and semi-blind equalization techniques to the GSM system. In [3] the authors address channel estimation and equalization in GSM receivers by exploiting a subspace based approach to perform channel identification/equalization based on small data records. In [4], an equalization concept for the Enhanced Data rates for GSM Evolution (EDGE) is described, considering suboptimum receivers based on delayed decision-feedback sequence estimation and reduced-state sequence estimation. The work in [5] is devoted to channel estimation under GSM-like environmental conditions, when a training sequence is arithmetically added to the information data. Semi-blind equalization techniques applied to GMSK-based mobile communication systems are addressed in [6], where a novel semi-blind block algorithm is presented to provide reliable communication through a typical mobile multi-path channel.

2. SEMI-BLIND BUSSGANG EQUALIZATION

In this Section we address the design of a fractional semi-blind Bussgang equalizer. We will assume the discrete-time model in (1), to design the linear equalizer according to the Minimum Mean Square Error (MMSE) criterion.

Generally speaking, the discrete-time model of a QAM communication link describes the relation between the complex QAM transmitted symbols s_k, and the samples $y[n]$ of the base-band representation of the received signal, possibly taken at fractional sampling interval T_s/P. The linear discrete-time model is:

$$b[n] = \sum_{k=-\infty}^{+\infty} s_k \cdot g[n-Pk]$$
$$x[n] = \sum_{l=-\infty}^{+\infty} h[l]b[n-l] \quad ; \quad y[n] = x[n] + v[n] \qquad (1)$$

where $g[n]$ is the impulse response of the pulse shaping filter, $h[n]$ is the channel impulse response and $v[n]$ is a realization of additive white noise, statistically independent of the symbols $s_n \in \mathcal{A}$, being $\mathcal{A}$ the discrete source alphabet. In this paper, we will address fractional sampling with sampling interval $T_s/2$, *i.e.* $P = 2$ in (1).

Let us denote as $f[l]$ the equalizer taps, and let the input symbols s_n be drawn from a random stationary serie. For FIR equalizer structure, the MMSE filter $f[l], l = 0, \cdots, 2L-1$ can be obtained as the solution of a suitable linear system [1]. Let us denote as $R_y[-m, k-m] \stackrel{\text{def}}{=} \mathrm{E}\{y[2n-k]\,\overline{y}[2n-m]\}$ the bi-argumental autocorrelation of the cyclostationary random process of the received signal samples $y[n]$, and by $R_{sy}[k] \stackrel{\text{def}}{=} \mathrm{E}\{s_n\,\overline{y}[2n-k]\}$ the cross-correlation between $y[n]$ and the stationary symbols s_n. Then, the FIR MMSE equalizer satisfies

$$\sum_{m=0}^{2L-1} R_y[-m, k-m] f[m] = R_{sy}[k], \qquad (2)$$

for $k = 0, \cdots, 2L-1$ or, in matrix form, the $(2L \times 2L)$ linear system $\mathbf{R}_{\rm y}\,\mathbf{f} = \mathbf{r}_{\rm sy}$ being $(\mathbf{R}_{\rm y})_{k,m} \stackrel{\text{def}}{=} R_y[-m, k-m]$, $(\mathbf{r}_{\rm sy})_k \stackrel{\text{def}}{=} R_{sy}[k]$ and $(\mathbf{f})_m \stackrel{\text{def}}{=} f[m]$.

A sample estimate of the autocorrelation of the measurements $R_y[m,k]$ can be obtained by averaging over the observed samples $y[n]$. Hence, the key problem in solving (2) is the estimation of the right hand term $R_{sy}[k]$.

In blind equalization, the system in (2) can be solved observing that, due to the orthogonality principle, $R_{sy}[k]$ can be indirectly estimated by substituting the input symbols s_n with their MMSE estimate given the measurements $y[n]$, *i.e.* with $\tilde{s}_n \stackrel{\text{def}}{=} \mathrm{E}\{s_n|\cdots,y[n-1],y[n],y[n+1]\cdots\}$ This approach requires the joint MMSE estimation of the equalizer taps $f[n]$ and of the input symbols s_n, and has prohibitive computational complexity. In [1], the authors design a Bussgang blind equalization scheme based on alternating MMSE estimation of the equalizer taps $f[n]$ and of the input symbols s_n.

In trained equalization $R_{sy}[k]$ could be directly estimated on the base of the training information, or estimated by means of training based channel identification, since $R_{sy}[k] = \overline{h}[-k]$. However, due the limited resources bandwidth devoted to training, these estimation techniques can be insufficient for equalization of long, bad channels.

Here, we analyze a semi-blind Bussgang equalization scheme that integrates these two approaches by importing the training information into the Bussgang algorithm to improve the estimate of the cross-correlation $R_{sy}[k]$.

First, and once for all the iterations, the least square estimation of the autocorrelation matrix $\mathbf{R}_\text{y}$ is performed. The training based equalizer initialization is given by $\mathbf{f}^{(0)} = \mathbf{R}_\text{y}^{-1}\mathbf{r}_\text{ty}$ where $(\mathbf{r}_\text{ty})_k = \frac{1}{N_T}\sum_{n\in\mathcal{T}} t_n\,\overline{y}[2n-k]$, and t_n are the N_T symbols corresponding to the training information located in the set of time indices $n \in \mathcal{T}$.

At the i-th iteration step:

- the observations $y[n]$ are filtered by the previous estimate of the MMSE filter $f^{(i-1)}[n]$, so obtaining the equalized sequence $\hat{s}_n^{(i)} = (f^{(i-1)} * y)[n]$;
- the MMSE estimate $\tilde{s}_n$ of the unknown input symbols s_n is evaluated through the application of a suitable nonlinear transform $\eta(\cdot)$;
- the estimate of the cross-correlation $R_{sy}[k]$ is refined by exploiting the nonlinearly estimated symbols $\tilde{s}_n^{(i)}$ together with the known training symbols t_n

$$(\mathbf{r}_\text{sy}^{(i)})_k = \frac{1}{N}\left(\sum_{n\notin\mathcal{T}} \tilde{s}_n\,\overline{y}[2n-k] + \sum_{n\in\mathcal{T}} t_n\,\overline{y}[2n-k]\right)$$

- the MMSE equalizer taps $f^{(i)}[n]$ are updated according to $\mathbf{R}_\text{y}\,\mathbf{f}^{(i)} = \mathbf{r}_\text{sy}^{(i)}$ (2)

After suitable convergence testing, the algorithm stops providing the MMSE equalizer $f^{(i_\text{max})}[n]$, the linearly estimated symbols $\hat{s}_n^{(i_\text{max})}$ and the corresponding symbols estimate $\tilde{s}_n^{(i_\text{max})}$.

The form of the nonlinearity $\eta(\cdot)$ depends on the symbol constellation and it can take into account also an eventual symbols statistical dependence [1].

3. A CASE STUDY: SEMI-BLIND EQUALIZATION OF THE GSM SIGNAL

The Global System for Mobile communication (GSM) encompasses either circuit switched or packet switched services, employing the Gaussian Minimum Shift Keying (GMSK) modulation at the access network. This radio access technology is also integrated in the UMTS network as the terrestrial interface GERAN, complementary to the CDMA interface UTRAN.

The GSM employees a periodically inserted training sequence to counteract the time-variant radio channel fading; the bandwidth fraction devoted to training depends on the logical channel. Namely, the GSM data at the physical layer are formatted in bursts. Each burst is built by $N_\mathcal{B}$ information bits b_k and $N_\mathcal{T}$ training bits t_k for equalization and synchronization, and consecutive bursts are separated by suitable guard times. Bursts are modulated using GMSK modulation.

During the k-th symbol interval $[kT_s, kT_s+T_s)$, the GMSK modulated signal $u(t)$ depends on the binary input symbol u_k and on the modulation state σ_k:

$$u(t) = A\cos[2\pi f_c t + \phi(t,u_k,\sigma_k)],\;\; kT_s \le t < (k+1)T_s$$

The phase $\phi(t,u_k,\sigma_k)$ depends on the base-band pulse $g(t)$

$$\phi(t) = 2\pi h\int_0^t \sum_{n=0}^{k} u_n g(\tau - nT_s)\,d\tau + \phi_0$$

being ϕ_0 the random phase at time $t=0$ and h the modulation index. The GMSK base-band pulse is defined as $g(t) \stackrel{\text{def}}{=} (h_1 * h_g)(t)$, where

$$h_1 = \frac{1}{2T_s}\text{rect}_{T_s}\left(t - \frac{T_s}{2}\right)$$

$$h_g(t) = \sqrt{\frac{2\pi}{\ln 2}}B_g\exp\left(\frac{2\pi^2 B_g^2}{\ln 2}t^2\right)$$

where $B_g \approx 81.1\,\text{KHz}$ is the half-power bandwidth of the Gaussian shaped filter $h_g(t)$. The shaping filter $g(t)$ has pulse width longer than the symbol interval T_s and it introduces a correlation among the transmitted symbols; in principles the optimum receiver requires Maximum Likelihood Sequence Estimation that can be realized resorting to the Viterbi algorithm.

For the purpose of equalizer design, the GMSK modulated signal is well approximated by a QPSK modulated signal with suitably correlated input symbols [2], [1]. Namely, the discrete time model in (1) can be applied with $\mathcal{A}_4 = \{e^{\pm j\pi/4}, e^{\pm j3\pi/4}\}$. In [2], the authors highlight that if the QPSK symbols s_k are generated from i.i.d. binary symbols $u_n = \pm 1$ according to the recurrence equation

$$s_n = j\cdot u_n \cdot s_{n-1},\; s_0 \in \mathcal{A} \qquad (3)$$

the samples b_n of the modulated signal approximate the samples of the GMSK modulated signal carrying the same information sequence $u_n \in \{-1,1\}$. In fact, the correlation between the input symbols s_n due to the generation model (3) causes forced transitions in the QPSK modulated signal, similar to those observed in the GMSK modulated signal $u(t)$.

According to this model of the GMSK signal, the semi-blind Bussgang algorithm derived in Sect.2 can be applied once we have derived the nonlinear estimator for the case at hand.

3.1 MMSE symbol estimation for the GSM signal

A critical step of the iterative Bussgang equalization algorithm is the evaluation of the nonlinear estimate $\tilde{s}_n^{(i)}$. Since i the QPSK approximation of the GSM signal the symbols are generated according to the recurrence equation (3), they cannot be considered statistical independent. Even though the correlation extends over all the symbols, as indicated in [1], we resort to the following approximation

$$\tilde{s}_n \approx \mathrm{E}\{s_n | \hat{s}_{n-1}, \hat{s}_n, \hat{s}_{n+1}\} \quad (4)$$

where the estimation of the symbol s_n is conducted taking as statistical observations only three consecutive samples.

Denoting with $\underline{\hat{s}} = [\hat{s}_{n-1}, \hat{s}_n, \hat{s}_{n+1}]^T$ the triplet of linearly equalized symbols, the MMSE estimator in (4) is written as :

$$\tilde{s}_n = \mathrm{E}_{\mathbf{s}|\hat{\mathbf{s}}}\{s_n\} = \int_{\mathbf{S}} s_n p_{\mathbf{s}|\hat{\mathbf{s}}}(\mathbf{s}|\hat{\mathbf{s}})\, d\mathbf{s} \quad (5)$$

Let us express the equalized symbol at the generic i-th iteration, as a noisy version the corresponding input symbol, *i.e.*

$$\hat{s}_n = s_n + w_n \quad (6)$$

being w_n the overall equalization error due to both then-residual Inter Symbol Interference (ISI), and the noise amplification. Let the vectors $\mathbf{s}$ and $\mathbf{w}$ be analogously defined, *i.e.* $\hat{\mathbf{s}} = \mathbf{s} + \mathbf{w}$.

To evaluate the estimator in (5) in closed form, the noise w_n is approximated as a realization of a stationary white complex Gaussian process with variance σ_w^2, statistically independent of the symbols s_n. Adopting the polar notations for the transmitted symbols, the linearly equalized symbols and the nonlinearly estimated symbols, respectively, *i.e.* $s_n \stackrel{\text{def}}{=} |s_n| e^{j\alpha_n}, \hat{s}_n \stackrel{\text{def}}{=} |\hat{s}_n| e^{j\beta_n}, \tilde{s}_n \stackrel{\text{def}}{=} |\tilde{s}_n| e^{j\gamma_n}$ and introducing the notation $\Delta\phi \stackrel{\text{def}}{=} \arg\{\hat{\mathbf{s}}^{\mathrm{H}}\mathbf{s}\} + \beta_n - \alpha_n$ the estimator in (5) is written as

$$\tilde{s}_n = E_{\mathbf{s}|\hat{\mathbf{s}}}(s_n) = e^{j\beta_n} \cdot \frac{\displaystyle\int_{\mathbf{S}} |s_n| e^{-\frac{\|\mathbf{s}\|^2}{\sigma_w^2}} e^{-j\Delta\phi} p_{\mathbf{S}}(\mathbf{s}) d\mathbf{s}}{\displaystyle\int_{\mathbf{S}} e^{-\frac{\|\mathbf{s}\|^2}{\sigma_w^2}} p_{\mathbf{S}}(\mathbf{s}) d\mathbf{s}}$$

Considering the 4-QAM constellation and the recurrence equation (3), we are faced only with $K = 16$ admissible and equiprobable triplets $\mathbf{s}^{(k)}$. After setting $\beta_n \stackrel{\text{def}}{=} \angle \hat{s}_n$, $\beta_0^{(k)} \stackrel{\text{def}}{=} \angle s_0^{(k)}$, we finally obtain:

$$\eta(\underline{\hat{s}}) = \left[\sum_{k=0}^{K-1} s_0^{(k)} e^{-\frac{|\mathbf{s}^{(k)}|^2}{\sigma_v^2}} e^{\frac{2|\hat{s}_n||s_0^{(k)}| M_k \cos(\Delta\phi_k + \beta_0^{(k)} - \beta_n)}{\sigma_v^2}} \right] \cdot \left[\sum_{k=0}^{K-1} e^{-\frac{|\mathbf{s}^{(k)}|^2}{\sigma_v^2}} e^{\frac{2|\hat{s}_n||s_0^{(k)}| M_k \cos(\Delta\phi_k + \beta_0^{(k)} - \beta_n)}{\sigma_v^2}} \right]^{-1} \quad (7)$$

where the normalized version of the scalar product $\underline{\hat{s}}^{\mathrm{H}} \cdot \mathbf{s}^{(k)}$ is defined as $M_k e^{j\Delta\phi_k} \stackrel{\text{def}}{=} \left[\underline{\hat{s}}^H \cdot \mathbf{s}^{(k)}\right] \left[\hat{s}_n s_0^{(k)}\right]^{-1}$.

Let us remark that the estimator (7)estimator (7) is substantially different from that described in [1]. In fact, since we have used the training symbols to design the first equalizer in the iterative equalization, we can assume "coherent" observations in (6), *i.e.* the unknown channel phase has been already recovered in the previous equalization stages, while in [1] the "incoherent" case has been addressed. In fact, the estimator (7) modifies both the phase and the magnitude of the linear estimate $\hat{s}_n$, not only as a function of the phase differences among the symbols of the triplet, as the estimator discussed in [1], but also as a function of the phase β_n of the central symbol $\hat{s}_n$.

Let us observe that when evaluated for the GMSK signal , the coherent estimator (7) tends to restore differences of $\pm\pi/2$ between input symbols, due to the correlation induced by (3); however, for estimated symbol phase tending to $\pm\pi/2$, the phase correction is reduced and only the magnitude is corrected on the basis of adjacent symbols.

Finally, when the Bussgang algorithm stops providing the input symbols estimates $\{\tilde{s}_n^{(i)\max}\}$, the binary information sequence u_n is extracted from the estimated QPSK symbols by inverting the recurrence (3), *i.e.*

$$\hat{u}_n = \tilde{s}_n^{(i)\max} / j\tilde{s}_{n-1}^{(i_{\max})}$$

3.2 Time-varying linear model of mobile radio channels

The transmission on mobile channels employees frequencies whose wavelength is typically comparable with terrestrial obstacles, such as buildings, cars, pedestrians. Hence, the received signal suffers for temporal spread, slow fading due to shadowing, fast fading due to random echoes , and Doppler effects due to the relative velocity between the mobile station and the base station.

The mobile channel is described by a time variant impulse response $c(\tau,t)$ that is modeled as a Gaussian Wide Sense Stationary Uncorrelated Scattering (GWSSUS) random process. The response of the channel to a single impulse allocated a time t is a sovrapposition of N_e echoes, delayed by a different interval τ_ν, rotated by a different phase shift θ_ν and affected by a Doppler shift $f_{d\nu}$. According to this model, the time varying impulse response is

$$c(\tau,t) \approx \frac{1}{\sqrt{N_e}} \sum_{\nu=1}^{N_e} e^{j(2\pi f_{d_\nu} t + \theta_\nu)} \delta(\tau - \tau_\nu) \quad (8)$$

The phases θ_ν are uniformly distributed random variables in $(0, 2\pi)$. The random delays τ_ν have probability density function proportional to the power delay profile of the channel. The Doppler frequencies f_{d_ν} are assumed to be random variables distributed according to the Doppler frequency power spectrum. The power delay profile and the Doppler frequency power spectrum referred in the following have been devised during the works of the COST-207 project. Both the Doppler frequency f_{d_ν} and the propagation delay τ_ν depend on the time. However, the channel variations occurs on an interval of $1/f_{d\max} = 1/225\text{Hz} \approx 4.4\,\text{ms}$ and the channel can be considered stationary on a limited number of symbols. In the equalizer design, we will assume the channel to be constant over the whole burst.

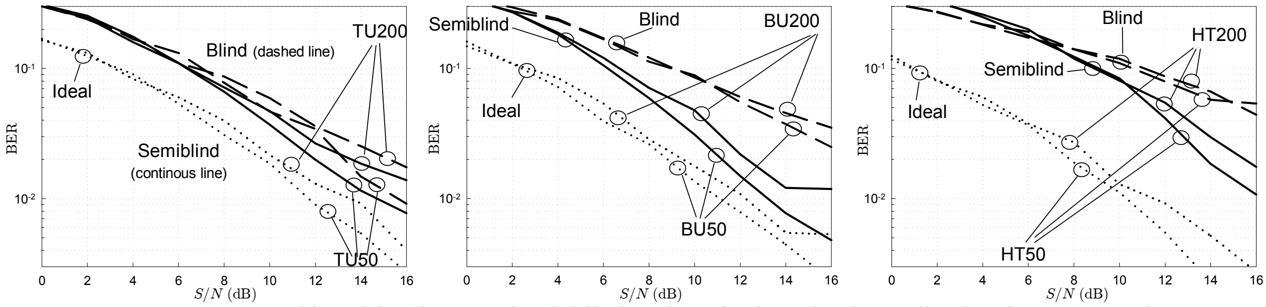

Figure 1: BER versus SNR achieved by linear trained, blind and perfectly trained equalization in $TU50$ and $TU200$ with $2L = 16$, $BU50$ and $BU200$ with $2L = 20$, $HT50$ and $HT200$ with $2L = 32$.

3.3 Experimental results

The transmitted signal is generated according to the format of a GSM normal burst, encompassing two groups of 58 information bits, a 26 bits training sequence and 3 guard bits at the beginning and at the end of the burst.

The equalizer is designed using a linear model of the GMSK signal as given by (1), with root raised cosine pulse shape of roll-off factor $\rho = 0.1$.

We report the equalization accuracy for the COST 207 time varying channels, namely the channels Typical Urban, Bad Urban e Hilly Terrain at maximum Doppler shift f_{dm} of 50Hz e 200Hz.

For comparison purposes, we have also reported results pertaining to the blind Bussgang algorithm in [1], and pertaining to an ideal, perfectly trained, linear equalizer of equal length. Since the ideal, semi-blind and blind algorithms share the basic equalizer structure, the results can be interpreted as achieved by the same equalizer in presence of different amount of training information.

Fig.1 summarizes the average BER observed in 500 Montecarlo runs, at different average Signal to Noise Ratio $\overline{S}/N$, in the TU, BU and HT channels. The dashed lines represent the blind equalizer, the solid lines represent the trained equalizer, while the dotted lines represent the ideal perfectly trained equalizer. The FIR equalizer length have been chosen as $2L = 16, 20, 36$ for the TU, BU, HT channels respectively. We observe that the here described semi-blind equalizer performs very well in both the TU and BU channels. In HT the performances degrade, although the degradation affects fast fading rather than slow fading conditions. However, let us observe that in HT, for fast fading conditions, the constant channel approximation limits the performances of a MLSE equalizer too, even for ideal channel estimate.

Interestingly enough, the results of the semi-blind algorithm are comparable with those achieved by the MLSE implemented by the Viterbi algorithm (reported in [7]). We remark that the Bussgang approach is more flexible. In fact, the Bussgang equalizer structure does not require a preliminary channel identification, so allowing for a more flexible adaptation to different amount of training information. Moreover, the adoption of constellations richer than QPSK, such as those of the EDGE system, only affects the number of sums implied by (7), and does not substantially modifies the computational complexity at the receiver.

4. CONCLUSION

In this paper we have devised a semi-blind Bussgang equalizer. The equalizer design can be applied in presence of either QAM or GMSK modulation, exploiting a linear approximation of the GMSK signal. The performance of the equalization technique is discussed in the case of GSM signals observed through typical mobile radio channels. The accuracy is comparable with MLSE implemented by the Viterbi algorithm at comparable computational cost, while it allows a flexible adaptation to different amount of training information as well as the use of higher order constellations.

REFERENCES

[1] G. Panci, P. Campisi, S. Colonnese, G. Scarano, “Blind equalization for correlated input symbols: a Bussgang approach”, *IEEE Trans. on Signal Processing*, vol.53, no. 5, May 2005.

[2] Z. Ding, G. Li, “Single-channel blind equalization for GSM cellular systems” *IEEE Journal on Selected Areas in Communications*,

[3] A. Ray, S. Prakriya, “A subspace based approach for channel estimation and equalization in GSM receivers” *IEEE Int.Conf. on Personal Wireless Communications*, December 2002, pp. 134 - 137.

[4] W.H. Gerstacker, R. Schober, “Equalization concepts for EDGE” *IEEE Trans. on Wireless Communications*, January 2002, pp. 190-199.

[5] A.G. Orozco-Lugo, M.M. Lara, “Channel estimation using implicit training” *IEEE Trans. on Signal Processing*, January 2004, vol.52, pp. 240-254.

[6] R. Gallego, A. Rouxel, F. Alberge, P. Duhamel, “Semi-blind equalization for GMSK-based mobile communications” *Proceedings of IEEE Int. Conf. onAcoustics, Speech, and Signal Processing, (ICASSP ’04)*, May 2004, vol.4, pp. 861-864.

[7] D. Boss, K.D. Kammeyer and T. Petermann, “Is Blind Channel Estimation feasible in Mobile Communication Systems? A Study based on GSM”, *IEEE Journal on Selected Areas in Communications*, vol.16, no. 8, pp. 1479-1492, October 1998.