14th European Signal Processing Conference (EUSIPCO 2006), Florence, Italy, September 4-8, 2006, copyright by EURASIP

# A NEW LDPC-STB CODED MC-CDMA SYSTEMS WITH SOVA-BASED DECODING AND SOFT-INTERFERENCE CANCELLATION

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## ABSTRACT

*This paper analyzes several interference cancellation scheme applied to LDPC-STB coded MC-CDMA systems. In these systems, a linear MMSE detector is conventionally used to reduce interference generated by multipath, multiuser, and multiple antennas propagation. To obtain further performance improvements, a more efficient SOVA-based iterative MMSE scheme is considered. This receiver performs soft-interference cancellation for every user based on a combination of the MMSE criterion and the turbo processing principle. It is shown that these block space-time schemes, concatenated with LDPC detectors can potentially provide significant capacity enhancements over the conventional matched filter receiver*[1].

## 1. INTRODUCTION

In mobile communications systems, diversity techniques have proved to be very useful to combat fading and to increase the capacity of the channel. Recently, new space diversity schemes have been developed in combination with channel coding and equalization, resulting in very efficient schemes for single carrier systems on frequency selective fading channels [1-3]. The Low Density Parity-Check (LDPC) codes proposed by Gallager have attracted much attention as good error correcting codes achieving error rate performance close to the Shannon limit [4]. In [5], a concatenation scheme of LDPC codes (with powerful error correction capabilities) and Space-Time Block Coding (STBC) based on Alamouti scheme (LDPC-STBC) was proposed. It has been shown in [6] that, when the block length is relatively large, the error rate performance of LDPC codes is better than that of turbo codes with almost the same block length and code rate. Furthermore, the decoding algorithm of LDPC codes has less complexity than that of turbo codes.

LDPC codes have been applied to Code Division Multiple Access (CDMA) [7, 8] and Orthogonal Frequency Division Multiplexing (OFDM) [9]. Also, as we stated before, Futaki *et al.* proposed a concatenation scheme LDPC codes and STBC based on the Alamouti's scheme. However, the LDPC-STBC applied to Multi-Carrier CDMA (MC-CDMA) has not been reported, neither using it with Minimum Mean Square Error (MMSE) detector nor by means of Soft Output Viterbi Algorithm (SOVA)-based soft-interference cancellation.

This paper discusses the performance of a LDPC-based STB coded MC-CDMA system over frequency selective Multiple-Input Multiple-Output (MIMO) channels with multiuser detection based MMSE and soft-interference cancellation by SOVA-based decoding.

## 2. LOW-DENSITY PARITY-CHECK CODES

The Low-Density Parity-Check (LDPC) codes and their iterative decoding algorithm were proposed by Gallager in 1962 [10, 11]. These codes have been almost forgotten for about thirty years, in spite of their excellent properties. However, LDPC codes are now recognized as good error correcting codes achieving near Shannon limit performance [12].

LDPC codes are linear block codes specified by a very sparse (containing mostly 0's and only a small number of 1's) random parity-check matrix, but are not systematic. The parity-check matrix of an LDPC is an $M \times N$ matrix $\mathbf{A}$, where $M$ is the number of parity bits, and $N$ is the transmitted block length ($N = K + M$, with $K$ as the source block lenght). The matrix $\mathbf{A}$ is specified by a fixed column weight $j$ and a fixed row weight $k = jN/M$ (in the MacKay's and Neal's codes $k$ is as uniform as possible [4]), and code rate $R = K/N$. In this case, we call the resulting code a regular Gallager code because the bipartite graph defined by its parity-check matrix is regular [4]. It has been reported that when the block length is relatively large, irregular LDPC codes with non-uniform column weight outperform turbo codes with almost the same block length and code rate [6].

LDPC codes can be decoded using a probability propagation algorithm known as the sum-product or belief propagation algorithm [13], which is represented by a factor graph that contains two types of nodes: the "bit nodes" corresponding to a column of the parity-check matrix, which also corresponds to a bit in codeword and the "check nodes" corresponding to a row of the parity-check matrix, which represents a parity-check equation. An edge between

[1] *This work has been partly supported by National Project TIC2003-09061-C03-01 and the IST program IST-2002 507039.*

a bit node and a check node exist if and only if the bit participates in the parity-check equation represented by the check node.

$$
\mathbf{A} = \left[\begin{array}{cccccccccccccccccccc}
1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&1&1&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1\\
\hline
1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&0&0&0&0\\
0&1&0&0&0&1&0&0&0&1&0&0&0&0&0&0&1&0&0&0\\
0&0&1&0&0&0&1&0&0&0&0&0&0&1&0&0&0&1&0&0\\
0&0&0&1&0&0&0&0&0&0&1&0&0&0&1&0&0&0&1&0\\
0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&1\\
\hline
1&0&0&0&0&1&0&0&0&0&0&1&0&0&0&0&0&1&0&0\\
0&1&0&0&0&0&1&0&0&0&1&0&0&0&0&1&0&0&0&0\\
0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&0&1&0\\
0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&1&0&0&0\\
0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1
\end{array}\right]
$$

Figure 1. LDPC matrix $(N = 20, j = 3, k = 4)$.

**2.1. Construction of LDPC Codes**

We construct an LDPC code with a method similar to that of [10, 11]. A parity-check matrix is divided into three submatrices, each containing a single 1 in each column. The first of these submatrices contains 1's in descending order; its $i$th row contains 1's in the columns $(i-1)k+1$ to $ik$. The other submatrices are merely column permutations of the first submatrix. The permutations of the second and the third submatrices are independently selected. After constructing the parity-check codes like this, we remove the four-cycle from the parity-check matrix by extracting the corresponding columns.

**2.2. Sum-Product Algorithm**

The decoding problem is to find the most probable vector $\mathbf{x}$ such that $\mathbf{A}\mathbf{x} \bmod 2 = 0$, with the likelihood of $\mathbf{x}$ given by $\Pi_n f_n^{x_n}$, where $f_n^0 = 1 - f_n^1$ and $f_n^1 = 1/\left(1+\exp\left(-2y_n/\sigma^2\right)\right)$ for AWGN channel or $f_n^1 = (y_n/\sigma^2)\exp\left[-y_n{}^2/2\sigma^2\right]$ for Rayleigh channel, and $y_n$, $\sigma^2$ represent the received bit and noise variance, respectively. We denote the set of bits, $n$, that participate in check $m$ as $\mathcal{N}(m) \equiv \{n : A_{mn} = 1\}$, where $A_{mn}$ represents the element of the $m$th row and $n$th column in the parity-check matrix. Similarly, we define the set of checks $m$ in which bit $n$ participates as $\mathcal{M}(n) \equiv \{m : A_{mn} = 1\}$. We denote a set $\mathcal{N}(m)$ with bit $n$ excluded as $\mathcal{N}(m) \setminus n$. The algorithm has two alternating parts, in which quantities $q_{mn}$ and $r_{mn}$ associated with each non-zero element in the matrix $\mathbf{A}$ are iteratively update. The quantity $q_{mn}^x$ is meant to be the probability that bit $n$ of $\mathbf{x}$ is $x$, given the information obtained via checks other than check $m$. The quantity $r_{mn}^x$ is meant to be the probability of check $m$ being satisfied if bit $n$ of $\mathbf{x}$ is considered fixed at $x$ and the other bits have a separable distribution given by the probabilities $\{q_{mn'} : n' \in \mathcal{N}(m) \setminus n\}$. The *a posteriori* probabilities for a bit are calculated by gathering all the extrinsic information from the check nodes that connect to it, which can be obtained by the following iterative sum-product procedure.

*Step 1:* Initialization

The variables $q_{mn}^0$ and $q_{mn}^1$, which are the probabilities sent from the $n$th bit node to the $m$th check node along a connecting edge of a factor graph, are initialized to the values $f_n^0$ and $f_n^1$, respectively.

*Step 2:* Horizontal Step (bit node to check node)

We define $\Delta q_{mn} \equiv q_{mn}^0 - q_{mn}^1$ and compute (1) and (2) for each $m$, $n$ and $x = 0,1$:

$$\Delta r_{mn} = \prod_{n' \in \mathcal{N}(m) \setminus n} q_{mn'} \tag{1}$$

$$
\begin{aligned}
r_{mn}^x &= \{1 + (-1)^x \Delta r_{mn}\}/2 \\
\Rightarrow r_{mn}^0 &= (1 + \Delta r_{mn})/2 \\
r_{mn}^1 &= (1 - \Delta r_{mn})/2
\end{aligned} \tag{2}
$$

Where, $r_{mn}$ represents the probability information sent from the $m$th check node to the $n$th bit node.

*Step 3:* Vertical Step (check node to bit node)

For each $n$, $m$ and $x = 0,1$ we update (3):

$$q_{mn}^x = \alpha_{mn} f_n^x \prod_{m' \in \mathcal{M}(n) \setminus m} r_{m'n}^x \tag{3}$$

Where, $\alpha_{mn}$ is a normalization factor chosen such that $q_{mn}^0 + q_{mn}^1 = 1$. We can also update the *a posteriori* probabilities $q_n^0$ and $q_n^1$, given by (4):

$$q_n^x = \alpha_n f_n^x \prod_{m \in \mathcal{M}(n)} r_{mn}^x \tag{4}$$

Where, $\alpha_n$ is a normalization factor chosen such that $q_n^0 + q_n^1 = 1$.

*Step 4:* Check stop criterion

Hard decision is made on the $q_n^1$. That is, if $q_n^1 > 0.5$ $\hat{x}_n = 1$ and if $q_n^1 < 0.5$ $\hat{x}_n = 0$. The resulting decoded vector $\hat{\mathbf{x}}$ is checked against the parity-check matrix $\mathbf{A}$. If $\mathbf{A}\hat{\mathbf{x}} = 0$, the decoder stops and outputs $\hat{\mathbf{x}}$. Otherwise, it repeats the procedure from the Step 2. The sum-product algorithm sets a maximum number of iterations: if the number of iterations reaches that maximum, the decoder stops and outputs $\hat{\mathbf{x}}$ as the results of the hard decision.

## 3. LDPC-STBC WITH SOFT-INTERFERENCE CANCELLATION

We proposed the LDPC-STBC for MC-CDMA with soft-interference cancellation based on an iterative MMSE-SOVA detector. The proposed system block diagram is shown in Figure 2, for the case of a downlink transmission. The binary information data $\{b^k\}$ for user $k$ are LDPC encoded and transmitted by $N_t$ antennas. In this paper, we use the Alamouti scheme [14], therefore $N_t = 2$. At the ST encoder output, each symbol is multiplied by the spreading code of the specific user, $\mathbf{c}^k = [c_1^k \dots c_f^k \dots c_{L_c}^k]^T$, where $c_f^k$ is the $f$th subcarrier of the spreading code of the $k$th user. The length $L_c$ of the spreading sequence is equal to the number $N_c$ of subcarriers. So, STBC is carried out on two adjacent OFDM symbols, and the receiver has to process two successive symbols as a whole block.

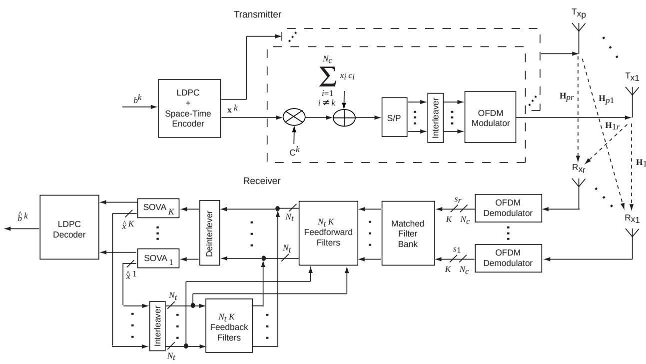


Figure 2. Block diagram of the proposed LDPC-STBC for MC-CDMA system with soft-interference cancellation.

The received signal at the $r$th receive antenna is equal to:

$$\begin{bmatrix} \mathbf{s}_r(t) \\ \mathbf{s}_r(t+1) \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{1r} & \mathbf{H}_{2r} \\ \mathbf{H}_{2r}^* & -\mathbf{H}_{1r}^* \end{bmatrix} \begin{bmatrix} \mathbf{C} & 0 \\ 0 & \mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{n}_r(t) \\ \mathbf{n}_r(t+1) \end{bmatrix} \tag{5}$$

Where, $\mathbf{s}_r(t) = [s_{r,1}(t) \dots s_{r,f}(t) \dots s_{r,Nc}(t)]^T$ is the vector of $N_c$ received signals, $\mathbf{H}_{pr}$ $(p \in \{1,2\}, r = 1)$ is a diagonal matrix, $\mathbf{C} = [\mathbf{c}^1 \mathbf{c}^2 \dots \mathbf{c}^K]$ is an $L_c$ order square matrix of users spreading codes, $\mathbf{x}_1$ and $\mathbf{x}_2$ are vectors of data symbols transmitted in a block by the $K$ active users, and $\mathbf{n}_r(t)$ is the AWGN vector (variance $E|n_{r,f}|^2 = N_0 \ \forall f, r$).

The resulting received signal $y = \left[\mathbf{y}_1^T \ \mathbf{y}_2^T\right]^T$ is the addition of the combined signals from all receives antennas:

$$y = \sum_{r=1}^{Nr} \mathcal{G}_r \mathcal{S}_r \text{ , with } \mathcal{G}_r = \begin{bmatrix} \mathbf{G}_{1r} & \mathbf{G}_{2r}^* \\ \mathbf{G}_{2r} & -\mathbf{G}_{1r}^* \end{bmatrix} \tag{6}$$

Where $\mathbf{G}_{pr}$ is a $2N_c \times 2N_c$ diagonal matrix containing the equalization coefficients for the channel $pr$ $(p \in \{1,2\}, r = 1)$.

After despreading and threshold detection, the detected symbols correspond to the sign of the scalar product of the received signals $y_{1,f}$, $y_{2,f}$ and the specific spreading code $\mathbf{c}^k$:

$$\hat{x}_{1,2}^k = sign[\langle \mathbf{y}_{1,2}, \mathbf{c}^k \rangle] = sign\left[\sum_{f=1}^{N_c} c_f^k y_{1,2,f}\right] \tag{7}$$

In order to improve the detection robustness, the interference terms of the MAI and ISI have to be suppressed without increasing the noise term. Hence, to combat the channel fading and the MAI, the detection schemes have to be optimized to extract the desired user's signal while reducing other users' interferences.

### 3.1 Iterative MMSE-SOVA Detector

We applied the iterative MMSE detector derived by Vucetic *et. al.* [15], with SOVA-based soft-interference cancellation.

There are two known Soft-Input Soft-Output (SISO) decoding methods: Maximum A Posteriori (MAP) decoder and SOVA. While the former provides the best performance in terms of minimizing the decoding errors, the latter has significantly lower complexity with only slight degradation in the decoding performance. Hence, SOVA is more suitable for hardware implementation. For the iterative detector, there are two different SOVA versions described in the literature; one proposed by C. Battail [16] and the other by Hagenauer [17]. The key difference between these two algorithms is the updating rule. Besides, in [18] it is shown that the performance of the SOVA approach of Battail is 0.5 dB better than that of Hagenauer. However, by comparing Battail and Hagenauer schemes it can be easily seen that they have the same basic structure but the latter has a simplified version of the updating rule, which from an implementation point of view is preferable since it does not need any mathematical manipulation, apart from a single comparator. For these reasons we selected Hagenauer-SOVA for our iterative receiver.

In an iterative receiver, the interference estimate for the $p$th transmit antenna of the $k$th user is formed by adding the regenerated signals of all users and all transmit antennas, except the one for the desired user $k$ and antenna $p$. After each decoding iteration, the soft decoder outputs, calculated by means of the tools given by Hagenauer in [19], are used to update the *a priori* probabilities of the transmitted symbols. These updated probabilities are used in the calculation of the MMSE filter feedforward and feedback coefficients. Assuming that $z_p^k(t)$ is the input to the $k$th user decoder corresponding to the $p$th transmit antenna at time $t$, it is represented by:

$$z_p^k(t) = \left(\mathbf{w}_{fp}^k(t)\right)^H s(t) + \left(\mathbf{w}_{bp}^k(t)\right)^H \underline{\hat{\mathbf{x}}}_p^k(t) \tag{8}$$

Where $\mathbf{w}_{fp}^k(t)$ is the $N_r N_c \times 1$ optimized feedforward coefficients vector, $\mathbf{w}_{bp}^k(t)$ is the $(N_t K - 1) \times 1$ feedback coefficients vector, $\underline{\hat{\mathbf{x}}}_p^k(t)$ is the $(N_t K - 1) \times 1$ vector representing the feedback soft decisions for all users and all transmit antennas except the decision corresponding to the $p$th transmit antenna of user $k$. Note that the feedback coefficients appear only through their sum in (8). Hence, we can assume, without loss of generality, that:

$$\omega_{bp}^k(t) = \left(\mathbf{w}_{bp}^k(t)\right)^H \underline{\hat{\mathbf{x}}}_p^k(t) \tag{9}$$

Where $\omega_{bp}^k(t)$ is a single coefficient that represents the sum of the feedback terms.

The coefficients $\mathbf{w}_{fp}^k(t)$ and $\omega_{bp}^k(t)$ are obtained by minimizing the mean square error value $\varepsilon$ between the data symbols and their estimates, given by:

$$\begin{aligned}\varepsilon &= E\left[\left|z_p^k(t) - x_p^k(t)\right|^2\right] \\ &= E\Big[\Big|\left(\mathbf{w}_{fp}^k(t)\right)^H \left\{\mathbf{h}_p^k x_p^k(t) + \underline{\mathbf{H}}_p^k \underline{\mathbf{x}}_p^k(t) + n(t)\right\} \\ &\quad + \omega_{bp}^k(t) - x_p^k(t)\Big|^2\Big]\end{aligned} \tag{10}$$

Where $\mathbf{h}_p^k$ is the $N_r N_c \times 1$ signature vector for the $p$th transmit antenna of the $k$th user, $\underline{\mathbf{H}}_p^k$ is an $N_r N_c \times (N_t K - 1)$ matrix composed of the signature vectors of all users and transmit antennas except the $p$th antenna of the $k$th user, and $\underline{\mathbf{x}}_p^k(t)$ is the $(N_t K - 1) \times 1$ transmitted data vector from all users and transmit antennas except the $p$th antenna of the $k$th user. The optimum feedforward and feedback coefficients, $\mathbf{w}_{fp}^k(t)$ and $\omega_{bp}^k(t)$, respectively, can be represented by the following expressions:

$$\mathbf{w}_{fp}^k(t) = \left(A + B + R_n - FF^H\right)^{-1} \mathbf{h}_p^k \tag{11}$$

$$\omega_{bp}^k(t) = -F^H\left(\mathbf{w}_{fp}^k(t)\right) \tag{12}$$

Where,

$$\begin{aligned} A &= \mathbf{h}_p^k \left(\mathbf{h}_p^k\right)^H \\ B &= \underline{\mathbf{H}}_p^k \Big[\mathbf{I}_{N_t K-1} - Diag\Big(\underline{\mathbf{x}}_{E_p}^k \left(\underline{\mathbf{x}}_{E_p}^k\right)^H\Big) \\ &\quad + \underline{\mathbf{x}}_{E_p}^k \left(\underline{\mathbf{x}}_{E_p}^k\right)^H\Big]\left(\underline{\mathbf{H}}_p^k\right)^H \\ F &= \underline{\mathbf{H}}_p^k \underline{\mathbf{x}}_{E_p}^k \\ R_n &= \sigma_n^2 \mathbf{I}_{N_r N_c} \end{aligned} \tag{13}$$

And $\mathbf{I}_N$ denotes the identity matrix of size $N$, $\underline{\mathbf{x}}_{E_p}^k$ is the $(N_t K - 1) \times 1$ vector of the expected values of the transmitted symbols from the other $N_t K - 1$ users and their transmit antennas.

In iterative detector, during the first decoding iteration, we assume that the *a priori* probabilities for all transmitting symbols are equal, and hence, $\underline{\mathbf{x}}_{E_p}^k = 0$. After each iteration, $\underline{\mathbf{x}}_{E_p}^k$ is updated from the soft outputs of the decoders, as derived in [19] by Hagenauer *et al*., and then used to generate the new set of filter coefficients.

## 4. SIMULATIONS RESULTS

We have evaluated, by simulations, the system performance, measured in terms of BER and FER versus $E_b/N_0$ and active users. In this system, both linear and iterative MMSE-SOVA detectors have been used. The different subcarriers are affected by independent frequency selective Rayleigh fading. We further assume that the signals from all different users are received with the same power, and that the receiver perfectly knows the channel response. Each frame is composed by 120 symbols that are transmitted from $N_t = 2$ transmit antennas, thus forming 60 transmission symbol blocks in each frame. The channels remain constant during the transmission of a whole symbol block. The $(120.64.3.109)$ LDPC matrix used is those obtained by MacKay and Neal [20]. We have assumed full system load, that is, there are $K = 4$ active users simultaneously in a cell with $L_c = 4$ spreading codes present in the system. Figure 3 and Figure 4 depict BER and FER performance for several MMSE detectors on frequency selective Rayleigh fading channels, respectively. We can see that iterative schemes perform significantly better when compared to the linear MMSE detector. Besides we found negligible differences between iterative schemes with varying iteration number for low SNR values, so we suggest to always use a single iteration equalizer in those situations. Also, the LDPC concatenated with STB coded system improves in more than 4 dB to the MC-CDMA counterpart without LDPC coding.

## 5. CONCLUSIONS

In this paper, we have evaluated the performance of a synchronous downlink in a multiuser LDPC-STB coded MC-CDMA system, operating over frequency selective Rayleigh channels. We have obtained significant BER and FER improvements when an iterative MMSE detector is used, and the LDPC concatenated with STBC improves the MC-CDMA performance. Furthermore, we propose the use of a single iteration equalizer when the SNR is low, because we noticed very minor improvements as we increased the number of iterations.

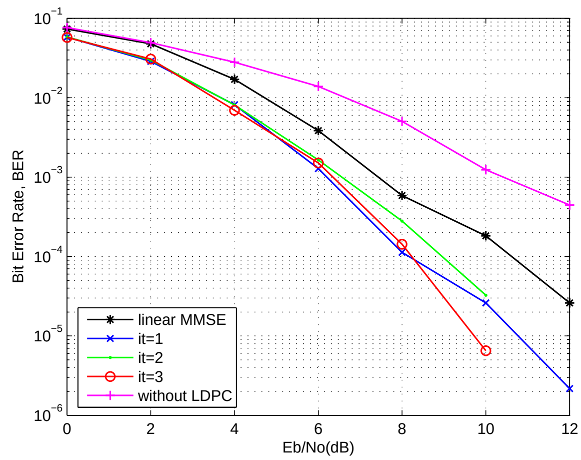


Figure 3. BER performance of a LDPC-STBC for MC-CDMA system with an iterative MMSE receiver.

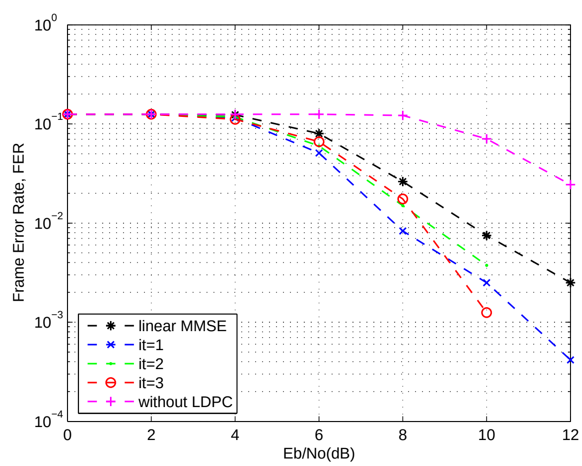


Figure 4. FER performance of a LDPC-STBC for MC-CDMA system with an iterative MMSE receiver.

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