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SLEPIAN PULSES FOR MULTICARRIER OQAM

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ABSTRACT[1]

OFDM/OQAM is spectrally efficient multicarrier (MC) system, robust to mobile channel (time and frequency) variability thanks to pulse shaping. In this contribution, pulse shaping with discrete non-orthogonal pulse based on Discrete Prolate Spheroidal Sequence (DPSS), is proposed. DPSS, being the most concentrated pulse out of all sequences of the same length, is the natural choice for pulse shaping. The novelty of the analysis lies in the discovery of quasi-orthogonality property of DPSS to its time and frequency shifts. DPSS pulse analysis shows that this property is valid in the case of short pulse of the single MC symbol length. MC/OQAM system performance with DPSS and other pulses types is analyzed by simulation confirming the quasi-orthogonality of DPSS. The advantage of using the most concentrated pulse is shown in terms of interference that two MC/OQAM systems in adjacent frequency bands cause one to another. Simulations also show that DPSS permits smaller frequency separation between systems that are allocated adjacent frequency bands.

1. INTRODUCTION AND STATE OF ART

To enable transmissions of high data rates over mobile channel, the chosen data modulation format must be resilient to wideband channel impairments reflected primarily in delay and frequency spread. One of the potential candidates is the MC scheme since it is immune to channel frequency selectivity. One variant of this scheme, OFDM/OQAM is resilient to channel time variability, which is possible thanks to the use of well-localized pulses. Nevertheless, system complexity is incremented with respect to the well-known OFDM scheme with Cyclic Prefix (CP). Pulses concentration permits greater system spectral efficiency and minimize the interference caused to systems in neighboring frequency bands.

Beside localization properties, the window function is designed to be orthogonal to its time and frequency shifts, in order to enable perfect reconstruction of symbols. OFDM/OQAM pulses are proposed for UMTS [4,5,6,7] where the initial continuous time pulse is the Gaussian function that has the best localization in the time and the frequency domain. This pulse is afterwards orthogonalized both in time and frequency domain with Isotropic Orthogonal Transform Algorithm (IOTA) transform, and therefore the system is denoted OQAM/IOTA [1]. The IOTA pulses are sampled and truncated to 3 or 4 OFDM symbol periods in order to perform discrete signal processing [5].

Generation of OFDM/OQAM pulses obtained by the orthogonalization with Zak transform of windows with good localization properties was analyzed in [11]. This approach starts with well-localized pulse and performs its posterior orthogonalization in the Zak domain. In order to perform the orthogonalization, the IFFT and FFT (of the pulse length) have to be calculated. However, it must be stressed that this orthogonalization procedure can not guarantee the preservation of good pulse localization.

One approach to pulse generation related to DPSSs (also known as Slepian pulses) is presented in [8], [9] and [10]. In these papers, a pulse is expanded either in prolate spheroidal wave functions (PSWFs) or in DPSSs series, depending if it is continuous or discrete. Only well-concentrated, symmetric DPSSs were used, guarantying the adequate localization of the pulse. The coefficients of the expansion were found with an optimization procedure that minimizes the interference between pulses. The main difference with respect to the idea presented here is that in [9] the chosen pulse is not the most concentrated, but only well-concentrated, and the emphasis is put on the minimization of interference. Therefore, the optimal pulse that minimizes interference could have very poor localization. The presented optimization procedure searches for the minimization of interference, but does not pursue the best pulse concentration. Pulses of several OFDM/OQAM symbol periods are used in order to be able to expand the pulse with DPSSs.

Bearing in mind that this system is supposed to transmit in dispersive channel that inevitable destroys the orthogonality, in this contribution the orthogonality condition is relaxed and replaced by minimum interference search. The design of pulses is performed in discrete time to

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take advantage of digital signal processing. The proposed pulse corresponds to DPSS that is the solution to maximum power concentration in the desired bandwidth. The pulse with single MC symbol duration is shown to have acceptable properties in terms of interference. Therefore the orthogonalization, that requires DFT and IDFT calculations of a matrix [11] is avoided. The filter length is reduced to single MC symbol duration, making the filtering process complexity lower than of those commonly used for OFDM/OQAM pulses, of several OFDM symbols.

This paper begins with the introduction of a discrete baseband OFDM/OQAM system model, followed by a short reminder about DPSS basic properties. The design of DPSS pulse and its analysis with simulations of realistic system configurations is presented in Sections 3 and 4. Eventually, the main conclusions are given.

2. OFDM/OQAM DISCRETE SYSTEM MODEL

Discrete OFDM/OQAM model for subcarrier *k* is shown in Fig 1.

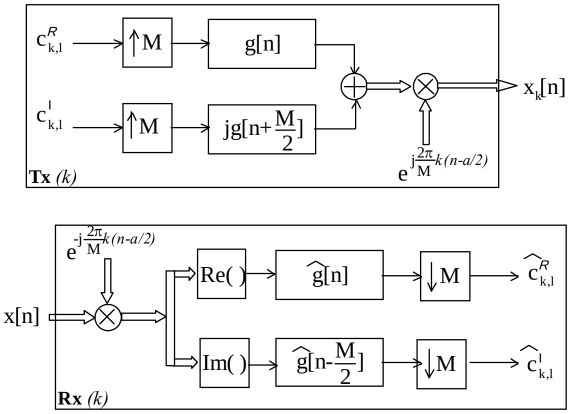

Fig 1. k^{th} subcarrier simplified OFDM/OQAM baseband model

The total number of modulated subcarriers is *N* out of *M* available. The value of *a* depends of the filter length L_g as $a = \mathrm{mod}\left(L_g + M/2 - 1, M\right)$. The OFDM symbol length is *M* chips, that is $T=MT_c$. The real valued symbols $c_{k,l}{}^{R}$ and $c_{k,l}{}^{I}$, per each subcarrier *k*, at OFDM symbol *l*, arrive at symbol rate *1/T*, and are up sampled by factor *M*. They are filtered with FIR filters $g[n]$ and $jg[n+M/2]$, that is the same filter but with offset in delay and phase. Two real valued symbols are transmitted per each subcarrier maintaining the same transmission rate as for OFDM/QAM. Summed filtered signals are then modulated to the corresponding subcarrier. The signal at the output of the transmitter, $x[n]$, is obtained as the sum of signals of all modulated subcarriers.

$$x[n] = \sum_{k=0}^{N-1} x_k[n] = \sum_{k=0}^{N-1}\left(\sum_{l=-\infty}^{\infty} c_{k,l}^{\Re} g(n-lM) e^{j\frac{2\pi}{M}k(n-a/2)} + \sum_{l=-\infty}^{\infty} c_{k,l}^{\Im} jg(n+M/2-lM) e^{j\frac{2\pi}{M}k(n-a/2)}\right) \tag{1}$$

At the receiver, in order to recover the transmitted symbol, demodulation with exponential function is performed per each subcarrier. Filtering with receiver filters $\hat{g}[n]$ ($\hat{g}[n]=g[-n]$) , and its delayed version $\hat{g}$ $[n+M/2]$, is done on in-phase and in-Quadrature part of the signal, respectively. Down sampling with factor *M* gives the estimation of sent symbols, $\hat{c}_{k,l}{}^{R}$ and $\hat{c}_{k,l}{}^{I}$. This scheme does not include the equalization of the received symbols which is necessary in the case of transmission over frequency selective channels. The channel equalization is performed per subcarrier with a complex channel frequency response, as suggested in [6].

The perfect recovery of data is possible if both ICI and ISI are cancelled (see [11]). This is accomplished by pulses that satisfy two conditions:

1) pulses must be even,

$$g[n] = g[a + (2r+1)M/2 - n] \tag{2}$$

with $r \in Z$ and $a \in [0, M-1]$,

2) they must satisfy eq. (3):

$$\frac{1}{2}\langle g, g_{l,-m}\rangle + \frac{1}{2}\langle g, g_{l,m}\rangle = \delta[l]\delta[m] \tag{3}$$

where $\langle \bullet \rangle$ is the scalar product and

$$g_{l,m} = g[n-lM]e^{j\frac{2\pi}{M/2}m(n-a/2)} \tag{4}$$

3. DPSS

The energy concentration of a band limited signal in the time domain was the initial study performed by Slepian and Pollack that resulted in PSWFs as the most concentrated functions [12]. Further research switched time and frequency domains, and started considering sequences. Slepian sequences, or DPSSs, are index-limited sequences that are the solution to the energy concentration problem in the desired bandwidth [13]. The measure of the concentration in the bandwidth (-*W*,*W*) is given by eq. (5) and DPSSs are the eigenvectors of the system of equations given by eq. (6), for *l*=0…*M*-1, and $n \in Z$:

$$\beta = \frac{\int_{-W}^{W} |G(\omega)|^2 d\omega}{\int_{-\infty}^{\infty} |G(\omega)|^2 d\omega} \tag{5}$$

$$2W\sum_{m=0}^{M-1}\mathrm{sinc}\left(2\pi W(n-m)\right)g_l[m]=\lambda_l g_l[n] \tag{6}$$

The DPSS parameter λ_l specifies the concentration ratio of the signal in the desired bandwidth and the total signal energy. The most concentrated signal is obtained with DPSS $g_0[n]$ associated with the maximum eigenvalue and maximizing eq. (5). This DPSS is the one that is used as a pulse in this contribution.

4. OQAM PULSE DESIGN

It is well known that DPSS are the optimum solution to the energy concentration problem of finite length sequences, and their application for pulse shaping is not novel. The pulse that is expanded in terms of truncated PSWFs or DPSSs series resulting in good frequency concentration is proposed in [8], [9] and recently in [10]. These pulses are optimized to minimize the interference and their duration is larger than a single MC symbol. The idea of that research is to find a well-localized pulse that can be expanded into DPSSs series. In order to have a larger number of well-localized DPSSs, the pulse length is chosen to be larger than *M*. Afterwards, the weights in this expansion are chosen in order to minimize the quadratic error function (the measure of orthogonality). Larger pulses lengths enable pulse expansion in DPSSs series and gave the authors more degrees of freedom for the pulse optimization. Nevertheless, the resulting pulse may not be well localized, as weighting of DPSSs with worse concentration may result in smaller pulse interference to its time and frequency shifts.

Here the opposite approach is taken, and the starting point is the best-concentrated pulse in a given bandwidth and a posteriori, the interference it causes is calculated with adequate measure. In this contribution, the interest is put in analysis of the short pulse, of length *M*, and it is found that no further optimization is needed, as the DPSS pulse is almost orthogonal to its shifts. System with short DPSS pulse is compared to short orthogonal pulses which confirms its quasi orthogonality.

The analysis is performed for *3* different, well localized windows: Gaussian, low pass filter analyzed in [11] that will be denoted FIR, and the window corresponding to DPSS.

The orthogonality of pulses of length *M* is given by [3]:

$$g^2[n]+g^2[n+M/2]=2/M \quad \text{for} \quad n=0,1,\ldots,M/2-1 \tag{7}$$

and the suitable measure of interference can be the pulse deviation from the orthogonality value:

$$I_g=\sqrt{\sum_{n=0}^{M/2-1}\left|g^2[n]+g^2[n+M/2]-2/M\right|^2} \tag{8}$$

Characterization of different windows with respect to this measure is given in logarithmic scale in Fig.2. The bandwidth parameter, *b*, refers to the bandwidth for which low-pass/ DPSS filters are designed. The distance between subcarriers is 1/*M* and *W*=*b*/*M* is the half bandwidth in which signal energy should be maximized (eq. (5)). The Gaussian pulse width is determined with parameter α, that is the reciprocal of the pulse standard deviation, [14]:

$$g[n]=\left(\frac{\alpha}{\pi}\right)^{1/4}e^{-\frac{1}{2}\left(\alpha\frac{n-M/2}{M/2}\right)^2} \tag{9}$$

It should be stressed that the Gaussian window is a truncated Gaussian pulse and therefore, it does not maintain the initial pulse property of the minimum time-bandwidth product.

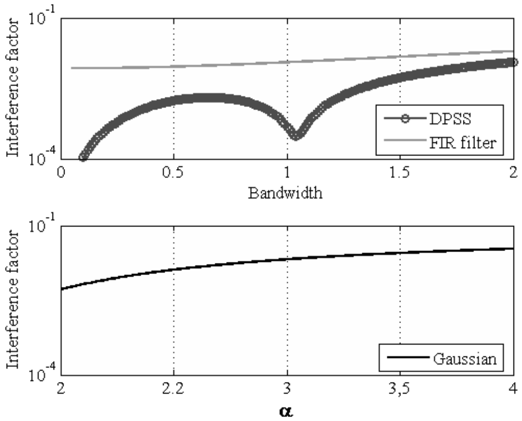

Fig.2 FIR, DPSS and Gaussian window design. FIR and DPSS shapes can be adjusted according to their bandwidth, and the Gaussian pulse according to reciprocal of standard deviation

It can be observed that the minimum interference of DPSS window is much lower than for low-pass FIR and Gaussian window functions. As the interference is increasing with bandwidth or α, in the case of low pass FIR filter and Gaussian window design, the optimization procedure is not possible. However, the DPSS window posses a minimum at *b*=1.03. As the DPSS corresponding to the sequence with the most concentrated spectrum between adjacent MC subcarrier is obtained for *b=1*, this interference minimum actually almost corresponds to the best localized pulse. It can also be observed that DPSS has two pronounced minimum of interference factor. Nevertheless, pulse standard deviation from the mean must be taken into account, as pulse with its limit value, zero, degenerates into a line. In terms of MC pulses, this would correspond to rectangular window that is orthogonal to its shifts, but is not immune to the channel frequency spread. The same happens when values of α are decreasing for Gaussian pulse, the pulse approaches rectangular window. In what follows the value $\alpha=2.5$ will be used, that is the value belonging to the interval of commonly used values (2.5-3.5, see [14]), and interference measure is minimized.

Therefore, out of all well-localized windows, only DPSS does not require orthogonalization, as the remaining interference with non-orthogonal DPSS will not produce great impact. Actually, the measured power of interference is 1.207*e*-4 without channel and 1.5*e*-3 in PedA channel for *16*QAM modulation alphabet.

Orthogonalization with Zak Transform is applied to all analyzed filters in order to compare the loss in performance when using non-orthogonal pulses. The non-orthogonality of pulses will lead to an error floor in system performances due to interference between the original pulse and its shifts in the time and frequency domain. This is the reason for the appearance of pronounced gap between performance of systems with orthogonal and non-orthogonal pulses.

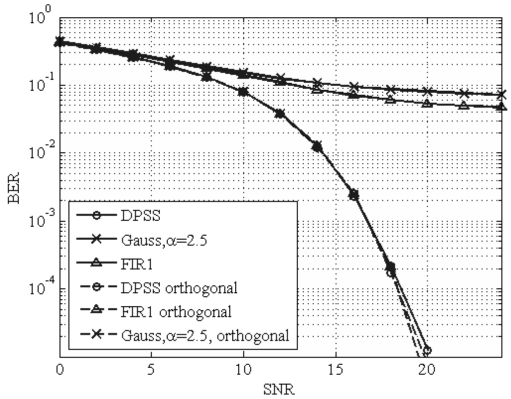

Fig.3 Performance with different pulses in AWGN channel. Clear degradation due to non-orthogonal pulses is observed

A system with *M*=1024 and *N*=666 16-QAM modulated carriers is considered, the scenario used in [7], but without channel coding. The EGC equalization per subcarrier was used. Channel coding is not considered in the initial analysis, as it can mask the error floor.

The pronounced difference in performance when orthogonal (and quasi-orthogonal) or non-orthogonal pulses are applied, can be seen in channel with AWGN, shown in Fig. 3, where BER is given for different SNR per subcarrier. It must be stressed that the performance with orthogonal pulses and quasi-orthogonal DPSS pulse is almost identical, as expected, due to the small amount of residual interference.

The system performance comparing 3 pulses is shown in Fig 4, where, for the sake of system completeness, channel coding was used, ½ rate convolutional coding (171 133). The simulations were carried out in Pedestrian A, multipath Rayleigh channel with mobile velocity 10km/h. Comparison of performances in this, more realistic scenario, shows the necessity for orthogonalization of low-pass FIR and Gaussian window. It can also be observed that the performance with quasi-orthogonal DPSS is the same as the performance of orthogonal pulses.

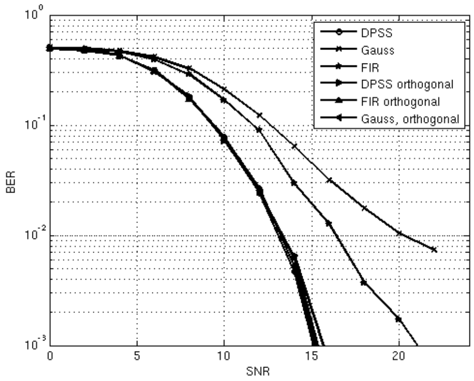

Fig 4. Performance of pulses in PedA channel with channel coding. SNR-loss or error floor appear with non-orthogonal pulses

Robustness of pulse-shaped MC/OQAM to mobile channel impairments is confirmed by simulation of a number of different realization of Pedestrian A channel with maximum Doppler frequency of *40*Hz and constant SNR. The exponential BER is shown in Fig. 5, both for MC/OQAM with DPSS and for classical OFDM with rectangular pulse shaping, without CP. It can be observed that MC/OQAM system reaches better performance more often, as for *15*% of all channel realization BER is below *1e-4*, while with OFDM system this value is only reached in *1*% of all cases.

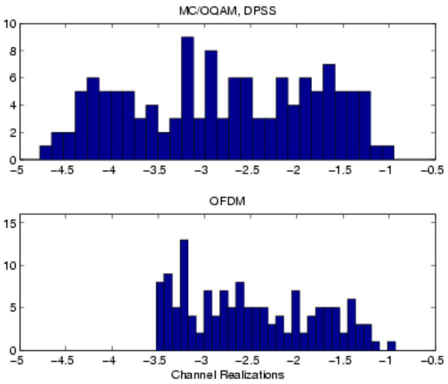

Fig 5. BER exponential value for different channel realizations for MC/OQAM/DPSS and OFDM/rectangular pulse systems

Although the transmission rates of OFDM and MC/OQAM that were simulated are the same, the twofold advantage of pulse shaping can be observed. By using better pulse shape, both intersymbol (ISI) and intercarrier (ICI) interference, that arose due to transmission over multipath channel, are decreased.

5. ADJACENT BANDS INTERFERENCE

Good pulse localization is fundamental for efficient radio spectrum sharing, as the interference that a pulse shaped system causes in adjacent frequency bands is minimized. In order to confirm the adequacy of DPSS pulses, two adjacent

UL MC/OQAM systems are simulated with different separation between the closest subcarriers, as shown in Fig 6. Two groups of subcarriers, assigned to different users, are shown. Usually in realistic systems (see system proposed for UL in [15], for example) one free subcarrier is left in order to separate users bands. Therefore, the separation between adjacent subcarriers of two users should be twice the separation between subcarriers of one user, ν. However, this can not be always accomplished due to different oscillator frequencies. For example, frequency misalignment might occur, as shown schematically in Fig.6.

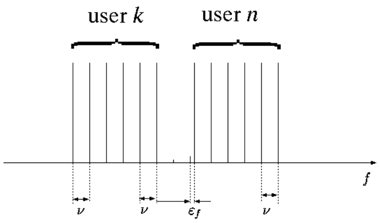

Fig 6. Frequency allocations of two UL MC/OQAM systems with frequency misalignment

BER on the subcarriers on the bands edges is shown with respect to systems separation. Different values of frequency bands separation were used, $\Delta = 2\nu(1+\varepsilon)$. Single path, noiseless channel was assumed, in order to measure only the errors due to interference of systems.

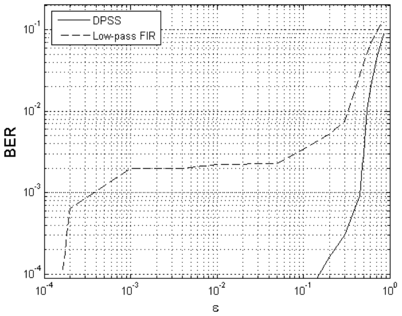

Fig 7. Performance of UL MC/OQAM systems with frequency misalignment

It can be observed that in order to have small degradation, smaller than BER 1E-4, the frequency misalignment for DPSS pulse should be better than 0.1ν, while in the case of low-pass orthogonal filter it must be smaller than $1E-4\times\nu$. This mean than even small misalignments in will result in degradation when the former pulse is applied in system. This confirms the robustness of MC/OQAM system to the frequency misalignment when using the best concentrated pulse, DPSS.

6. CONCLUSION

Well-localized short DPSS pulse for MC OQAM system is presented. Its single MC symbol length makes it attractive for the implementation, as the number of filter taps is low. Additionally, due to the short detection delay, the system latency is decreased. Negligible interference of pulse with its shifts in the time and frequency domains eliminates the need for its orthogonalization, thus preserving good localization properties of the pulse. Therefore, the complexity of pulse generation is reduced, as the calculation of its Zak transform and inverse Zak Transform (several IFFT and FFT of pulse length) are not needed. Tests in realistic scenario confirm its good localization which makes it resilient to mobile channel spread in the time and in the frequency domain.

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