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FUZZY METRICS AND PEER GROUPS FOR IMPULSIVE NOISE REDUCTION IN COLOR IMAGES

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ABSTRACT

A new method for removing impulsive noise pixels in color images is presented. The proposed method applies the *peer group* concept defined by means of fuzzy metrics in a novel way to detect the noisy pixels. Then, a switching filter between the identity operation and the *Arithmetic Mean Filter* (AMF) is defined to perform a computationally efficient filtering operation over the noisy pixels. Comparisons in front of classical and recent vector filters are provided to show that the presented approach reaches a very good relation between noise suppression and detail preserving.

1. INTRODUCTION

Digital Color Images are frequently disturbed by the presence of the so-called impulsive noise [1, 2]. In this context, the filtering process becomes an essential task to avoid possible drawbacks in the subsequent image processing steps.

When the images are contaminated with impulsive noise the switching approaches are widely used due to their sufficient performance and proven computational simplicity. On the basis of the classical vector filters as the *Arithmetic Mean Filter* (AMF) [1], the *Vector Median Filter* (VMF) [3] , the *Basic Vector Directional Filter* (BVDF) [4, 5], or the *Distance Directional Filter* (DDF) [6] the switching approaches aims at selecting a set of pixels of the image to be filtered leaving the rest of the pixels unchanged. A series of methods for selecting the noise-likely pixels have been proposed to date [7, 8, 10, 12, 13, 14, 15, 16, 17]. In [7] the authors propose to determine if the vector in consideration is likely to be noisy using cluster analysis. The standard deviation, the sample mean and various distance measures are used in [8, 9] to form de adaptive noise detection rule. [10] proposes to use an statistical test and [11] uses statistical confidence limits. In [12, 13, 14] a neighborhood test is applied. Then, the filtering operation is performed only when it is necessary. In a similar way, in [18] a genetic algorithm is used to decide in each image position to perform the VMF operation, the BVDF operation or the identity operation. In [15, 16, 17], it is proposed to privilege the central pixel in each filtering window to reduce the number of unnecessary substitutions.

The *peer group* concept using classical metrics is employed in the approaches introduced in [13, 14] to detect impulsive noisy pixels by checking the size of the central pixel *peer group* . In this paper, the *peer group* concept is adapted to the use of a certain fuzzy metric. The use of this fuzzy metric is considered instead of the classical metrics since this fuzzy metric has provided better results than classical metrics in impulsive noise filtering [19, 16]. The proposed *peer group* concept is employed to define a switching filter between the AMF and the identity operation. Experimental results for performance comparison are provided to show that the proposed approach outperforms the classical vector filters and the recent approaches listed above.

The paper is organized as follows. In Section 2 the fuzzy metric is described. Section 3 presents the *peer group* concept. The method to detect the noisy pixels is defined in Section 4. Section 5 contains experimental results and discussion. Finally, conclusions are presented in Section 6.

2. AN APPROPRIATE FUZZY METRIC

One of the most important problems in fuzzy topology is to obtain an appropriate concept of fuzzy metric. In [20] a particular class of fuzzy metrics in the George and Veeramani's sense, [21], called stationary fuzzy metrics, were defined; for simplicity they will be referred as fuzzy metrics. From now on, and according to [21, 20] a fuzzy metric space is an ordered triple $(X,M,*)$ such that X is a (nonempty) set, $*$ is a continuous t-norm and M is a fuzzy set of $X \times X$ satisfying the following conditions for all $x,y,z \in X$:

(FM1) $M(x,y) > 0$
(FM2) $M(x,y) = 1$ if and only if $x = y$
(FM3) $M(x,y) = M(y,x)$
(FM4) $M(x,z) \geq M(x,y) * M(y,z)$

$M(x,y)$ represents the degree of nearness of x and y and according to (FM2) $M(x,y)$ is close to 0 when x is far from y. If $(X,M,*)$ is a fuzzy metric space we will say that $(M,*)$ is a fuzzy metric on X.

The authors proved in [21] that every fuzzy metric $(M,*)$ on X generates a Hausdorff topology on X. Actually, this topology is metrizable as it was proved in [22, 23], and so the above definition can be considered an appropriate concept of fuzzy metric space.

The next proposition will be established to be applied in next sections when working with color pixels $\mathbf{x}_i$ that are characterized by terns of values in the set $\{0,1,\dots,255\}$. This proposition is a particular case of the one used in [16], inspired in [24], and its proof will be omitted.

Proposition. *Let X be the set $\{0,1,\dots,255\}$ and let $K > 0$. Denote by $(x_i(1),x_i(2),x_i(3))$ the element $\mathbf{x}_i \in X^3$. The function M given by*

$$M(\mathbf{x}_i,\mathbf{x}_j) = \prod_{l=1}^{3} \frac{min\{x_i(l),x_j(l)\}+K}{max\{x_i(l),x_j(l)\}+K} \qquad (1)$$

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for all $\mathbf{x}_i, \mathbf{x}_j \in X^3$*, is a fuzzy metric on* X^3*, where the t-norm* $*$ *is the usual product in* $[0,1]$.

In this way from now on,

$$M(\mathbf{x}_i, \mathbf{x}_j) \tag{2}$$

will be the fuzzy distance between the color image vectors $\mathbf{x}_i$ and $\mathbf{x}_j$. According to [19, 16], an appropriate value for K when comparing RGB color vectors is $K = 1024$.

3. PEER GROUPS IN THE FUZZY CONTEXT

A color RGB image is commonly represented as a multi-dimensional array $N_1 \times N_2 \times 3$, where every pixel $\mathbf{x}_i, i = 1, 2, \ldots, N_1N_2$ is a three component vector in X^3, as mentioned above. The *peer group* concept introduced in [25, 26] have been used in various RGB color image filter designs. In this work, the notion of *peer group* given in [13] will be adapted to the context of fuzzy metrics. So, attending to the concept of nearness (see axiom FM2 in the definition of fuzzy metric given in Section 2), for a central pixel $\mathbf{x}_i$ in a 3×3 filtering window W and fixed $d \in]0,1]$, we denote by $\mathscr{P}(\mathbf{x}_i, d)$ the set

$$\{\mathbf{x}_j \in W : M(\mathbf{x}_i, \mathbf{x}_j) \geq d\}$$

that is, $\mathscr{P}(\mathbf{x}_i, d)$ is the set of pixels of the filtering window W whose fuzzy distance to $\mathbf{x}_i$ is not less than d. Obviously, $\mathscr{P}(\mathbf{x}_i, d)$ is not empty for each $\mathbf{x}_i$, since $\mathbf{x}_i \in \mathscr{P}(\mathbf{x}_i, d)$.

Now, employing the same terminology used in [13], given a natural number m, we denote by $\mathscr{P}(\mathbf{x}_i, m, d)$ a subset of $\mathscr{P}(\mathbf{x}_i, d)$ constituted by $\mathbf{x}_i$ and other m elements of $\mathscr{P}(\mathbf{x}_i, d)$, which will be called a *peer group* of m elements (associated to $\mathbf{x}_i$). Clearly, for each $\mathscr{P}(\mathbf{x}_i, d)$ we can find a *peer group* for $m = 0$, but it could not exist for $m \geq 1$.

4. PROPOSED FILTERING TECHNIQUE

As it was commented in Section 1, the filters in [13, 14] determine a *noise-free* pixel when its *peer group* reaches a minimum size m (see Section 3). An appropriate value of m for any type and density of noise is difficult to find. In homogeneous regions, higher values of m would present a robust and proper performance. In edges or details, lower values of m are needed to properly preserve the uncorrupted data, however, the robustness is lower as m decreases.

In this work, an iterative algorithm aimed at solving the selection of m is proposed. First, a higher value of m is required to determine in a robust way *noise-free* pixels in homogeneous regions far from borders and details. Second, to reach an appropriate performance in edges and detailed regions, the value of m is iteratively decremented. Then the required *peer group* size is lower but the restriction of all the members of the *peer group* to be previous *noise-free* pixels is added. The intuitive underlying idea in this second step is that if a pixel is similar to some (m) *noise-free* pixels it should be *noise-free*, as well. Using the notation in Sections 2-3 the proposed algorithm for detection and removal of impulsive noise pixels which will be called *Iterative Peer Group Switching Arithmetic Mean Filter* (IPGSAMF) is defined as follows.

1. Every pixel $\mathbf{x}_i$ in the image for which it can be found a peer group $\mathscr{P}(\mathbf{x}_i, 4, d)$ is declared as *noise-free* . The rest of the pixels are declared as *non-assigned*.
2. For each *non-assigned* pixel $\mathbf{x}_i$ in the image, if a peer group $\mathscr{P}(\mathbf{x}_i, 3, d)$ where all the pixel members of the peer group excepting $\mathbf{x}_i$ are *noise-free* can be found, then $\mathbf{x}_i$ is declared as *noise-free* .
3. Repeat step 2 but searching peer groups $\mathscr{P}(\mathbf{x}_i, 2, d)$.
4. Repeat step 2 but searching peer groups $\mathscr{P}(\mathbf{x}_i, 1, d)$.
5. Each pixel declared as *non-assigned* is now declared as *noisy* .
6. The *noisy* pixels in the image are now substituted performing the filtering operation. Each *noisy* pixel is replaced by the output of the AMF (mimicking [13]) performing over the *noise-free* neighbors of the pixel in substitution in a 3×3 window. Notice that a *noisy* pixel may not have any *noise-free* neighbor in its 3×3 neighborhood. If it is the case, the window must be enlarged in size until at least one *noise-free* neighbor is included. The AMF is applied because of its computational simplicity, however other suitable filters as the VMF could be applied in the above conditions, as well.

The proposed filter methods performs by detecting in step 1 a set of pixels which can be declared as *noise-free* with a high reliability since they are *similar* to, at least, the half of their neighbors. On the basis of these *noise-free* pixels, new *noise-free* pixels are detected in the steps 2,3 and 4 by relaxing the required condition respect to the number of pixels members of the peer group but requiring them to be previous *noise-free* pixels. Finally, the procedure leaves the impulsive noise pixels isolated (step 5). The *noisy* pixel substitution is finally performed in step 6.

Figure 1 shows a noisy image along with the pixels declared as *noise-free* after each step and the final output of the filter. Notice that the performance of the filter critically depends on the first selection of *noise-free* pixels performed in step 1 for which the value of d is quite important. The appropriate value of this parameter is studied in Section 5 and it is proposed a robust setting for it.

5. EXPERIMENTAL RESULTS

In this section, the impulsive noise model for the transmission noise, as defined in [1], has been used to add noise to some tests images in order to assess the performance of the proposed filter in front of the classical and recent filters in Table 1. For simplicity, it has been used the common objective quality measure PSNR defined as [1]

$$PSNR = 20 \log \left(\frac{255}{\sqrt{\frac{1}{NMQ} \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{q=1}^{Q} \left(F^q(i,j) - \hat{F}^q(i,j) \right)^2}} \right), \tag{3}$$

where M, N are the image dimensions, Q is the number of channels of the image ($Q = 3$ for color image), and $F^q(i,j)$ and $\hat{F}^q(i,j)$ denote the q^{th} component of the original image vector and the filtered image, at pixel position (i,j), respectively.

The results of the assessment using some details of the images Peppers and Baboon are shown in Tables 2 and 3. Similar results can be obtained for other common objec-

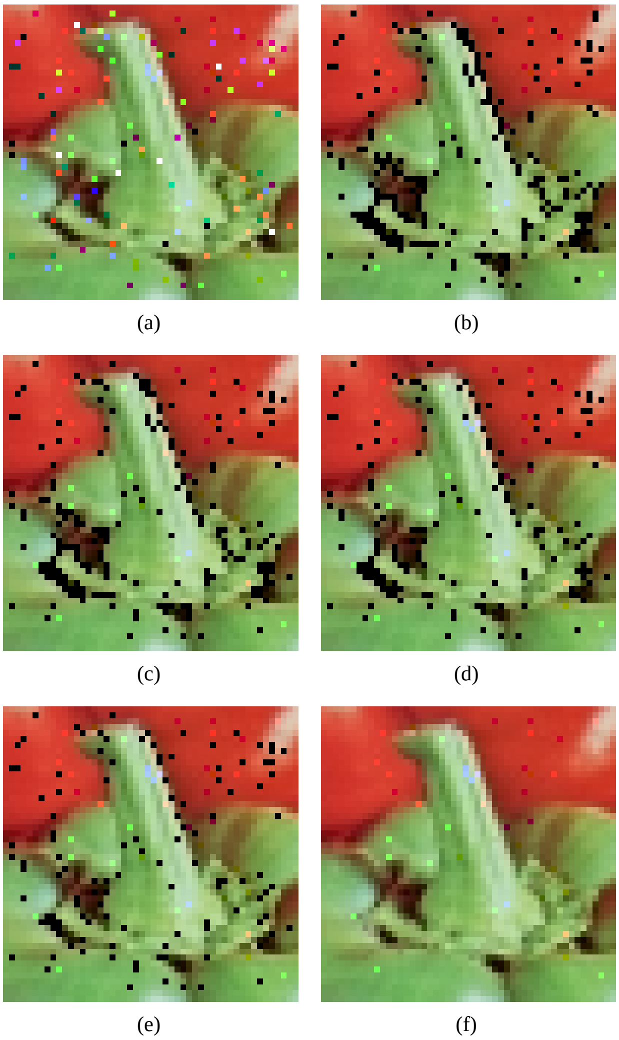

Figure 1: Sample of performance of the proposed filter (*noisy* pixels in black, *noise-free* pixels colored): (a) Detail of Peppers image with 5% impulsive noise, (b) *noise-free* pixels after step 1, (c) *noise-free* pixels after step 2, (d) *noise-free* pixels after step 3, (e) *noise-free* pixels after step 4, (f) IPGSAMF output,

Table 1: Filters taken for performance comparison and notation.

Notation	**Filter**
AMF	Arithmetic Mean Filter [1]
VMF	Vector Median Filter [3]
BVDF	Basic Vector Directional Filter [5]
DDF	Directional Distance Filter [6]
FIVF	Fast Impulsive Vector Filter [16]
PGSAMF	Peer Group Switching AMF [13, 14]
AVMF	Adaptive Vector Median Filter [10]
IPGSAMF	Iterative Peer Group Switching AMF

Table 2: Comparison of the performance in terms of PSNR using a detail of the Baboon image contaminated with different densities of impulsive noise.

Filter	5%	15%	25%
None	23.41	17.75	15.32
AMF	22.90	21.77	20.73
VMF	23.45	23.30	22.85
BVDF	21.33	20.99	20.31
DDF	23.37	23.07	22.58
FIVF	28.33	24.86	23.08
SAMF	27.08	25.34	22.84
AVMF	25.93	25.58	23.25
IPGSAMF	29.90	25.59	23.56

tive quality measures as MAE or NCD. Some outputs of the filters in comparison are shown in Figures 2 and 3.
As it was commented in Section 4 the filter performance is influenced by the value of the d parameter. It has been found that appropriate values of d are in the range $[0.900, 0.940]$. The optimal value of d for a particular image is directly proportional to the density of the contaminating noise. However, a robust setting of d presenting a good performance for several images and noise densities is $d = 0.925$.

6. CONCLUSIONS

In this paper, the *peer group* concept has been adapted to the use of a certain fuzzy metric. Using this concept, a novel filter for impulsive noise removal has been introduced. The usefulness of the *peer groups* and fuzzy metrics for impulsive noise detection and removal is shown. The proposed filtering method is able to properly isolate and remove impulsive noise pixels while preserving the uncorrupted image structures. The classical vector filters are significantly outperformed and the presented performance is competitive respect to recently introduced vector filters with good detail-preserving ability, outperforming them in many cases.

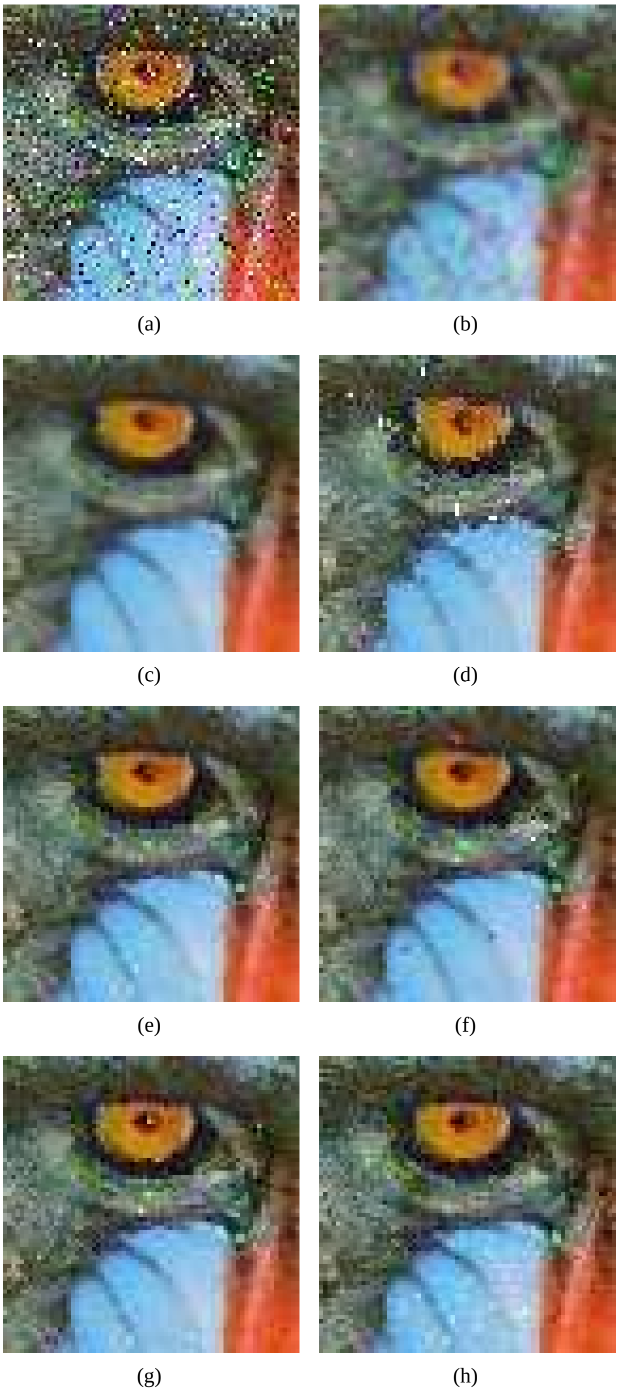

Figure 2: Performance comparison: (a) Detail of Baboon image with 25% impulsive noise, (b) AMF output, (c) VMF output, (d) BVDF output, (e) FIVF output, (f) SAMF output, (g) AVMF output, (h) IPGSAMF output.

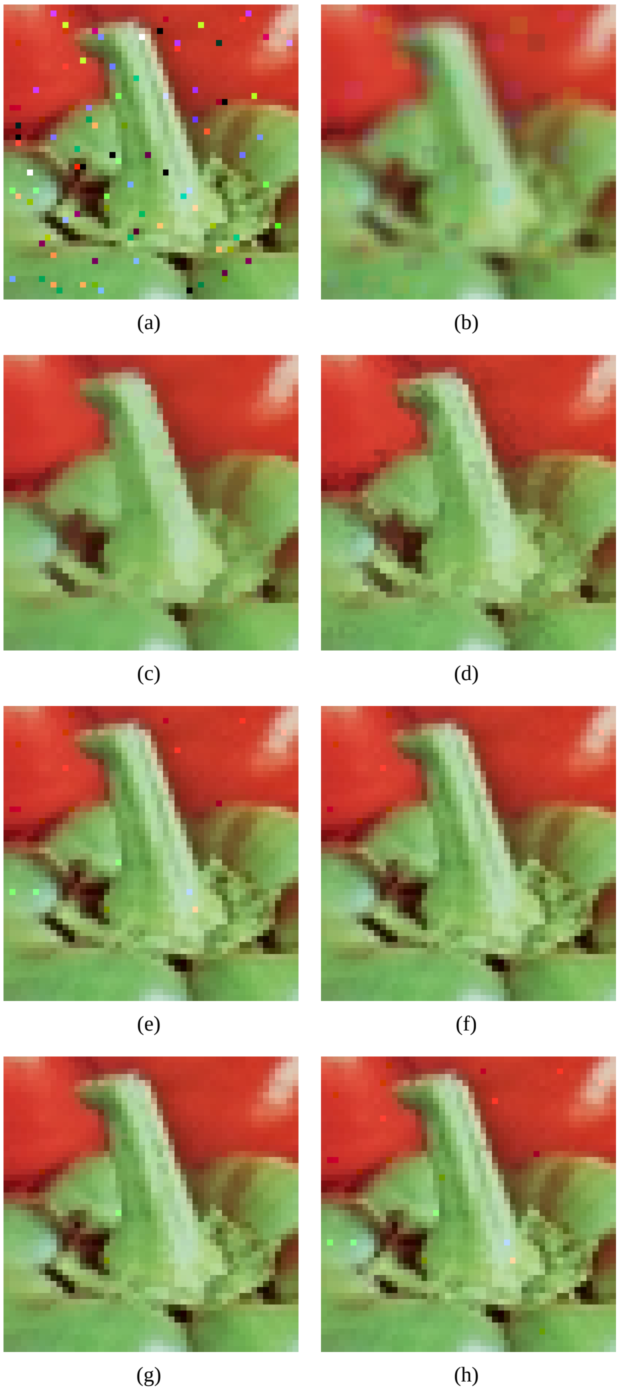

Figure 3: Performance comparison: (a) Detail of Peppers image with 5% impulsive noise, (b) AMF output, (c) VMF output, (d) BVDF output, (e) FIVF output, (f) SAMF output, (g) AVMF output, (h) IPGSAMF output.

Table 3: Comparison of the performance in terms of PSNR using a detail of the Peppers image contaminated with different densities of impulsive noise.

Filter	5%	20%	30%
None	23.74	17.15	14.76
AMF	25.91	23.36	21.76
VMF	28.10	27.06	26.72
BVDF	27.32	25.75	24.00
DDF	28.08	26.46	25.56
FIVF	32.98	28.91	26.51
SAMF	32.93	27.67	24.95
AVMF	32.85	28.23	24.34
IPGSAMF	33.48	27.44	24.13

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