14th European Signal Processing Conference (EUSIPCO 2006), Florence, Italy, September 4-8, 2006, copyright by EURASIP

PERFORMANCE OF THE SUPER STABLE ORBITS BASED SPREADING SEQUENCES IN A DS-CDMA SYSTEM WITH A MMSE RECEIVER

Zouhair Ben Jemaa, Sylvie Marcos*, and Safya Belghith+*

*Laboratoire des Signaux et Systems, Supélec, CNRS UMR8506
Plateau du Moulon 3, rue Joliot-Curie 91192 Gif-sur-Yvette cedex
phone: + 33 [0]1 69 85 12 12, email: ben-jemaa@lss.supelec.fr
+Laboratoire SysCom , Ecole Nationale d'Ingénieurs de Tunis
Campus Universitaire, 1060 Tunis

ABSTRACT

It has been shown in [12] that quantized selected Super Stable Orbits (SSO) of the logistic map allow better performance in terms of the Average Interference Parameter (AIP) criterion than those allowed by Gold Sequences, when a conventional or decorrelator receiver is considered in a DS-CDMA system. In this paper we analyse the performance of the MMSE receiver when these orbits are considered. The motivation is that the performance of the MMSE receiver in term of error rate depends on the AIP. We find that the performance of the MMSE can be improved by considering non classical spreading sequences.

1. INTRODUCTION

It is well known, in the literature of spread spectrum systems that the performance of the receivers depends strongly on the correlation properties of the used pseudo-noise sequence set. Many kinds of classical sequences have been proposed, but their properties in terms of correlation functions are different, i.e Maximum and Gold sequences have good autocorrelation properties but their intercorrelation is poor while it is the inverse for the Golay and Hadamar sequences. In this context, some researchers have been interested during the last decade in exploiting statistical properties of sequences generated by chaotic systems. It has been shown in previous works [3][10][11] that using truncated and quantized chaotic sequences generated by a one dimension map as spreading sequences in a DS-CDMA system, allows a better ensemble averaging performance in terms of bit error rate (BER) compared to the classical sequences.

In [8], unstable periodic orbits (UPO) were considered to provide good spreading sequences. But the task of finding the set of UPO is difficult especially when the periods are large. In [10], similarities between the Super Stable Orbits (SSO) set and the UPO or quantized chaotic sequences set of the logistic map were presented and the motivation for using SSOs was given. These orbits have been considered for two receivers in the DS-CDMA system : the conventional [11] and the decorrelator [12] receivers. In both cases, it was found that the selected SSOs-based sequences allow better performance than Gold sequences.

In this paper, we consider binary sequences corresponding to SSOs of the logistic map in the MMSE multiuser detection receiver. As done in [11-12], the SSOs are chosen to have a better average interference parameter (AIP) than the Gold sequences. We here analyze the effect of these sequences on the MMSE receiver performance.

The paper is organized as follows. In the second section, the DS-CDMA system and the relation between the AIP and the MMSE are briefly recalled. In section 3, the interest of using chaotic systems to generate spread spectrum sequences is resumed. Section 4 is devoted to the motivation of using SSOs based spreading sequences. The performance of the MMSE receiver with the proposed SSOs sequences is analyzed in section 5. Section 6 is the conclusion.

2. PERFORMANCE OF THE MMSE RECEIVER IN TERMS OF AIP

2.1 The DS-CDMA model

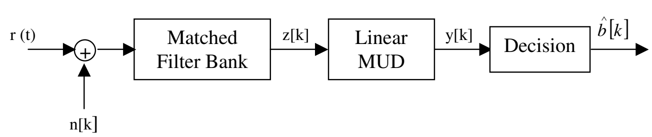

Figure 1: DS-CDMA Model.

The received signal of a Direct Sequence – Code Division Multiple Access (DS-CDMA) system is

$$r(t) = \sum_{k=0}^{K} r_k(t) + n(t) \tag{1}$$

where K is the number of users, $n(t)$ is the noise introduced by the channel and supposed to be an AWGN of zero mean and variance σ^2, $r_k(t)$ is the received signal corresponding to user k and given by

$$r_k(t) = \sqrt{2P} b_k(t-\tau_k) a_k(t-\tau_k) exp(j\omega_0(t-\tau_k)+\phi_k)$$

τ_k is the delay introduced by the channel for user k and P is the common signal power of all users,

$$a_k(t) = \sum_{j=-\infty}^{+\infty} a_k[j] P_{T_c}(t - jT_c)$$

$$b_k(t) = \sum_{j=-M}^{+M} b_k[j] P_{T_b}(t - jT_b)$$

$2M+1$ is the transmitted symbol number, $b_k[j]$ is the information signal of user k at time j and $a_k[j]$ is the spread spectrum code; periods T_b and T_c are related by $T_b = NT_c$, N is the spreading factor.

For the MMSE receiver, the estimated symbol $\widehat{b}_k[j]$ is obtained by minimizing the mean square error

$$MSE_k = E[(b_k[j] - y_k[j])^2] \qquad (2)$$

where $y_k[j]$ is the output of the multiuser detector (MUD) receiver at time j matched to user k. This minimum is noted by $MMSE_k$.

2.2 AIP criterion

By averaging (2) over all users and delays, it has been proved in [6] that

$$\overline{MMSE} \leq \frac{\sigma^2}{1+\sigma^2} + \frac{1}{(1+\sigma^2)^2}\frac{K-1}{3N^3}AIP \qquad (3)$$

where the AIP is defined by

$$AIP = \frac{1}{K(K-1)} \sum_{k} \sum_{i \neq k} \left(2\mu_{k,i}(0) + \mu_{k,i}(1)\right) \qquad (4)$$

$$\mu_{k,i}(n) = \sum_{l=1-N}^{N-1} C_{k,i}(l) C_{k,i}(l+n)^*$$

$C_{k,i}$ is the aperiodic autocorrelation (AAC) function of the sequences a_k and a_i

$$C_{k,i}(l) = \begin{cases} \sum_{j=0}^{N-l-1} a_k[j] a_i[j+l]^* \; if \; 0 \leq l \leq N-1 \\ \sum_{j=0}^{N+l-1} a_k[j-l] a_i[j]^* \; if \; 1-N \leq l \leq -1 \\ 0 \qquad otherwise \end{cases}$$

From (3), it is obvious that the minimization of $\overline{MMSE}$ may be obtained by minimizing the AIP criterion.

3. CHAOTIC SYSTEMS BASED SPREADING SEQUENCES

The idea of using chaotic systems to generate spread spectrum sequences has been treated in previous works [2-4,7-11]. In particular, it was shown in [3,10] that truncated and quantized chaotic sequences generated by a particular piecewise map allow average performance that is superior to that of Gold sequences. In [2,10], the shape of the (AAC) function which optimizes the AIP criterion has been determined, it is of the form

$$C(n) = (N - |n|)(-r)^n \qquad (5)$$

where $r = 2-\sqrt{3}$. The AIP of a sequence set with this (AAC) function is given by

$$AIP_{opt} = \frac{\sqrt{3}}{3N} \qquad (6)$$

For independent identically distributed (iid) sequences, i.e. for which the (AAC) function is null outside zero, the AIP is

$$AIP_{iid} = \frac{2}{3N} \qquad (7)$$

From formulae (6) and (7) it is found that we win $15,4$ percent by using sequences with aperiodic auto-correlation function (5) compared to using i.i.d. sequences. This result shows that it is possible to improve the average of the performance in term of AIP. Two important results have been shown in [2,10],

- for an N-Markov map defined by

  $$f_n(x) = (nx)mod(1)$$

  the average over all sequences of the AAC function is null outside zero and thus the corresponding sequences have the same average correlation properties as i.i.d. sequences.
- if we consider a (n,m)-Markov map defined by

  $$f_{n,m}(x) =$$

  $$\begin{cases} [(n-m)x]mod(\frac{n-m}{n}) + \frac{m}{n}, \;\; if \; 0 \leq x \leq \frac{n-m}{n} \\ m[x - \frac{n-m}{n}]mod(\frac{m}{n}), \;\; otherwise \end{cases}$$

  the average of the AAC function is equal to the optimum given by formula (5), and thus the AIP is the same as (6)

The difficulty of this approach consists in finding good initial conditions, i.e. allowing performance better or equal to the average. The AIP of a randomly chosen set of sequences generated by a Markov map is also random, it depends strongly on initial conditions as will be shown later.

Due to the similarity between the truncated chaotic sequence set and the UPO set the later has been considered rather than the former [8]. But the task of finding the set of UPOs is difficult even when the periods are small. This is due to the fact that the set of UPOs is dense in the strange attractor and is of measure zero. Moreover, high precision is required to locate them. Therefore it is useful to find another set of periodic solutions that has the same properties and is easier to manipulate. In this work, we considered the SSOs of the logistic map. In the next section we recall some properties of the logistic map SSOs and give the motivation of using them instead of UPOs.

4. SSOS BASED SPREADING SEQUENCES

Let us begin by recalling the non linear system defined by the logistic map

$$x_{k+1} = f_a(x_k) \qquad (8)$$

where

$$f_a(x) = 1 - ax^2, 0 \leq a \leq 2 \qquad (9)$$

a is the bifurcation parameter.

4.1 Periodic Orbits of the logistic map

Let $x_k,\ k = 0,1,...$ be the sequence defined by the initial condition x_0 and the recurrence (8). A p-periodic orbit is a set of p ordered points $X = x_0 x_1 ... x_{p-1}$ satisfying $f_a(x_i) = x_{i+1}$ for all $0 \leq i \leq p-2$ and $f_a(x_{p-1}) = x_0$. So, beginning by any point x_i, the sequence defined by (8) is the repetition of the periodic orbit X; it is noted by $\overline{x_0 x_1 ... x_{p-1}}$.

4.2 Quantized Super Stable Orbits

The stability of a p periodic orbit $X = \overline{x_0 x_1 ... x_{p-1}}$ is defined by its Floquet multiplier (FM) [1]:

$$\lambda(X) = \frac{d f_a^p}{dx}(x_i) = \prod_{k=0}^{p-1} f_a'(x_k)$$

where f_a' is the derivative of f_a' and f_a^p is the p times composition of f_a.

We have the following three possible situations:

- $|\lambda(X)| < 1$, the orbit X is stable
- $|\lambda(X)| > 1$, the orbit X is unstable
- $|\lambda(X)| = 1$, the orbit X can be stable or unstable

When $\lambda(X) = 0$, X is said to be super-stable. It is obvious that a periodic orbit is super-stable if and only if it contains the critical point 0. So a SSO is always of the form $\overline{1x_1 ... x_{p-2}0}$, i.e the initial condition is always $x_0 = 1$. The SSOs differ by their bifurcation parameter a as will be shown in the next sections. In this work we consider binary sequences corresponding to SSOs using the following rule

$$\begin{cases} S_i = 1 \; if \; x_i > 0 \\ S_i = -1 \; if \; x_i \leq 0 \end{cases} \tag{10}$$

4.3 Motivation of using SSOs

For the logistic map, there exist two types of bifurcations; the tangential bifurcation (TB) and the period doubling bifurcation (PDB) (fig. 2). For the TB (point T), two periodic orbits appear at some value of the parameter a both with a FM equal to $+1$, for one orbit the FM increases by increasing a and it becomes instable (dashed line), for the other the FM decreases to achieve 0 (S_1), this is equivalent to say that one point of the periodic orbit crosses the critical point 0, the stable orbit becomes superstable, and by increasing a further the FM decreases to achieve -1, where the second type of bifurcation occurs. For the PDB (points D_1 and D_2) two things happen. The p-periodic orbit looses its stability and becomes instable with a FM < -1 and a $2p$-periodic orbit appears with a FM equal to $+1$ which will decrease and we find the same situation as above. Note that when an unstable p-periodic orbit appears for a value a_* of a, it will continue to exist but remains unstable when a increases, and it will have a trace in the strange attractor corresponding to every value $a_c > a_*$ of a where the map is chaotic. Thus, using these remarks, it is possible to associate to every UPO existing for a value a_c where the logistic map is chaotic a SSO from which it was born. It has been shown in [12] that a SSO and the corresponding UPO have the same correlation properties especially when they are quantized. This is the motivation of using quantized SSOs of the logistic map as spreading codes.

5. PERFORMANCE OF THE SELECTED QUANTIZED SSOS

We selected the SSOs that will be quantized and used as spreading codes as follows. We calculated a great number (1000) of SSOs using the method called *word lifting technic* described in [1], then we kept the 100 best ones with respect to the figure of merit (FOM). The FOM of a sequence $a_k[j]$

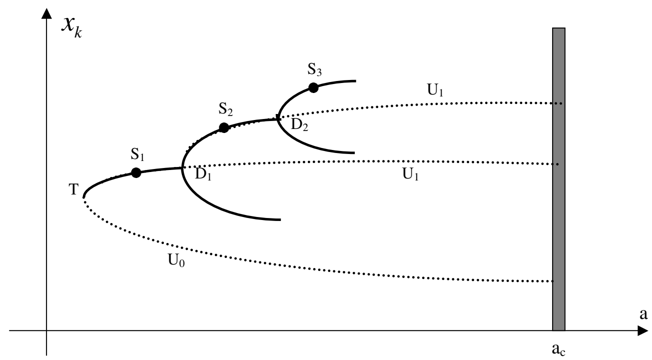

Figure 2: Bifurcations of the logistic map

is defined by

$$FOM_k = \frac{C_k(0)^2}{\sum_{j=1}^{N-1} 2C_k(j)^2} \tag{11}$$

C_k is the AAC function of a_k. Then we considered randomly two sequences among the 100 selected ones and added the sequence among the others which gives the minimum AIP, then we added to the three obtained sequences a new sequence which gives the minimum AIP, and so on. To compare our results to the results obtained when using classical Gold sequences and quantized Marcov sequences we ordered them using AIP as we did with quantized SSOs, i.e. to each subset of sequences we add the sequence selected from the remaining ones and which minimizes the AIP of the new subset. In figure 3, are plotted the AIPs of theses different sequences, the horizontal continue line corresponds to the optimum average AIP given by formula (6). When we choose randomly $(10,2)$ Markov sequences, the AIP oscillates around this value. We can see in the figure that Gold sequences behave better than the sequences with AAC (6) which are optimal in the mean sense, when the number of users is less than 13, but become poor after this value. This could be explained by the fact that Gold sequences are optimized with respect to the periodic correlation function. From the curve corresponding to the $(10,2)$ Markov ordered with respect to the AIP as described in the beginning of this section, we can see that it is possible to obtain a set of sequences with AIP much less than the average optimal one given by (6). We can see also that quantized SSOs allow the best AIP when compared to the other considered sequences, especially when the number of user is reduced. Indeed, these orbits were selected in the beginning such that they have good FOM which as can be seen from formula (11), is intuitively related to the AIP. To see the influence of these results on the performance of the MMSE receiver, we computed its BER for the four types of sequences, Gold, randomly chosen $(10,2)$ Markov sequences, Markov Sequences and quantized SSOs. In figure 4 are plotted the BER curves of these sequences, the randomly chosen $(10,2)$ Markov sequences give the worse BER and quantized selected SSOs the best one, this can be explained by the results shown in figure 3.

6. CONCLUSION

Quantized SSOs have been considered as spreading sequences in a DS-CDMA system. These orbits were selected such that they allow a good FOM and are ordered with respect to the AIP criterion as described in section 5. To compare the performance of the MMSE receiver when using the proposed SSOs, to the performance when using Gold sequences and sequences generated by the $(10,2)$ Markov chaotic system, we ordered theses sequences as done for SSOs. It has been shown that the MMSE receiver performance in term of BER is improved when using SSOs. In a further work we will study the performance of these orbits from the synchronization point of view.

REFERENCES

[1] B. L. Hao, *Elementary Dynamic Symbolics and Chaos in Dissipative Systems.* World Scientific, 1989

[2] R. Rovatti and G. Mazzini, "Interference Minimization based on Correlation Shaping in Asynchrone DS-CDMA," *Electronic Letters*, vol. 35 N13, 1998.

[3] G. Mazzini, G. Setti ans R. Rovatti, "Chaotic Complex Spreading Sequences for asynchronous DS-CDMA-Part I: System Modeling and Results," In *IEEE. Trans. Circ. Syst. -Part I*, Vol. 44. pp. 937–947. Oct. 1997

[4] T. Yang and L. O. Chua, "Chaotic digital code division multiple access (CDMA) communication systems," In *Int. J. Bifurcat. Chaos*, Vol. 7, n. 12, pp. 2789–2805. 1997

[5] M. V. Poor, S. Verdu, "Probability of Error in MMSE Multiuser Detection," *IEEE Transaction on Information Theory*, vol. 43, No. 3 pp. 858–871, May 1997.

[6] H. Elder-Boll, A. Busboom, and H. Schotten, "Spreading Sequences for Zero-Forcing DS-CDMA Multi-user Detectors," in *Proc. IEEE PIMRC Helsinky, Finland*, 1997 pp. 53–57,

[7] D. S. Broomhead, J. P. Huke and M. R. Muldoon, "Codes for spread Spectrum applications generated using chaotic dynamical systems," In *Dynam. Stab. Syst*, Vol. 14, n. 1, pp. 95–105, March 1999

[8] T. L. Caroll, "Spread spectrum Sequences for Unstable Periodic Orbits," In *IEEE. Trans. Circ. Syst. -Part I*, Vol. 47, n. 4, pp. 443–447, April. 2000

[9] C-C Chen, K. Yao, K. Umeno and E. Biglieri, "Design of Spread Systems Using Chaotic Dynamical Systems and Ergodic Theoty," In *IEEE. Trans. Circ. Syst. -Part I*, Vol. 48. N9. pp. 1110–1114. sept. 2001

[10] M. Khanfouci, "Séquences chaotiques pour les systèmes CDMA," In *Thesis*, University Paris XI, Orsay, January. 2005

[11] M. Khanfouci and S. Marcos, "PLM Sequences for the Performance Optimization of Linear Multi-user Detector," In *EUSIPCO'2005 Proceeding*, Septembre. 2005

[12] Z. Ben Jemaa and S. Belghith, "Exploring Unstable Periodic Orbit Using Super Stable Ones," In *Nonlinear Dynamics of Electronic Systems NDES'2002 Proceeding*, pp. 3-21–3-24, June. 2002

[13] Z. Ben Jemaa and S. Belghith, "Correlation properties of Binary Sequences Generated by the Logistic map-Application to DS-CDMA," In *Nonlinear Signal and Image Processing NSIP'2003 Proceeding*, June. 2003

[14] Z. Ben Jemaa and S. Belghith, "Performance of Super Stable Orbits in Decorrelating Receiver DS-CDMA System," In *Nonlinear Dynamics of Electronic Systems NDES'2003 Proceeding*, June. 2003

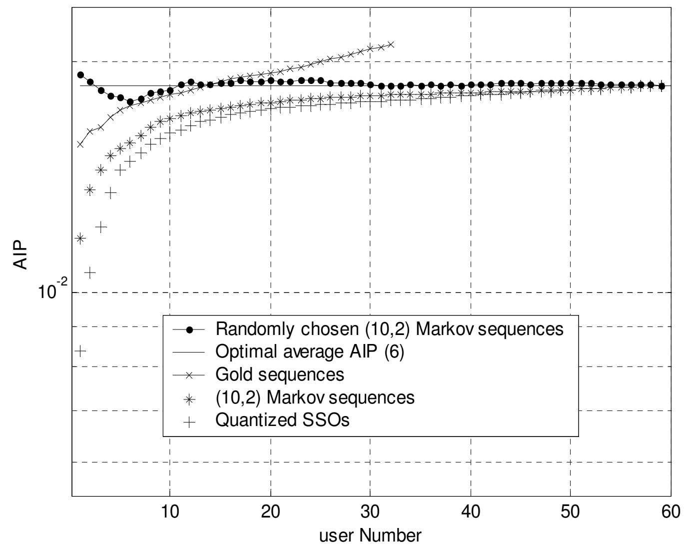

Figure 3: **AIP versus user number for N=31**

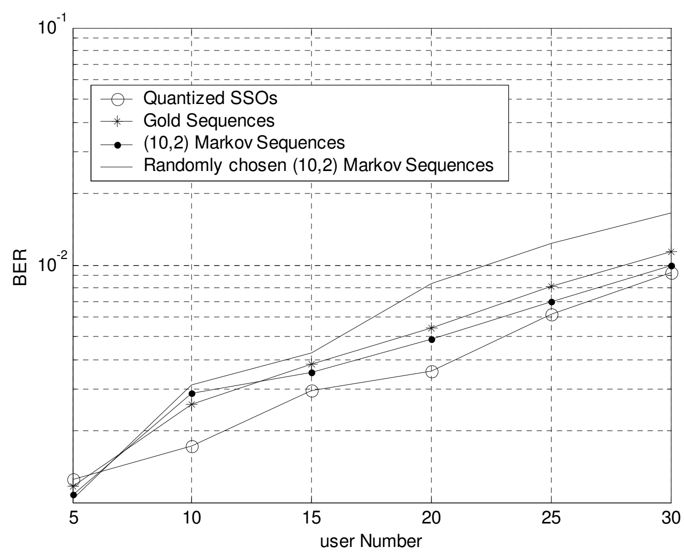

Figure 4: **BER of the MMSE receiver versus user number for N=31**