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DETECTION OF ANOMALOUS EVENTS IN BIOMEDICAL SIGNALS BY WIGNER ANALYSIS AND INSTANT-WISE RÉNYI ENTROPY

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ABSTRACT

Rényi entropy is receiving an important attention as a data analysis tool in many practical applications, due to its relevant properties when dealing with time-frequency representations (TFR). Rényi entropy is characterized for providing generalized information contents (entropy) of a given signal. In this paper we present our results from applying the Rényi entropy to a 1-D pseudo-Wigner distribution (PWD) of a biomedical signal. A processed filtered signal is obtained by the application of a Rényi entropy measure to the instant-wise PWD of the given biomedical signal. The Rényi entropy allows individually identify, from an entropic criterion, which instants have a higher amount of information along the temporal data. Our method makes possible accurate localization of normal and pathological events in biomedical signals; hence early diagnosis of diseases is facilitated this way. The utility of our method is illustrated with examples of application to phonocardiograms.

1. INTRODUCTION

Biomedical signals contain important information about the healthy condition of human beings. Anomalous events in these signals are commonly associated to diseases. Today's technology offers the opportunity to digitize and analyze biomedical signals. Consequently, the anomalous events can be automatically detected in real time and the results conveniently transmitted elsewhere for further processing, if necessary. Studies on the application of time-frequency analysis techniques, including entropy measures, have been carried out to show the time-varying properties of the biomedical signals [1, 2]. In such time-frequency analysis, the Rényi entropy appears to be a suitable measure to identify information performances. However, the Rényi entropy can be approached under different normalizations, originating different representations. This paper proposes one specific normalization, under an instantaneous-wise basis, as the most suitable among others for digital analysis of biomedical signals. The aim of our method is to accurately detect anomalies through the discordances observed between a normal regular pattern of the signal and its abnormalities. This discrimination is accomplished by the use of mathematical tools such as the pseudo-Wigner distribution (PWD) and the Rényi entropy.

An important feature of the PWD is that it can provide a straightforward instantaneous-wise time-frequency representation of a given signal, especially suitable for non-stationary signal analysis. On the other hand, generalized Rényi entropy measurements can provide a quantitative criteria to evaluate the importance of the information in a given moment *t* that can be used for adaptive and automatic parameter's selection in time-frequency analysis [3]. In this paper, we describe the application of this measure to the problem of detecting anomalous events in heart sound recordings, through the instantaneous frequency information provided by the PWD of the biomedical signal.

This paper is structured as follows. A mathematical background is outlined in section 2. The method is described in section 3. Section 4 presents the experimental results obtained by using the described method. Conclusions and future research work in this area are drawn in section 5.

2. THEORETICAL BACKGROUND

2.1 The pseudo-Wigner distribution

Our method is based on the use of a time-recorded digitized input of a biomedical signal. Frequency based information of the signal is obtained by associating to a given instant *t* a vector containing its discrete pseudo-Wigner distribution (PWD). Subsequently, the Rényi entropy of the instantaneous PWD indicates the degree of information contents of the processed signal.

The following discrete approximation to the Wigner distribution [4] proposed by Claasen and Mecklembräuker [5], similar to Brenner's expression [6], is used here:

$$W(n,k) = 2\sum_{m=-\frac{N}{2}}^{\frac{N}{2}-1} z(n+m)z^*(n-m)e^{-i2\pi k(\frac{2m}{N})} \quad (1)$$

Equation 1 can be interpreted as the discrete Fourier transform (DFT) of product $r(n,m)= z(n+m)z^*(n-m)$. Here z^* indicates the complex-conjugate of signal z. This equation is limited to a spatial interval [$-N/2,N/2-1$]. In Eq. (1), n and k represent the time and frequency discrete variables respectively, and m is a shifting parameter, which is also discrete. $W(n,k)$ is a matrix where every row is a vector representing the instant-wise PWD of instant n. By scanning the temporal signal with a 1-D window of N data, i.e., by shifting the window to all possible positions along the signal, the full instant-wise PWD of the signal is obtained.

Figure 1 shows an example of the graphical representation of the PWD at a given instant n of a temporal signal. The PWD will have different magnitude coefficients at every position $t = n$ due to the data value changes along the time signal. A way to measure such differences in the PWD is to define an instant-wise measure such as, for example, the Rényi entropy of such instantaneous PWD.

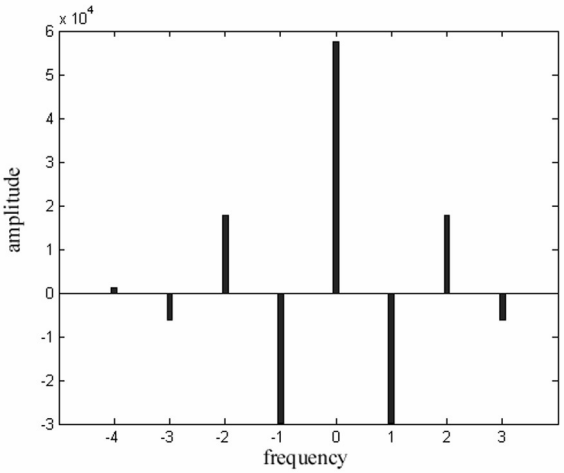

Figure 1. PWD of a given signal at time $t = n$

2.2 Rényi entropy measures

The entropy measure was initially proposed independently by Shannon [7] and Wiener [8] as a measure of the information contents per symbol, coming from a stochastic information source. Later, Rényi [9] extended this notion to yield the generalized entropy. Time-frequency distribution measures are defined in analogy to the Rényi entropy measure and have been introduced in the time-frequency analysis area by *Williams et al.* [10-13], with a significant contribution of *P. Flandrin et al.* in establishing the properties of this measure [14]. The use of entropy measures in positive time-frequency representations was introduced by Pitton et al. [15]. The Rényi entropy measure applied to a time-frequency distribution $P(t,\omega)$ has the form

$$R_\alpha = \frac{1}{1-\alpha} log_2\left(\sum_n\sum_k P^\alpha(n,k)\right) \quad (2)$$

Here n is again the temporal discrete variable and k is the frequency discrete variable. $\alpha \geq 2$ are recommended values for time-frequency distribution measures [14]. Although the Rényi measures of time-frequency distributions formally resemble the original entropies, they do not have the same properties, conclusions and results derived from classical information theory. The positivity, $P(t,\omega) \geq 0$ will not be always preserved, along with the unity energy condition, $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(t,\omega)dtd\omega = 1$. In order to reduce a distribution to the unity signal energy case, a normalization procedure must be considered [10]. The normalization can be performed in various ways, leading to a variety of possible definitions of such measure [3, 14]:

a) Normalization with the signal energy

This normalization is given by,

$$RE_\alpha = \frac{1}{1-\alpha} log_2\left(\frac{\sum_n\sum_k P^\alpha(n,k)}{\sum_n\sum_k P(n,k)}\right) \quad (3)$$

with $\alpha \geq 2$.

The behavior of this measure is quite similar to the non-normalized measure form, except in its magnitude. This type of normalization is important for comparison among various distributions, or within the same distribution when the energy is not unbiased.

b) Normalization with the distribution volume

This normalization is given by,

$$RV_3 = -\frac{1}{2} log_2\left(\frac{\sum_n\sum_k P^3(n,k)}{\sum_n\sum_k |P(n,k)|}\right) \quad (4)$$

Such volume-normalized form has been used for adaptive kernel design [10]. Note that the term within the logarithm is just the ratio of norms L_3 and L_1, while the logarithm is a monotonic function. Thus, measure (b) can be considered as L_3 / L_1, and can be reduced to the general case.

c) Quantum normalization

Quantum Mechanics [16] inspires a normalization by assimilating the PWD of a given instant $t = n$ with a *wave function* and derive a *probability density function* by means of $\breve{P}(n,k) = PWD(n,k)PWD^*(n,k)$, followed by a normalizing step to satisfy the condition

$\sum_n \sum_k \breve{P}(n,k) = 1$. The general case in Eq. 2 with $\alpha = 3$, gives

$$\breve{R}_3 = -\frac{1}{2} log_2 \left(\sum_n \sum_k \breve{P}^3(n,k) \right) \quad (5)$$

This measure can be interpreted in a instantaneous basis as follows

$$\breve{R}_3(n) = -\frac{1}{2} log_2 \left(\sum_k \breve{P}^3(n,k) \right) \quad (6)$$

The term $\breve{P}$ in Eq. 6 has to be also normalized in an instantaneous way as follows,

$$Q(n,k) = PWD(n,k) PWD(n,k)^* \quad (7)$$

$$\breve{P}(n,k) = Q(n,k) \Big/ \sum_k Q(n,k) \quad (8)$$

in order to fulfill the normalizing condition,

$$\sum_k \breve{P}(n,k) = 1, \qquad \forall n : 1 \le n \le M \quad (9)$$

being M the size of the data and $-N/2 \le k \le N/2$.

Empirically this normalization has shown to be the most suitable procedure in the case of phonocardiograms, and it was effectively used in the experimental examples described therein.

3. DESCRIPTION OF THE METHOD

A phonocardiogram (PCG) is a graphical representation of the sounds produced by the heart. The generation of these sounds is caused not only by the opening and closure of the cardiac valves, but also by the vibrations of the whole cardiovascular system originated by blood pressure gradients [17]. A normal cardiac cycle has two main sounds, called S1 and S2, which define the start of systole and diastole, respectively. However, abnormalities and problems in the cardiac valves or walls generate turbulences in the blood flow, which are heard as *murmurs*. These murmurs are often pathologic and must be detected in order to provide proper treatment to the patient.

A common problem in the processing of the PCG is to detect the main events of the cardiac cycle (S1, S2, murmurs…) in order to diagnose the valvular state of the heart. Different methods have been developed to treat this problem [18], usually by applying a threshold and detecting events that are above the threshold. However, in pathological recordings, the delineation of the individual events is a difficult task, because the events are usually very close in time, and their separation may be not clear using threshold techniques. Thus, new methods have been developed [19], which are capable of accurately detecting the limits of the events, even though two or more events appear together or if murmurs have greater amplitude than S1 or S2.

The method that we propose provides valuable information about the delineation of the cardiac events, since it reveals their positions in an unprocessed signal. This method makes easier the task of event detection, because the simple analysis of the amplitude of the signal can originate confusing information about the situation of the events.

The biomedical signal to be treated requires a conditioning step, which consists in applying some bias to the signal in order to set up all values over zero. Signal downsampling is also possible for reducing the computational time. Once the biomedical signal has been digitized and biased, the PWD of the data is obtained through Eq. 1, by considering a sliding window of N values. Normally N will be a small number (e.g.: N = 8, 10, etc.), in order to ascertain temporal localization of events. Nevertheless, the exact value of this parameter will depend on the particular characteristics of the signal and the sampling frequency used to record it. For the experiments presented in the sequel, the window size for calculating the PWD was set to N=8.

After producing the PWD of the temporal signal, every instant n can be associated to a vector of N components, which represents its instantaneous PWD. Then, the Rényi entropy can be measured in a instantaneous basis, by following the *quantum normalization*, shown in Eq. 6.

This procedure originates a new temporal sequence, $\breve{R}_3(n)$, of data where normal and anomalous events can be better detected and visualized, by providing a more discriminative distinction between them. It is important to remark that parameters such as the window size or the bias amount given to the signal will originate changes in the shape and in the absolute value or the resulting entropy. The observed results have a notable meaning when comparing different signals submitted to identical processes. Additionally, it is essential to point out that the *quantum normalization* is the key of the powerful discriminative response of this method.

4. EXPERIMENTAL RESULTS

In this section some examples are presented to illustrate the way the proposed method works. Here the signals were biased to set their minimum values to "+1" and the window used to calculate the PWD was N = 8, as mentioned above.

Figure 2A presents two cardiac cycles of a normal PCG record. Fig. 2B shows the filtered signal obtained after applying the method to the phonocardiographic signal. Heart beatings are then identified as clean and independent events. The strength of our method lies in its capability to remark the events, while the rest of the signal (ambient

noise and internal noise) shows very low values, thus making easier the detection of events.

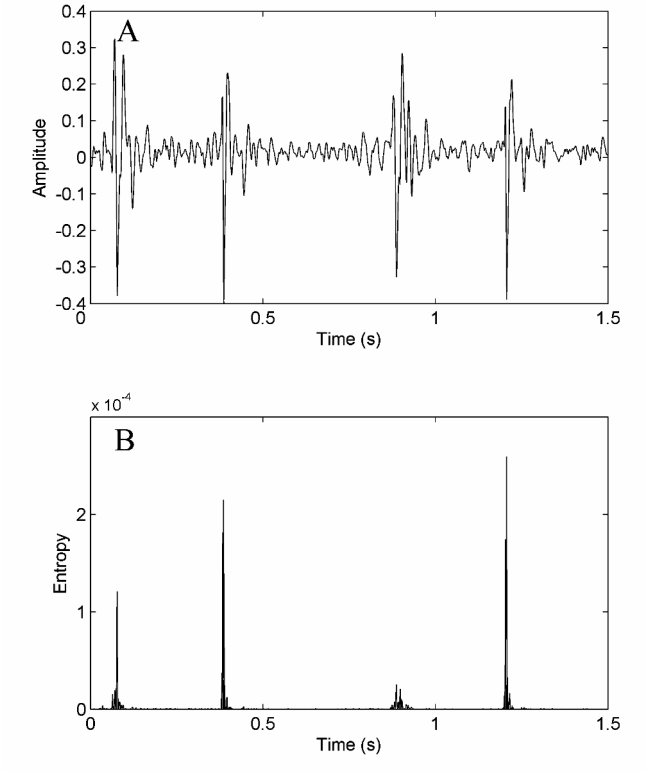

Figure 2. Example of events detection in a phonocardiographic signal belonging to a normal person. Two cardiac cycles are represented. A) Phonocardiographic signal. B) Events detected by the described method.

Another example is presented in Fig. 3. Here the figure represents two cardiac cycles of the PCG signal of a patient affected by aortic stenosis, showing a systolic murmur between S1 and S2 (Fig. 3A). The murmur has greater amplitude (and energy) than S1 and S2, thus a threshold-based method to detect events would not work properly.

Fig. 3B shows the detected events, S1 and S2, after filtering out the signal with our method. S1 appears joined to the systolic murmur, whereas the values for the rest of the signal are very near zero (silent periods). The amplitude in the silent periods due to internal and external noise is higher than the signal of Fig. 2A, but the filtering has set the values again to practically zero. This result is clearly different from the result in Fig. 2B. Murmurs extend through silent periods when compared with a normal signal as the one in Fig. 2B, for example. Differences between anomalous and normal signals are obvious from the results provided by our method, and this constitutes the basic result to the diagnosis of diseases by an expert trained clinician and provides an automatic device for diseases detection and identification.

The height of the resulting events seems to be related to their frequency contents. For example, S2 contains higher frequencies than S1. This is shown in Fig. 2B with S2 events higher than S1 ones, thus making easier to distinguish between S1 and S2 in PCG signals. Likewise, the systolic murmur due to aortic stenosis contains higher frequencies than S1 and S2. Fig. 3B shows this effect, with the amplitude of the murmur in the filtered signal being higher than the amplitude of S2 and S1.

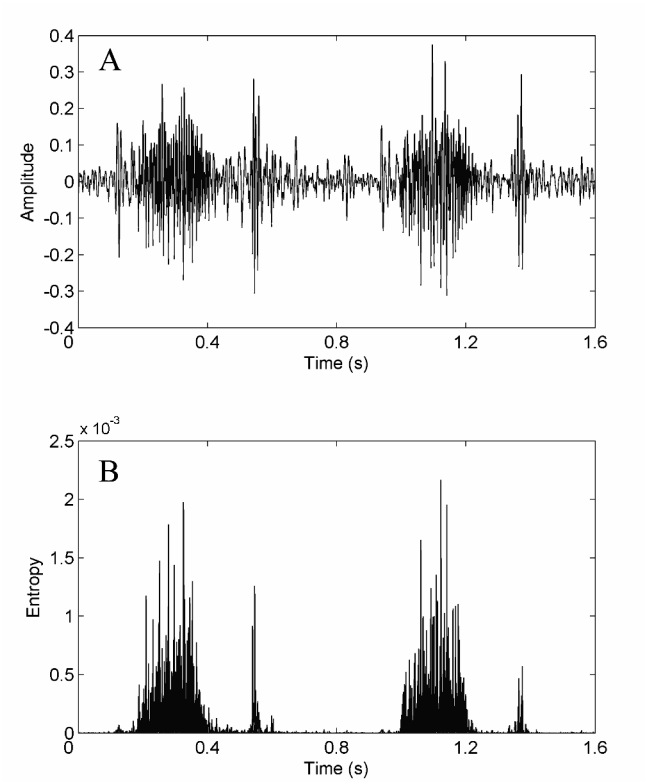

Figure 3. Example of events detection in a phonocardiographic signal belonging to a patient affected by a murmur due to aortic stenosis. Two cardiac cycles are represented. A) Phonocardiographic signal. B) Events detected by the described method.

5. CONCLUSIONS

Biomedical signals are characterized by presenting special instants with highly significant information identified as *events*. Detection of these events requires the most localized analysis. A new method of analysis based on a generalized entropy to the problem of events detection in biomedical signals has been presented in this paper. Examples show that this method can provide a way of identification and discrimination of normal and abnormal events in biomedical signals, useful for expert diagnosis or as input information to an automatic discriminating system. Future work, which will be published in due course, is focused on determining how the amplitude of the filtered signal is related to the amplitude and frequency of the original signal.

5. ACKNOWLEDGEMENTS

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