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Soft-Output Detection of Differentially Encoded M-PSK Over Channels with Phase Noise

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***Abstract*— We consider a differentially encoded M-PSK signal transmitted over a channel affected by phase noise. For this problem, we derive the exact maximum a posteriori (MAP) symbol detection algorithm. By analyzing its properties, we demonstrate that it can be implemented by a forward-backward estimator of the phase probability density function, followed by a symbol-by-symbol completion to produce the a posteriori probabilities of the information symbols. To practically implement the forward-backward phase estimator, we propose a couple of schemes with different complexity. The resulting algorithms exhibit an excellent performance and, in one case, only a slight complexity increase with respect to the algorithm which perfectly knows the channel phase. The application of the proposed algorithms to repeat and accumulate codes is assessed in the numerical results.**

I. Introduction

In the last few years, several iterative detection algorithms have been designed to decode powerful channel codes, such as turbo codes or low-density parity-check codes, transmitted over channels affected by a time-varying phase [1]–[8]. In particular, in [8] the authors developed an algorithm with a very low complexity and a practically optimal performance. Some of these algorithms require the insertion of pilot symbols to solve the phase ambiguity problem which arises in phase-uncertain channels and to make the iterative decoder bootstrap, especially in the case of strong phase noise and long codeword lengths [8].

A classical alternative to pilot symbols, that avoids the decrease of the effective information rate due to pilot insertion, is represented by the use of an inner differential encoding [9]. An inner differential encoder, or by using an equivalent terminology, an inner accumulator, is also a component of repeat and accumulate (RA) codes [10].

In principle, the algorithms described in [4]–[8] can be used for differentially encoded signals. However, they have a main drawback: they perform separate detection and phase tracking, that is, at every iteration an instance of a soft-input soft-output (SISO) algorithm for the differential code, for example implemented by means of a BCJR algorithm [11], the execution of the code-aware phase tracking algorithm (which takes advantage from the a posteriori probabilities coming from the BCJR), and finally another execution of the BCJR, are performed. Hence, they are characterized by an higher latency and do not exploit the code structure but only the soft-outputs produced by the decoder, thus possibly requiring the insertion of (a minimal amount of) pilot symbols to bootstrap or speed-up the convergence process.

On the contrary, the algorithms in [1]–[3] can be designed to jointly perform the decoding of the differential code and the detection in the presence of the unknown time-varying phase. In [1], after a proper discretization of the phase space, a super-trellis, taking into account the differential code and the phase model, is built and the BCJR algorithm is run over it. In [2], the channel phase is a priori averaged out, but the resulting algorithm still works on an expanded trellis. Finally, the algorithm in [3] can work on the trellis of the differential encoder or on an expanded trellis and *multiple* non-Bayesian phase estimators are used in the forward and backward recursions of the algorithm.

In this paper, we consider the problem of a differentially encoded M-ary phase shift keying (PSK) signal transmitted over a channel affected by phase noise. The approach is Bayesian, i.e., the channel phase is modeled as a stochastic process with known statistics. Although the implementation of the exact maximum a posteriori (MAP) symbol detection algorithm is impractical, we analyze its properties, finding that it can be implemented by using a *single* forward-backward estimator of the phase probability density function, followed by a symbol-by-symbol completion to produce the a posteriori probabilities of the information symbols. This algorithm obviously works in a joint decoding/phase tracking fashion and does not require the insertion of pilot symbols. Then, by using the canonical distribution approach [12] we develop a couple of practical schemes to implement the forward-backward estimator. The resulting algorithms may be used as SISO blocks for iterative detection/decoding in concatenated schemes.

II. System Model

We consider the transmission of a sequence of complex modulation symbols $\mathbf{d} = \{d_k\}_{k=0}^{K}$, belonging to an M-PSK alphabet $\left\{e^{j\frac{2\pi}{M}i}, i = 0, 1, \ldots, M-1\right\}$, over an additive white Gaussian noise (AWGN) channel affected by an unknown time-varying phase. Symbols $\{d_k\}$ are obtained from information sequence $\mathbf{c} = \{c_k\}_{k=1}^{K}$, assumed independent, but not identically nor uniformly distributed, and belonging to the same M-PSK alphabet, through differential encoding, i.e.,

$$d_k = d_{k-1} c_k \,. \tag{1}$$

The initial symbol d_0 is assumed unknown to the receiver.[1] Assuming Nyquist transmitted pulses, matched filtering, phase variations slow enough so as no intersymbol interference arises, the discrete-time baseband received signal is given by

$$r_k = d_k e^{j\theta_k} + w_k, \quad k = 0, 1, \ldots, K \tag{2}$$

where the noise samples $\mathbf{w} = \{w_k\}_{k=0}^K$ are independent and identically distributed (i.i.d.), complex, circularly symmetric Gaussian random variables, each with zero mean and variance $2\sigma^2$.

In the derivation of the proposed algorithms, for the time-varying channel phase θ_k we assume a random-walk (Wiener) model:

$$\theta_{k+1} = \theta_k + \Delta_k \tag{3}$$

where $\{\Delta_k\}$ are real i.i.d. Gaussian random variables with zero mean and standard deviation σ_Δ,[2] and θ_0 is uniformly distributed in $[0, 2\pi)$. The value of σ_Δ is assumed known to the receiver. The sequence of phase increments $\{\Delta_k\}$ is supposed unknown to both transmitter and receiver and statistically independent of $\mathbf{d}$ and $\mathbf{w}$. The assumption on the phase noise model will be relaxed in the numerical results.

III. MAP SYMBOL DETECTION OF DIFFERENTIALLY ENCODED PSK SIGNALS

We derive here the exact MAP symbol detection algorithm for the considered problem by using a properly defined factor graph (FG) and the sum-product algorithm (SPA) [13].

Let us consider the joint distribution of vectors $\mathbf{c}$, $\mathbf{d}$, and $\boldsymbol{\theta} = \{\theta_k\}_{k=0}^K$ given $\mathbf{r} = \{r_k\}_{k=0}^K$:[3]

$$p(\mathbf{c}, \mathbf{d}, \theta | \mathbf{r}) \propto P(\mathbf{c}) P(\mathbf{d}|\mathbf{c}) p(\boldsymbol{\theta}) \prod_{k=0}^K p(r_k | d_k, \theta_k) \tag{4}$$

where

$$p(r_k|d_k,\theta_k) = \frac{1}{2\pi\sigma^2} \exp\left\{ -\frac{|r_k - d_k e^{j\theta_k}|}{2\sigma^2} \right\} . \tag{5}$$

We can further factor the terms $P(\mathbf{c})$, $P(\mathbf{d}|\mathbf{c})$, and $p(\boldsymbol{\theta})$ in (4) as

$$P(\mathbf{c}) = \prod_{k=1}^K P(c_k) \tag{6}$$

$$P(\mathbf{d}|\mathbf{c}) = P(d_0) \prod_{k=1}^K I(d_k, d_{k-1}, c_k) \tag{7}$$

$$p(\boldsymbol{\theta}) = p(\theta_0) \prod_{k=1}^K p(\theta_k|\theta_{k-1}) \tag{8}$$

where $I(d_k, d_{k-1}, c_k)$ is an indicator function, equal to 1 if c_n and the differential symbols d_k and d_{k-1} respect the constraint (1), and to zero otherwise. Since the SPA is defined up to scaling its messages by positive factors, independent of the variables represented in the graph, from now on we take the notational liberty of using the equality symbol "$=$" instead of the proportionality symbol "$\propto$". Substituting (6), (7), and (8) into (4), *clustering* [13] the variables d_k and θ_k, we obtain the FG in Fig. 1. Since this FG does not contain cycles, the application to it of the SPA, with *non-iterative* forward-backward schedule, produces the *exact* a posteriori probabilities of symbols $\{c_k\}$. Taking into account the probabilistic meaning of the messages in the graph, defining $\mathbf{r}_{k_1}^{k_2} = \{r_k\}_{k=k_1}^{k_2}$ (hence $\mathbf{r} = \mathbf{r}_0^K$), the forward and backward recursions and the completion necessary to compute the a posteriori probabilities of symbol c_k (or, equivalently, the extrinsic information $\frac{P(c_k|\mathbf{r})}{P(c_k)}$) are, respectively

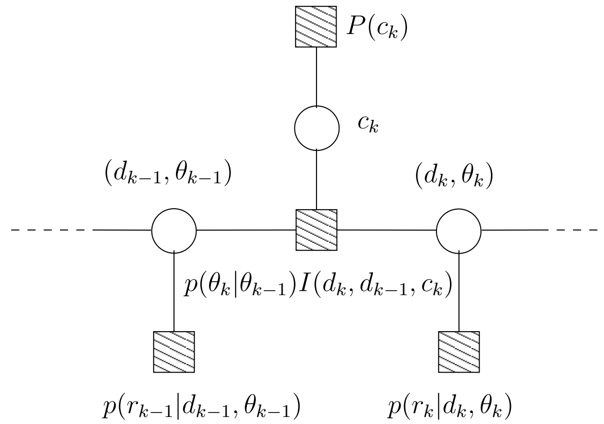

Fig. 1. Factor graph for the considered problem.

$$p(d_k, \theta_k | \mathbf{r}_0^k) = p(r_k|d_k,\theta_k) \sum_{c_k} P(c_k) \cdot \int p(\theta_k|\theta_{k-1}) p(d_{k-1} = d_k c_k^*, \theta_{k-1} | \mathbf{r}_0^{k-1}) d\theta_{k-1} \tag{9}$$

$$p(d_{k-1}, \theta_{k-1} | \mathbf{r}_{k-1}^K) = p(r_{k-1}|d_{k-1},\theta_{k-1}) \sum_{c_k} P(c_k) \cdot \int p(\theta_k|\theta_{k-1}) p(d_k = d_{k-1} c_k, \theta_k | \mathbf{r}_k^K) d\theta_k \tag{10}$$

$$\frac{P(c_k|\mathbf{r})}{P(c_k)} = \sum_{d_{k-1}} \iint p(d_{k-1}, \theta_{k-1}|\mathbf{r}_0^{k-1}) \cdot p(d_k = d_{k-1}c_k, \theta_k | \mathbf{r}_k^K) p(\theta_k|\theta_{k-1}) d\theta_k d\theta_{k-1} . \tag{11}$$

The initializations for the forward and backward recursions are $p(d_0, \theta_0|r_0) = p(r_0|d_0,\theta_0)$ and $p(d_K, \theta_K|r_K) = p(r_K|d_K,\theta_K)$. In the case of a different rotational invariant code [14], these considerations can be extended to the FG in which the encoder state and the channel phase are clustered.

Let us now consider (9). We may decompose

$$p(d_k, \theta_k | \mathbf{r}_0^k) = p(\theta_k | d_k, \mathbf{r}_0^k) P(d_k | \mathbf{r}_0^k) . \tag{12}$$

The first term of the right hand side considers the distribution of the unknown phase, at time k, given the past and present received samples and the state of the differential encoder, while the second term is the state probability, that is exactly the same probability mass function (pmf) evaluated in case of

[1] Since the transmission over a channel affected by phase noise will be considered, we may assume that the initial symbol d_0 is unknown to the receiver due to the initial channel phase uncertainty.

[2] Note that, since the channel phase is defined modulo 2π, the probability density function (pdf) $p(\theta_{k+1}|\theta_k)$ can be approximated as Gaussian in θ_{k+1}, with mean θ_k and variance σ_Δ^2, only if $\sigma_\Delta \ll 2\pi$.

[3] We still use the symbol $p(.)$ to denote a continuous pdf with some discrete probability masses.

detection in the presence of a constant known phase. In practice, the algorithm performs a *per-state Bayesian estimation* of the channel phase during the forward recursion. A similar decomposition can be clearly accomplished for the backward pdf (10).

A proof of the following three properties is omitted for a lack of space.

Property 1. Despite the values of the a priori information $\{P(c_k)\}$, it results that $P(d_k|\mathbf{r}_0^k) = const.$ and $P(d_k|\mathbf{r}_k^K) = const.$, for each value of k. Hence, it is not necessary to evaluate them.

Property 2. The pdfs $p(\theta_k|d_k, \mathbf{r}_0^k)$, for different values of d_k, differ for a shift of a multiple of $\frac{2\pi}{M}$, i.e.,

$$p(\theta_k|d_k = e^{j\frac{2\pi}{M}i}, \mathbf{r}_0^k) = p(\theta_k + \frac{2\pi}{M}i|d_k = e^{j0}, \mathbf{r}_0^k)\,. \quad (13)$$

An identical result holds for the backward pdf $p(\theta_k|d_k, \mathbf{r}_k^K)$.

Property 3. The summation over d_{k-1} in the completion (11) disappears because all the M terms of the summation are equal. Hence, only one of them needs to be evaluated.

From these three properties, it follows that, for each time epoch k, only the pdfs $p(\theta_k|d_k = 1, \mathbf{r}_0^k)$ and $p(\theta_k|d_k = 1, \mathbf{r}_k^K)$ need to be evaluated. By defining $\alpha_k(\theta_k) = p(\theta_k|d_k = 1, \mathbf{r}_0^k)$ and $\beta_k(\theta_k) = p(\theta_k|d_k = 1, \mathbf{r}_k^K)$, the forward-backward algorithm described by (9), (10), and (11) simplifies to

$$\alpha_k(\theta_k) = p(r_k|d_k = 1, \theta_k) \sum_{i=0}^{M-1} P\left(c_k = e^{j\frac{2\pi}{M}i}\right) \cdot \int \alpha_{k-1}\left(\theta_{k-1} - \frac{2\pi}{M}i\right) p(\theta_k|\theta_{k-1}) d\theta_{k-1} \quad (14)$$

$$\beta_{k-1}(\theta_{k-1}) = p(r_{k-1}|d_{k-1} = 1, \theta_{k-1}) \sum_{i=0}^{M-1} P\left(c_k = e^{j\frac{2\pi}{M}i}\right) \cdot \int \beta_k\left(\theta_k + \frac{2\pi}{M}i\right) p(\theta_k|\theta_{k-1}) d\theta_k \quad (15)$$

$$\frac{P(c_k = e^{j\frac{2\pi}{M}i}|\mathbf{r})}{P(c_k = e^{j\frac{2\pi}{M}i})} = \iint \alpha_{k-1}(\theta_{k-1})\beta_k\left(\theta_k + \frac{2\pi}{M}i\right) \cdot p(\theta_k|\theta_{k-1}) d\theta_k d\theta_{k-1}\,. \quad (16)$$

Hence, we have a single forward-backward estimator of the phase probability density function and a final completion.

This exact MAP symbol detection strategy involves integration and computation of continuous pdfs, and it is not suited for direct implementation. A solution for this problem is suggested in [12] and consists of the use of *canonical distributions*, i.e., the pdfs $\alpha_k(\theta_k)$ and $\beta_k(\theta_k)$ computed by the algorithm are constrained to be in a certain "canonical" family, characterized by some parameterization. Hence, the forward and backward recursions reduce to propagating and updating the parameters of the pdf rather than the pdf itself. In the next section, two low-complexity algorithms based on this approach will be described.

IV. LOW-COMPLEXITY ALGORITHMS

A. First Algorithm

A very straightforward solution to implement (14) and (15) is obtained by discretizing the channel phase [1], [8]. In this way, the pdfs $\alpha_k(\theta_k)$ and $\beta_k(\theta_k)$ become probability mass functions (pmfs) and the integrals in (14), (15), and (16) become summations. When the number L of discretization levels is large enough, the resulting algorithm becomes optimal (in the sense that its performance approaches that of the exact algorithm).[4] Hence, it may be used to obtain a performance benchmark and will be denoted to as "discretized-phase algorithm" (*dp-algorithm*).

B. Second Algorithm

By observing that the Tikhonov distribution ensures a very interesting performance with a low complexity when used as a canonical distribution in detection algorithms for phase noise channels, as demonstrated in [8], pdfs $\alpha_k(\theta_k)$ and $\beta_k(\theta_k)$ are constrained to have the following expressions

$$\alpha_k(\theta_k) = \sum_{m=0}^{M-1} q_{f,k}^{(m)} t\left(a_{f,k} e^{j\frac{2\pi}{M}m}; \theta_k\right) \quad (17)$$

$$\beta_k(\theta_k) = \sum_{m=0}^{M-1} q_{b,k}^{(m)} t\left(a_{b,k} e^{j\frac{2\pi}{M}m}; \theta_k\right) \quad (18)$$

where, for each time index k, $\{q_{f,k}^{(m)}, m = 0, 1, \ldots, M-1\}$ ($\{q_{b,k}^{(m)}, m = 0, 1, \ldots, M-1\}$) and $a_{f,k}$ ($a_{b,k}$) are, respectively, M real coefficients and one complex coefficient, and $t\,(z;\theta)$ is a Tikhonov distribution with complex parameter z defined as

$$t\,(z;\theta) = \frac{e^{\mathrm{Re}[ze^{-j\theta}]}}{2\pi \mathrm{I}_0(|z|)} \quad (19)$$

$\mathrm{I}_0(x)$ being the zero-th order modified Bessel function of the first kind.

Three approximations are now introduced in order to derive a low complexity detection algorithm:

i. the convolution of a Tikhonov and a Gaussian pdf is still a Tikhonov pdf, with a modified complex parameter [15], i.e.,

$$\int t(z;x) g(x,\rho^2;y) dx \simeq t\left(\frac{z}{1+\rho^2|z|}; y\right) \quad (20)$$

where $g(x,\rho^2;y)$ represents a Gaussian pdf in y with mean x and variance ρ^2;

ii. since, for large arguments, $\mathrm{I}_0(x) \simeq e^x$, we approximate

$$e^{\mathrm{Re}[ze^{-j\theta}]} \simeq 2\pi e^{|z|} t(z;\theta) \quad (21)$$

iii. let z be a complex number, $\{u_m, m = 0, 1, \ldots, M-1\}$ a set of complex numbers, and $\{q_m, m = 0, 1, \ldots, M-1\}$ a set of real numbers such that $\sum_m q_m = 1$, then the following approximation holds, especially when $|z|$ is sufficiently larger than each $|u_m|$ or when there is a $\overline{m}$ such that $q_{\overline{m}} \gg q_m, \forall m \neq \overline{m}$:

$$\sum_m q_m t\left(z e^{j\frac{2\pi}{M}m} + u_m; \theta\right) \simeq \sum_m q_m t\left(w e^{j\frac{2\pi}{M}m}; \theta\right) \quad (22)$$

where $w = z + \sum_\ell q_\ell u_\ell e^{-j\frac{2\pi}{M}\ell}$.

We now derive the reduced-complexity forward recursion. Substituting (5) into (14), assuming that $\alpha_{k-1}(\theta_{k-1})$ has the

[4] To avoid any performance loss, in [1] the authors proposed the following rule of thumb: the number of quantization levels has to be at least $L = 8M$.

canonical expression (17), and using approximation (20), we obtain

$$\alpha_k(\theta_k) = e^{\frac{1}{\sigma^2}\mathrm{Re}\left[r_k e^{-j\theta_k}\right]} \sum_{i=0}^{M-1}\sum_{m=0}^{M-1} P\left(c_k = e^{j\frac{2\pi}{M}i}\right) q_{f,k-1}^{(m)} \cdot t\left(a'_{f,k-1};\theta_k - \frac{2\pi}{M}(m+i)\right) \quad (23)$$

where $a'_{f,k-1} = \frac{a_{f,k-1}}{1+\sigma_\Delta^2|a'_{f,k-1}|}$. By now changing the first summation index in $n = (i+m)_{\mathrm{mod}M}$, using (19) and (21), and discarding irrelevant multiplicative factors, we have

$$\alpha_k(\theta_k) = \sum_{n=0}^{M-1}\left[\sum_{i=0}^{M-1} P\left(c_k = e^{j\frac{2\pi}{M}i}\right) q_{f,k-1}^{(n-i)_{\mathrm{mod}M}}\right] \cdot e^{\,a'_{f,k-1}e^{j\frac{2\pi}{M}n}+\frac{r_k}{\sigma^2}}\; t\left(a'_{f,k-1}e^{j\frac{2\pi}{M}n} + \frac{r_k}{\sigma^2};\theta_k\right) . \quad (24)$$

This resulting $\alpha_k(\theta_k)$ is not in the constrained form (17). However, by applying the approximation (22), we obtain the following updating equations for the parameters of the canonical distribution (17)

$$q_{f,k}^{(m)} \propto \left[\sum_{i=0}^{M-1} P\left(c_k = e^{j\frac{2\pi}{M}i}\right) q_{f,k-1}^{(m-i)_{\mathrm{mod}M}}\right] \cdot e^{\,a'_{f,k-1}e^{j\frac{2\pi}{M}m}+\frac{r_k}{\sigma^2}} , \quad m = 0,\ldots,M-1 \quad (25)$$

$$a_{f,k} = a'_{f,k-1} + \frac{r_k}{\sigma^2}\sum_m q_{f,k}^{(m)} e^{-j\frac{2\pi}{M}m} . \quad (26)$$

It is worth noticing that, before the evaluation of the coefficient $a_{f,k}$, the M real coefficients $q_{f,k}^{(m)}$ evaluated through (25) have to be normalized so that their sum is 1. Moreover the following initial values of the recursive coefficients result

$$q_{f,0}^{(m)} = \delta_m \qquad a_{f,0} = \frac{r_0}{\sigma^2} \quad (27)$$

where δ_m represents the Kronecker delta.

Similarly, it is also possible to find the backward recursive equations. Due to the lack of space, we report here only the final expressions

$$q_{b,k-1}^{(m)} \propto \left[\sum_{i=0}^{M-1} P\left(c_k = e^{j\frac{2\pi}{M}i}\right) q_{b,k}^{(m+i)_{\mathrm{mod}M}}\right] \cdot e^{\,a'_{b,k}e^{j\frac{2\pi}{M}m}+\frac{r_{k-1}}{\sigma^2}} , \quad m = 0,\ldots,M-1 \quad (28)$$

$$a_{b,k-1} = a'_{b,k} + \frac{r_{k-1}}{\sigma^2}\sum_m q_{b,k-1}^{(m)} e^{-j\frac{2\pi}{M}m} . \quad (29)$$

having defined $a'_{b,k} = \frac{a_{b,k}}{1+\sigma_\Delta^2|a_{b,k}|}$. The initial values of the backward coefficients are

$$q_{b,K}^{(m)} = \delta_m \qquad a_{b,K} = \frac{r_K}{\sigma^2} \quad (30)$$

Finally, substituting (17) and (18) into (16) and discarding irrelevant constants, the extrinsic information is evaluated as

$$\frac{P(c_k = e^{j\frac{2\pi}{M}i}|\mathbf{r})}{P(c_k = e^{j\frac{2\pi}{M}i})} \propto \sum_{m=0}^{M-1}\sum_{\ell=0}^{M-1} q_{f,k-1}^{(m)} q_{b,k}^{(\ell)} \cdot \mathrm{I}_0\left(\left|a'_{f,k-1} + a'_{b,k}e^{j\frac{2\pi}{M}(\ell-m-i)}\right|\right) . \quad (31)$$

In summary, this detection algorithm is based on three steps: a forward recursion in which, for each time epoch k, M real and one complex coefficients are evaluated based on (25) and (26), a backward recursion, based on (28) and (29), which proceeds similarly, and finally a completion (31), which consists of the sum of M^2 terms (although only a small amount of them is numerically significant, particularly from the second iteration ahead). This algorithm entails a minor complexity increase with respect to the known-phase MAP symbol detector [11] and will be denoted to as "algorithm based on Tikhonov parameterization" (*Tikh-algorithm*).

V. Numerical Results

In this section, the performance of the proposed algorithms is assessed by computer simulations in terms of bit error rate (BER) versus E_b/N_0, E_b being the received signal energy per information bit and N_0 the one-sided noise power spectral density. The sequence $\mathbf{c}$ is now assumed a codeword, possibly interleaved, of a channel code $\mathcal{C}$ constructed over the M-PSK modulation constellation. In particular, in Fig. 2 we consider a serially concatenated scheme composed by a convolutional code (CC), an interleaver, and a differentially encoded BPSK. The CC is a rate-1/2 non-recursive systematic code with 4 states and generators $(5,7)$. A uniform puncturing of its parity bits is used to obtain a rate-$2/3$ code. The codewords are composed of 16200 bits. Iterative detection/decoding is applied at the receiver using the proposed schemes to perform joint detection and decoding of the differential code. A maximum of 15 iterations is allowed and the phase noise affecting the channel is modeled as a Wiener process with $\sigma_\Delta = 6$ degrees. From Fig. 2 it can be observed that, despite the presence of this strong phase noise, the low-complexity *Tikh-algorithm* exhibits only a negligible performance loss with respect to the known-phase case. The performance of the *dp-algorithm* is practically coincident with that of the *Tikh-algorithm*. For comparison purposes, we also show (curve labeled *Tikh-sep-algorithm*) the performance of the algorithm in [8] also based on Tikhonov parameterization but performing a phase tracking separate from the decoding of the differential code. In order for this algorithm to bootstrap, a pilot symbol every 20 code symbols has been inserted in the frame, thus decreasing the effective information rate. This results in an increase in the required signal-to-noise ratio of about 0.21 dB.

In Fig. 3 we consider a non-systematic rate-1/2 irregular repeat and accumulate (RA) code [10] mapped on a QPSK modulation before the differential encoder. The codewords have length 16200 symbols. The RA code is defined by the degree distributions (see [16] for details) reported in Table I and the code has been designed following the approach in [16]. The differentially encoded QPSK symbols are transmitted over a channel affected by the DVB-S2 compliant phase noise model assuming a baud rate of 10 MBaud [8]. Although

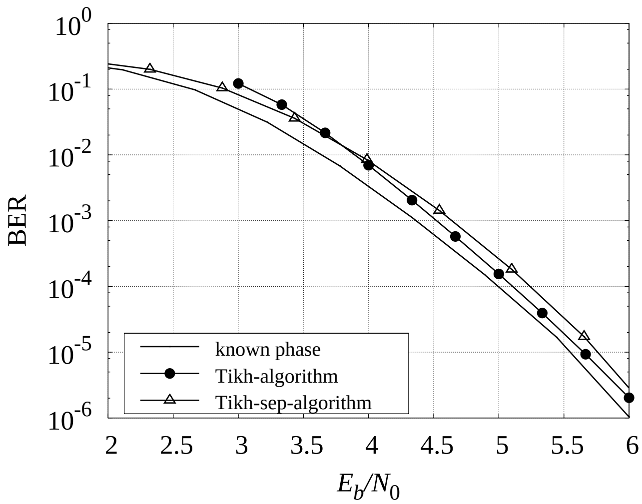

Fig. 2. Performance for BPSK signals

TABLE I

DEGREE DISTRIBUTIONS OF THE EMPLOYED RA CODE.

$d_{v,1} = 3$	$a_{v,1} = 0.8126$	$d_{c,1} = 1$	$a_{c,1} = 0.2000$
$d_{v,2} = 4$	$a_{v,2} = 0.1375$	$d_{c,2} = 3$	$a_{c,2} = 0.8000$
$d_{v,3} = 15$	$a_{v,3} = 0.0294$		
$d_{v,4} = 20$	$a_{v,4} = 0.0204$		

the Wiener model does not apply to this case, the proposed algorithms work well with a properly optimized value of $\sigma_\Delta = 0.6$ degrees. From the figure it can be observed that the *Tikh-algorithm* has about the same performance of the much more complex *dp-algorithm* and the two algorithms exhibit a loss with respect to the known phase case of only 0.1 dB. We would like to point out that this very good result has been obtained without the insertion of pilot symbols. As in Fig. 2, a performance curve (labeled *Tikh-sep-algorithm*) for separate differential decoding and phase tracking based on the algorithm in [8] is reported, where one pilot every 40 code symbols has been inserted to allow the algorithm bootstrap.

As a term of comparison, we considered only the *Tikh-sep-algorithm* since, as shown in [8], it outperforms those in [4]–[7]. Computer simulations, not reported here for a lack of space, also show that our algorithms outperform those in [2], [3]. Regarding the algorithm in [1], our *dp-algorithm* exhibits, for a same number of quantization levels, the same performance but a much lower complexity.

VI. CONCLUSIONS

In this paper, the problem of MAP symbol detection for differentially encoded M-PSK signals transmitted over a channel affected by phase noise has been faced. A simplified, although exact, version of the algorithm has been derived based on a forward-backward single estimation of the phase probability density function and a final completion. For the practical implementation of the forward-backward estimator, two algorithms have been proposed. The first one is based on the phase discretization and becomes optimal for a large enough number of discretization levels. To reduce the computational complexity, some approximations have been introduced in order to derive a new algorithm which exhibits a very good

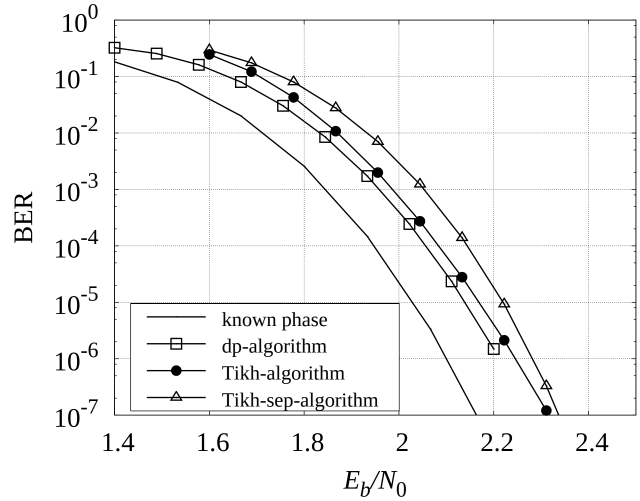

Fig. 3. Performance for QPSK signals

performance and a very low complexity, with only a minor increase with respect to the case of a perfectly-known phase.

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