DOWNLINK BEAMFORMING UNDER EIRP CONSTRAINT IN WLAN OFDM SYSTEMS

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Abstract

Range extension by means of downlink beamforming in a wireless local area network (WLAN) orthogonal frequency-division multiplexing (OFDM) system is addressed. A maximum ratio (MR) combining beamformer scaled according to the equivalent isotropic radiated power (EIRP) constraint is compared to the conventional total power (TP) restricted solution in typical propagation conditions. It is shown that the EIRP-scaled MR beamformer demonstrates significant performance degradation compared to the TP case. A joint optimization problem for the beamforming weights over all the sub-carriers subject to the EIRP constraint is formulated based on maximization of the minimum signal-to-noise ratio (SNR) at the receiver. A two-stage sub-optimal solution is proposed that exploits optimization over groups of non-adjacent sub-carriers and normalization to maintain the overall EIRP constraint. Its efficiency is illustrated in the IEEE 802.11 environment.

1. INTRODUCTION

Downlink beamforming is one of smart antenna [1] techniques that allows range extension using knowledge of the propagation channel. In Time Division Duplex (TDD) systems like IEEE 802.11 [2] the channel knowledge can be obtained from channel reciprocity [3]. Recently, commercial smart antenna products became available promising significant range extension for WLAN systems.

In [4], [5], downlink beamforming is considered under different restrictions including the TP and EIRP constraints. This problem is important because different regulations define different constraints for different countries. For example, in the 2.4 GHz band the USA regulations [6] define the TP constraint subject to some restrictions on the antenna gain. The European regulations [7] specify 100 mW EIRP without any separate constraints on the power and antenna gain. In a free space environment this significantly limits range extension possibility. In the case of multipath propagation, the conventional EIRP-scaled beamforming still may be effective compared to the single antenna case depending on the environment [4].

This paper addresses downlink beamforming in WLAN TDD OFDM systems such as IEEE 802.11a/g [2]. An access point (AP) equipped with multiple antennas and a single antenna terminal are considered. By means of comparison of the conventional MR combining solution under the TP and EIRP constraints in the typical IEEE 802.11 propagation conditions [9], it is demonstrated that although the scaled according to the EIRP constraint MR beamformer can improve the performance compared to the conventional single antenna AP, there is a significant performance degradation compared to the TP restricted MR beamformer. The EIRP constraint for an OFDM system is defined and a joint optimization problem for the antenna weights over all the sub-carriers subject to this constraint is formulated. A sub-optimal solution is proposed that is based on grouping of non-adjacent sub-carriers with low channel correlation. Its efficiency is compared to the scaled MR beamformer as well as to the narrowband SOCP based solution from [5] applied to each sub-carrier separately.

The problem formulation is given in Section 2. A joint optimization problem over all the sub-carriers subject to the EIRP constraint is presented in Section 3. A sub-optimal two-stage solution is given in Section 4 and its complexity is analyzed in Section 5. The simulation results are presented in Section 6. Section 7 concludes the paper.

2. PROBLEM FORMULATION

The system model consisting of the AP with N transmit antennas and a single antenna terminal is illustrated in Figure 1

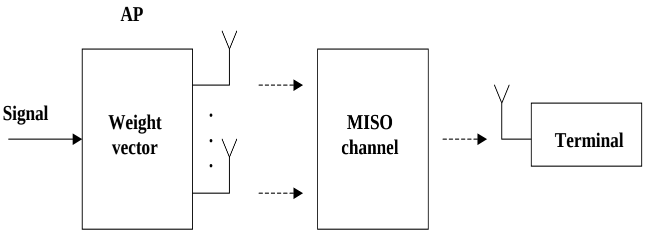

Figure 1: System model

A transmitted signal $\mathbf{x}(f)$ via the antenna of N elements is given by

$$\mathbf{x}(f) = \mathbf{w}(f)s(f), \quad (1)$$

where $\mathbf{w}(f)$ is the $N \times 1$ weight vector, $s(f)$ is the unit power transmitted symbol, $f = 1, \ldots, F$ is the sub-carrier number and F is the total number of working sub-carriers.

A conventional MR combining solution subject to the TP constraint can be formulated as follows:

$$\mathbf{w}_{\mathrm{TP}}(f) = \alpha(f)\sqrt{P_{\mathrm{T}}}\tilde{\mathbf{h}}^{*}(f), \quad (2)$$

$$\tilde{\mathbf{h}}(f) = \frac{\mathbf{h}(f)}{||\mathbf{h}(f)||}, \quad (3)$$

Part of this work has been done in the context of the IST 6FP OBAN project.

$$\sum_{f=1}^{F}\alpha^2(f)=1, \tag{4}$$

where $\mathbf{w}_{\text{TP}}(f)$ is the $N\times 1$ weight vector representing the TP solution, $\mathbf{h}(f)$ is the $1\times N$ vector representing the propagation channel at the fth sub-carrier, $\tilde{\mathbf{h}}(f)$ is the normalized channel vector, P_{T} is the total power constraint and $\alpha(f)$ is a power loading function between sub-carriers.

For simplicity, in this paper we address only uniform power distribution between sub-carriers for all the considered algorithms. In the TP case this means that $\alpha(f)=1/\sqrt{F}$.

Let us formulate the EIRP constraint for an OFDM system:

$$\max_{\Theta}\sum_{f=1}^{F}|\mathbf{w}^*(f)\mathbf{a}(\Theta,f)|^2<\text{EIRP}, \tag{5}$$

where $\mathbf{a}(\Theta,f)$ is the $N\times 1$ vector of array manifold depending on the antenna configuration, Θ is a direction-of-arrival (DOA) parameter, EIRP is the constraint. An example of the array manifold for a uniform linear array of omnidirectional elements is as follows:

$$\mathbf{a}(\Theta,f)=\left\{\begin{array}{c} 1 \\ e^{j\frac{2\pi d f_0(f)\sin(\Theta)}{c}} \\ \vdots \\ e^{j\frac{2\pi(N-1)d f_0(f)\sin(\Theta)}{c}} \end{array}\right\}, \tag{6}$$

where d is the distance between array elements, $f_0(f)$ is the fth sub-carrier frequency and c is the speed of light.

The simplest EIRP-restricted solution can be obtained by means of scaling the conventional TP beamformer (2) according the EIRP constraint:

$$\mathbf{w}_{\text{SMR}}(f)=\sqrt{\frac{\text{EIRP}}{\max_{\Theta}\sum_{f=1}^{F}|\mathbf{w}_{\text{TP}}^*(f)\mathbf{a}(\Theta,f)|^2}}\,\mathbf{w}_{\text{TP}}(f) \tag{7}$$

for $f=1,\ldots,F$.

This solution will be referred to the scaled MR (SMR) beamformer.

The main SMR disadvantage is that it is based on the TP solution optimized without taking into account the EIRP constraint. This means that in some propagation conditions the antenna pattern may have sharp peaks along some DOA's leading to the corresponding reduction of the total power and significant performance degradation. This situation is illustrated in Figure 2, which shows typical sub-carrier and total SMR antenna patterns for the AP with a uniform linear antenna array of two and four elements. One wavelength distance between antennas and "E"-channel [9], [10] in 2.4.GHz band are simulated.

One can see that sharp co-located sub-carrier beams are formed leading to significant total power reduction (TP=65 mW and TP=40 mW for two and four-antenna AP respectively) because of the EIRP constraint.

A direct optimization of the beamforming weights subject to the EIRP constraint is introduced in [5] in the narrowband case:

$$\mathbf{w}_{\text{EIRP}}=\arg\max_{\mathbf{w}}\text{Re}(\mathbf{h}\mathbf{w}) \tag{8}$$

subject to

$$|\mathbf{w}^*\mathbf{a}(\Theta_l)|^2<\text{EIRP},\, l=1,\ldots,L, \tag{9}$$

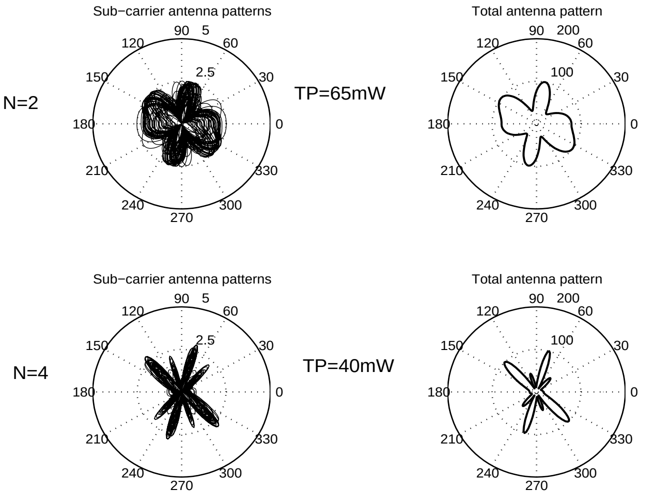

Figure 2: Typical example of the SMR antenna patterns for the two and four-antenna AP

where L is the number of controllable directions. It is pointed out in [5] that this is a convex SOCP problem that can be efficiently solved by the available numerical techniques such as the Interior Point algorithm [8].

Similar to (8), (9) optimization over separate sub-carriers could be considered as a direct OFDM extension of this narrowband solution.

The problem is to compare the existing EIRP-based solutions in terms of range extension possibility and to find a robust and relatively simple EIRP-based downlink beamformer for a WLAN OFDM system.

3. JOINT OPTIMIZATION OVER ALL SUB-CARRIERS

The idea is to formulate a general optimization problem subject to the EIRP constraint taking into account the uniform power loading assumption. Different optimization criteria can be exploited such as maximization of the average SNR or maximization of the minimum SNR over all the sub-carriers. Their efficiency may be different for different signalling, coding schemes and propagation conditions. In this paper we select maximum of the minimum SNR over all the sub-carriers as an optimization criterion. Following this approach, the joint constraint optimization problem can be formulated as follows:

$$[\mathbf{w}_{\text{EIRP-OFDM}}(f),\, f=1,\ldots,F]=$$
$$\arg\max_{\mathbf{w}(f)}\min_{f}\text{Re}[\tilde{\mathbf{h}}(f)\mathbf{w}(f)] \tag{10}$$

subject to

$$\sum_{f=1}^{F}|\mathbf{w}^*(f)\mathbf{a}(\Theta_l,f)|^2<\text{EIRP},\, l=1,\ldots,L, \tag{11}$$

where SNR at the receiver $|\tilde{\mathbf{h}}(f)\mathbf{w}(f)|^2$ as an optimization function is replaced with a real value similarly to (8).

Since the optimization function in (10) is linear and constraints in (11) are quadratic, this is again a convex SOCP problem [8].

One can expect that if the weight vectors defined by (10), (11) are found, then an average antenna pattern close to the omnidirectional one can be obtained leading to higher sub-carrier SNR's compared to the simple SMR solution.

A very high number of variables and constraints make this solution impractical. Some complexity reduction can be achieved by means of the conventional grouping of adjacent sub-carriers with highly correlated channels in some propagation conditions, but this still may be too complicated for on-line implementation.

4. JOINT OPTIMIZATION OVER GROUPS OF SUB-CARRIERS

A sub-optimal solution with reduced computational complexity can be obtained by means of decomposition of the general problem presented in Section 3. The idea is to apply a joint EIRP-restricted optimization similar to (10), (11) over separate groups of sub-carriers. The proposal for selection of the groups is to use sub-carriers with low channel correlation in one group. This allows us to achieve diversity and avoid a singularity problem. The overall EIRP constraint can be maintained by means of an additional normalization stage similar to the SMR normalization in (7) implemented after group-based optimization.

One possible grouping with a constant frequency shift in a group is as follows:

$$\Phi_j = [j, J+j, \ldots, (T-1)J+j],\ j=1,\ldots,J, \tag{12}$$

where Φ_j is the jth group of sub-carriers, T is the number of sub-carriers in a group and $J = F/T$ is the total number of groups. For example, according to the IEEE 802.11a/g specification [2] only $F = 52$ out of 64 sub-carriers are used for transmission, hence grouping for, e.g., $T = 4$, is as follows:

$$\Phi_j = [j, 13+j, 26+j, 39+j],\ j=1,\ldots,13, \tag{13}$$

Following this approach, a two-stage stage algorithm can be formulated as follows:

- *Optimization stage*

$$[\mathbf{w}_j(f),\ f\in\Phi_j] = \arg \max_{\mathbf{w}(f), f\in\Phi_j} \min_{f\in\Phi_j} \mathrm{Re}[\tilde{\mathbf{h}}(f)\mathbf{w}(f)] \tag{14}$$

  subject to

$$\sum_{f\in\Phi_j} |\mathbf{w}(f)^{*}\mathbf{a}(\Theta_l, f)|^2 < \nu,\ l=1,\ldots,L, \tag{15}$$

  where $j=1,\ldots,J$ and ν is an arbitrary positive constant controlling uniform EIRP distribution between groups of sub-carriers.

- *Normalization stage*

$$[\tilde{\mathbf{w}}_{\mathrm{SG}}(f),\ f=1,\ldots F] = [\gamma\mathbf{w}_j(f),\ j=1,\ldots J,\ f\in\Phi_j], \tag{16}$$

  where

$$\gamma = \sqrt{\frac{\mathrm{EIRP}}{\max_{\Theta}\sum_{j=1}^{J}\sum_{f\in\Phi_j}|\mathbf{w}_j^{*}(f)\mathbf{a}(\Theta, f)|^2}}, \tag{17}$$

This solution will be referred to the scaled grouped (SG) EIRP-based beamformer.

One can see that for one sub-carrier in a group, i.e. $\Phi_j = f$, $j = f = 1,\ldots,F$ and $J = F$, the SG algorithm can be considered as a direct OFDM extension of the narrow-band (8), (9) solution:

- *Optimization stage*

$$\mathbf{w}(f) = \arg\max_{\mathbf{w}(f)} \mathrm{Re}[\tilde{\mathbf{h}}(f)\mathbf{w}(f)] \tag{18}$$

  subject to

$$|\mathbf{w}^{*}(f)\mathbf{a}(\Theta_l, f)|^2 < \nu,\ l=1,\ldots,L. \tag{19}$$

- *Normalization stage*

$$\tilde{\mathbf{w}}_{\mathrm{SG}}(f) = \gamma\mathbf{w}(f),\ f=1,\ldots F, \tag{20}$$

  where

$$\gamma = \sqrt{\frac{\mathrm{EIRP}}{\max_{\Theta}\sum_{f=1}^{F}|\mathbf{w}^{*}(f)\mathbf{a}(\Theta, f)|^2}}, \tag{21}$$

If all the sub-carriers are included in one group, i.e., $\Phi_1 = [1,\ldots,F]$ and $J = 1$, the SG algorithm is equivalent to the joint solution presented in Section 3. In this case $\nu = \mathrm{EIRP}$ in (15) can be selected and the normalization stage is not required.

Therefore, selection of a group size allows us a trade off between performance and computational complexity.

5. COMPUTATIONAL COMPLEXITY

Let us estimate a number of real multiplications required for calculation of the SG-based transmit weight vector at the AP. The most complicated operation is SOCP optimization required for each of $J = F/T$ groups of sub-carriers. The dimension of the optimization vector in each group is $K = NT$. Taking into account that the conventional Least Squares (LS) solution requires $2(K^3 + K^2)$ real multiplications and the SOCP operation can be implemented with approximately 30 times higher complexity [8], the number of real multiplications required for the basic operation of the EIRP-based beamformer can be estimated as

$$Q(N,T) = 60\frac{F}{T}(N^3T^3 + N^2T^2), \tag{22}$$

or

$$Q(N,T) \approx 3\cdot 10^3 N^3T^2, \tag{23}$$

for IEEE 802.11a/g system, taking into account that $F = 52$ and typically $NT \gg 1$.

6. SIMULATION RESULTS

A uniform linear antenna array at the AP is simulated with "E"-channel propagation model (100 ns RMS delay spread) defined in [9], [10] for 2.4 GHz frequency band. For the optimization in (14), (15), MATLAB routines from the Optimization Toolbox are applied, although special algorithms and software are also available for SOCP [8].

Typical SG antenna patterns are given in Figure 3 for $T = 4$ sub-carriers in a group defined in (13). Figure 3 shows all 52 sub-carriers and the total antenna patterns in the two and four-antenna configurations with one wavelength distance between antenna elements. One can see that the proposed solution forms spatially distributed beams leading to the omnidirectional total pattern and TP≈100 mW in both cases. Comparison with the SMR antenna patterns and the reduced TP shown in Figure 2 suggests that better SNR and range extension performance can be expected for the proposed beamformer compared to the SMR case.

Figures 4 and 5 show Cumulative Distribution Functions (CDF) estimated over 1000 channel realizations for SNR gain at the receiver for $N = 2$ and $N = 4$ with one wavelength distance between antenna elements. The SG algorithm for $T = 1, 2, 4$ and 52 is presented together with the TP and SMR beamformers. The SNR gain $G(f)$ is calculated for each sub-carrier separately compared to the single antenna case:

$$G(f) = 10\log\frac{|\mathbf{h}(f)\mathbf{w}(f)|^2}{|h_1(f)|^2}, \qquad (24)$$

where $h_1(f)$ represents the propagation channel of the first antenna.

It is worth emphasizing that direct comparison of the TP and the EIRP-based solutions like SMR and SG is not fair because TP does not satisfy the EIRP constraint. So, the TP results are presented in Figures 4, 5 and below just for illustration of the performance degradation under the EIRP constraint.

One can see in Figures 4 and 5 that the increased number of sub-carriers in a group leads to the higher SNR gain, although, the improvement is not significant for $T > 4$. In overall, up to 1 dB and 2 dB SNR gain compared to the SMR beamformer can be observed for two and four-antenna AP with the SG algorithm ($T = 4$) respectively. It is important to note that the simplest optimization over separate sub-carriers ($T = 1$) is not effective in the considered environment especially in the two-antenna case.

A range extension possibility depends not only on beamforming, but also on signalling, coding and other system parameters, and can be illustrated by means of packet-error-rate (PER) and throughput performance.

PER and throughput are estimated for randomly located terminals at the given distance over 10 ms non-interrupted downlink transmission sessions for 16-QAM signaling, 3/4 code rate and 35 OFDM symbols in a slot (4320 information bits plus overhead). Each successful reception of at the receiver is followed with an acknowledgement (ACK) burst according to the IEEE 802.11 specification [2]. For simplicity we assume that re-transmissions use the basic (non-growing) back-off interval since according to the scenario shown in Figure 1, erroneous packets are not connected with interference from other terminals. Throughput is estimated as the total number of bits successfully transmitted over the 10 ms interval. ACK bursts are recovered to register successful transmissions, but ACK data bits are not taken into account for throughput estimation. Channel estimation is performed once at the beginning of each session. The performance is estimated over at least 500 independent sessions.

Figures 6 - 8 present the PER and throughput performance for two and four antennas at the AP with two wavelength distance between antenna elements. As expected, all the beamforming algorithms demonstrate significant range extension compared to the single-antenna case. One can see in Figure 6 that in the four-antenna case, the EIRP-based SMR solution shows about 22% performance degradation at 1% PER compared to the basic beamformer with the total power restriction. At the same PER level, the proposed SG solution demonstrates about 17% improvement compared to the SMR case.

The throughput CDF performance is given in Figures 7 and 8 for 100 m and 140 m distances respectively. Again, one can see that in the EIRP restricted scenario, SG demonstrates significant performance improvement compared to the SMR case.

7. CONCLUSION

It has been demonstrated that a conventional MR beamformer scaled according to the EIRP constraint in a WLAN OFDM system with a multiple antenna AP can be significantly improved by means of the proposed EIRP-based solution in the typical IEEE 802.11 propagation conditions. The proposed beamformer may be applied for range extension in WLAN OFDM systems working under the EIRP constraints, Additional spectral density EIRP constraints may be defined for some frequency bands. They can be taken into account similarly to the considered average EIRP constraint.

8. ACKNOWLEDGEMENT

The author would like to thank Prof. Y. I. Abramovich for participating in many fruitful discussions in the course of this work.

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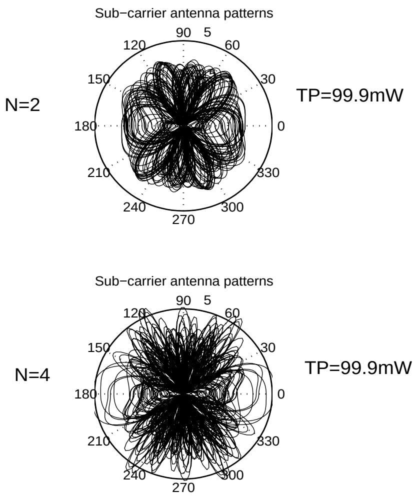

Figure 3: Typical example of the SG ($T = 4$) antenna patterns for the two and four-antenna AP

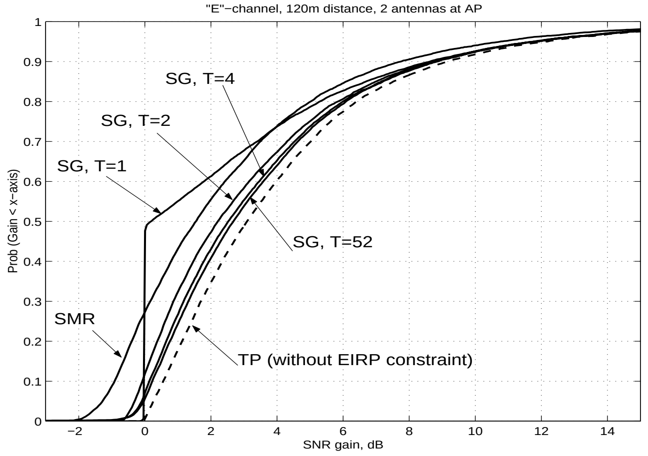

Figure 4: CDF of the SNR gain for the two-antenna AP

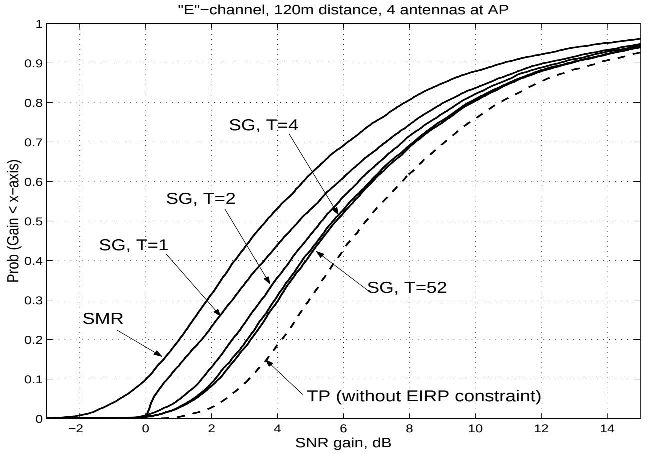

Figure 5: CDF of the SNR gain for the four-antenna AP

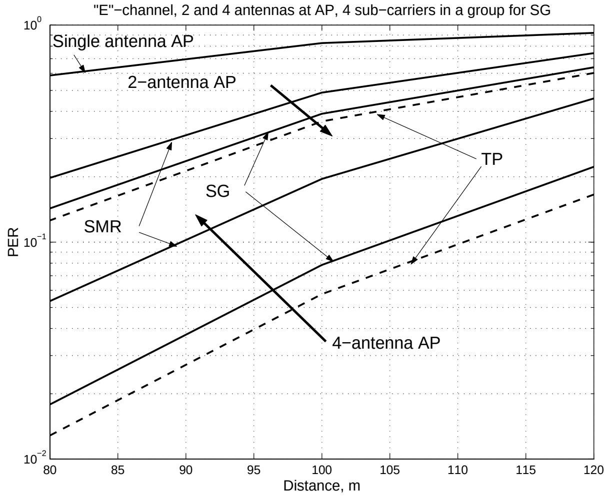

Figure 6: PER performance for the two and four-antenna AP

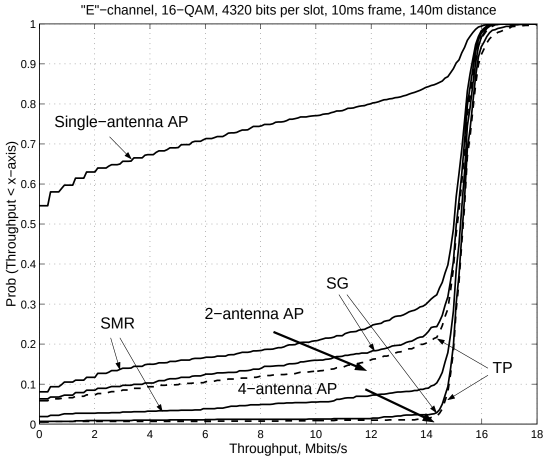

Figure 7: Throughput CDF the two and four-antenna AP, 100m distance, $T = 4$

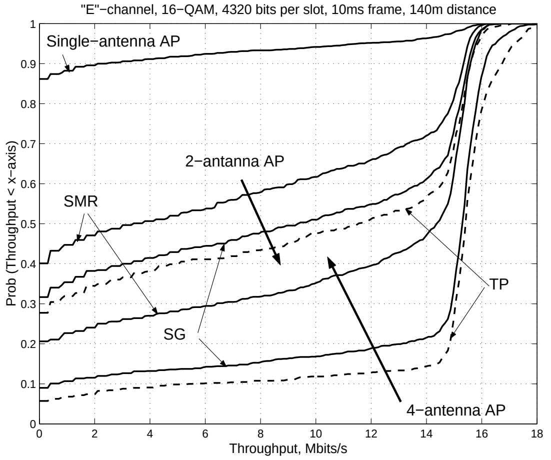

Figure 8: Throughput CDF the two and four-antenna AP, 140m distance, $T = 4$