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PERFORMANCE EVALUATION OF *K* OUT OF *N* DETECTOR

Yaser Norouzi[1], Maria S. Greco[2], Mohammad M. Nayebi[3]

1,3- Department of Electrical Engineering, sharif University of Technology
Azadi Ave. P. O. Box. 11365, Tehran, Iran
2- Department of Ingegneria del Informazione, University of Pisa
Via Caruso, 56122, Pisa , Italy
1-phone: + (39) 050 221 75 78, fax: + (39) 050 221 75 22, email: y_norouzi@ee.sharif.edu
2- phone: + (39) 050 221 76 20, fax: + (39) 050 221 75 22, email: m.greco@iet.unipi.it
3- phone: + (98) 912 307 51 80, fax: + (98) 21 88 77 13 68, email: nayebi@sharif.ir
web: http://ee.sharif.edu/~y_norouzi

ABSTRACT

In this paper the problem of k out of n detection, between sequence of M random bits is addressed. The main application of the result is in a radar system, when we want to detect a target (with unknown time of arrival (TOA)), using binary integration. Another application is in ESM systems, when the system wants to detect the existence of a swept jammer or a gated noise jammer. But as any other mathematical problem, it may have some other applications.

In this paper some simple equations for P_{fa} and P_d calculation of a detector which detects the existence of sequence of n ones between M random bits, is derived. These equations are then used to find the optimal detector structure in some special cases.

Besides, an approximate equation is derived for general case of k out of n detector, and it is shown that, the approximation is almost accurate.

KEY WORDS

k out of n detection, binary integration (BI), time of arrival(TOA)

1. INTRODUCTION

In 1950s and 60s some radar researchers suggested the binary integration to be used in radar systems [1,2]. In this technique all samples of received signal are converted to zero or one by comparing them with some fix or adaptive threshold. Then a moving window slides on these bits and a detection is declared when there exist at least *k* ones in the sliding window. Figure (1) shows this architecture.

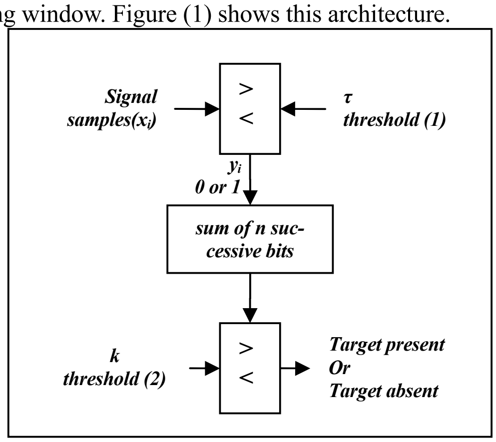


Figure (1) Binary Integration detection system

The authors who suggested the binary integration were aware that, under Gaussian noise assumption, binary integration is not as powerful as the integration of the true samples amplitudes [3], but they argue that the suggested detector has a very simple structure and therefore it is very easy to be implemented[4].

Nowadays the hardware problem is not so important as in the past but more investigations shows that, sometimes, binary integration is more powerful than a conventional adder. It is especially true when the probability density function (p.d.f.) of noise has a long tail [5]. As a result the binary integration is a good means of detection (for example see [6,7]).

For a *k* out of *n* detector, if we assume that in noise only case, the probability of a sample crossing the threshold (i.e.: having a bit equal to 1) is p_N and the probability of complementary event (i.e.: a bit equal to zero) is q_N, then the probability of declaring a detection in the noise only case (i.e.: P_{fa}) is calculated as:

$$P_{fa} = \sum_{i=k}^{n} \binom{n}{i} p_N^i q_N^{n-i} \tag{1-1}$$

But as it is mentioned in [8], the equation is true only when the time of arrival of the signal (i.e.: target echo) is known, which almost in all cases is not a true assumption.

In a real scenario we have a sequence of *M* bits. A lot of the bits of this sequence are results of the noise only process. And the target echo may be occur anywhere through this stream. In this case the calculations are not so simple as equation (1-1) indicates.

We use the notation $y_i \in N$ to show that i^{th} bit is from noise process, and use $y_i \in S$ to show the bit belongs to signal plus noise process. Using these notations, the hypothesis testing problem is as:

$$\begin{cases} H_0: & y_i \in N \quad i = 1,2,\ldots,M \\ H_1: & \begin{cases} y_i \in S & i = l+1, l+2, \ldots, l+n \\ y_i \in N & otherwise \end{cases} \end{cases} \tag{1-2}$$

$$\text{where } l \text{ is unknown but between } 1 \text{ and } M-n$$

A good detector for this problem is an sliding window with *k* out of *n* detection. In this case a false alarm will occur if under the noise only case a detection is declared anywhere in the sequence.

In this paper we will derive a precise solution for above system when the value of k is equal to n, and we will use this equation to find the optimum threshold values, in the case of n out of n detector. For general case of k out of n we will derive only some approximate formula.

In our writings, the problem is explained for detection of target in a radar system, but it is valuable to mention that, we have a similar problem, when in an ESM system we want to detect a swept jammer or a gated noise jammer [9,10]with unknown time of arrival.

The remaining parts of this paper are organized as follows. In section 2 we restate the problem, in order to clarify what is assumed and what we want to calculate. The problem is divided in two parts. The first part which is a n out of n detector is treated in section 3. In section 4 we focused on the next part of the problem which is a general k out of n detection.

2. PROBLEM STATEMENT

For more clarity we will explain the problem, here, in detail. There exist M samples of received signal (x_i $i=1,2,\ldots,M$). Under assumptions of noise only and signal plus noise, all the samples are independent. First all the samples are compared with a threshold (τ) and each sample is converted to a single bit (y_i $i=1,2,\ldots,M$).

Under the noise alone assumption, the probability that a bit is equal to one is p_N and the probability of a zero bit is q_N. Under signal plus noise assumption these probabilities are p_S and q_S respectively.

In this problem, the probability of false alarm is defined as: ***the probability of having at least one detection in sequence of M bits, in noise only situation.*** And the probability of detection is defined as: ***the probability of having at least one detection when the detection window is completely or at least in half, placed on the signal plus noise samples.***

We first derive the equations for n out of n detector, and then focus on general case of k out of n detector.

3. *N* OUT OF *N* DETECTION

3 .1. P_{fa} calculation

Defining P_m as the probability of (at least one) detection in first m samples, we can write:

P_m=P(detection in first m samples | detection in first $m-1$ samples) (3-1-1)
× P(detection in first $m-1$ samples)
+ P(detection in first m samples ∩ no detection in first $m-1$ samples)

$$\Rightarrow P_m = 1 \times P_{m-1} + P(A) \tag{3-1-2}$$

A= detection in first m samples ∩ no detection in first $m-1$ samples

Therefore we should calculate probability of event A in order to calculate P_m. If we want event A to occur, the stream of zeros and ones should be as shown in figure 2.

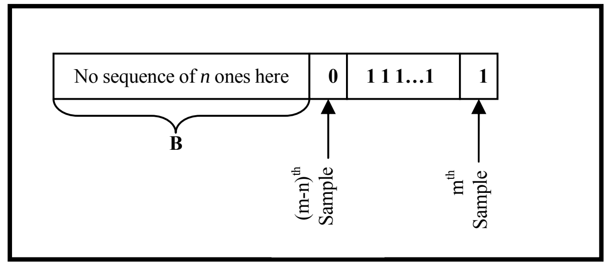


Figure (2) True sequence for occurrence of event A

Therefore we have:

$$P(A) = P(B) \times q_N \times p_N{}^n \tag{3-1-3}$$

But we know that:

$$P(B) = 1 - P_{m-n-1} \tag{3-1-4}$$

Therefore we have:

$$P_m = P_{m-1} + (q_N \times p_N{}^n) \times (1 - P_{m-n-1}) \tag{3-1-5}$$

Here we obtained in a linear difference equation of order ($n + 1$) which can be solved using methods introduces in several mathematical books [11,12]. To solve this equation, we need the first ($n + 1$) initial values. These initial values are simply as below:

$$\begin{cases} P_m = 0 & m = 1, \cdots, n-1 \\ P_n = p_N{}^n \\ P_{n+1} = (1 + q_N)\, p_N{}^n \end{cases} \tag{3-1-6}$$

Now, it is obvious that P_{fa} of this system equals P_m for m equal to the number of total samples. (i.e.: M) or:

$$P_{fa} = P_M \tag{3-1-7}$$

3.2. P_d calculation

Assume that there exist L successive samples of signal plus noise process, some where between M bits. In general case, L may or may not be equal to the length of detection window (i.e.: n). The situation is shown in figure 3.

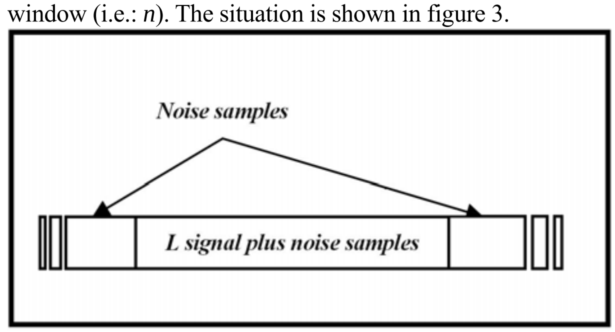


Figure (3) *L* signal plus noise samples, between noise samples

In this case, detection may occur anywhere in the stream of M samples, but detection is acceptable only if it is occurred near the signal samples. Therefore the detection probability depends on our definition about an acceptable detection.

If we define an acceptable detection as a detection when the detection window (which contains n samples) is completely placed on signal plus noise samples, then P_d may be calculated directly from equations (3-1-5) and (3-1-6) by replacing p_N and q_N with p_S and q_S, and then we have:

$$P_d = P_L \qquad \text{(3-2-1)}$$

But a better definition may be to accept detection when at least half of detection window is placed on signal plus noise samples. In this case we should modify equations (3-1-5) and (3-1-6) in order to be usable here. First, equation (3-1-6) is changed as below:

$$\begin{cases} P_m = 0 \quad m = 1,\dots,n-1 \\ P_n = p_N^{\frac{n}{2}} p_S^{\frac{n}{2}} \\ P_{n+1} = p_N^{\frac{n}{2}} p_S^{\frac{n}{2}} + q_N p_N^{\frac{n}{2}-1} p_S^{\frac{n}{2}+1} \end{cases} \qquad \text{(3-2-2)}$$

Then we should correct equation (3-1-5) as below:

$$\text{(3-2-3)}$$

$$P_m = P_{m-1} + q_s p_s^n \left(1 - P_{m-n-1}\right)$$
$$for\ m \le L + n/2$$
$$P_m = P_{m-1} + q_s p_s^{n-m+L} p_N^{m-L} \left(1 - P_{m-n-1}\right)$$
$$for\ \ L + n/2 < m \le L + n + 1$$

This equation is linear, but time varying. However we can solve it easily, using a simple computer program. Here, we have:

$$P_d = P_{L+n} \qquad \text{(3-2-4)}$$

It should be mentioned that, these equations need some modification for the case of odd values of n, as well as the situation of the target samples are placed at the beginning or the end of the sequence of samples.

Here, we have found two different equations for P_d, but it is almost obvious that, in practical situations there is a little difference between two results. Because, in a practical systems, the thresholds are set so that the probability of detection become extremely low, when the detection window is on the noise samples (P_{fa}).

In what follows, we show how to use the equations derived in sections 3-1 and 3-2 to find optimum n out of n detector for a problem:

3.3. Designation of a sample detection system

Assume that, we have 200 samples in each observation, and we want to detect target signals of duration of at least 10 samples with a SNR of at least 10 dB. Besides we need the P_{fa} not to be greater than 10^{-4}. Also we know that the probability density function of the noise and signal plus noise samples are Rayleigh and all samples are independent of others. Without loss of generality we can assume that the variance of noise samples is unity, and then if we select a threshold value equal to τ, we have:

$$p_N = \exp(-\tau^2/2) \qquad \text{(3-3-1)}$$

$$p_S = \exp(-\tau^2/2\sigma_S^2) \qquad \text{(3-3-2)}$$

$$\Rightarrow p_S = \left(p_N\right)^{\frac{1}{SNR}} \qquad \text{(3-3-3)}$$

Figure 3 shows the relation between p_N and p_S for a SNR of 10dB.

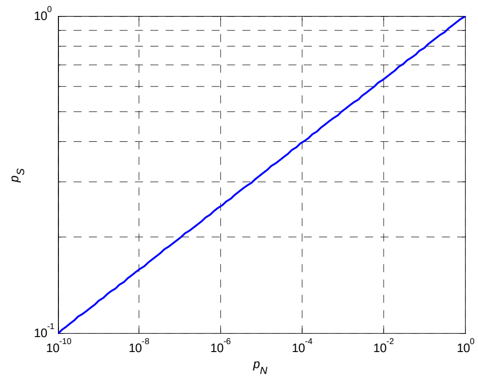


Figure (4) relation of p_N and p_S for a SNR equal to 10dB, when noise and signal distributions are Rayleigh

The next step to solve the problem is to find the required value of p_N corresponding to each value of n (the length of detection window) in order to achieve a P_{fa} value of 10^{-4} . But first we should remember that as long as we need to detect a target of length of 10 samples, using a window length of several times greater than 10 is not of any use. Therefore, we examine the values of n between 1 and 14. Using equations (3-1-5) and (3-1-6), the required values of p_N are listed in table 1. Corresponding values of p_S can be calculated directly from equation (3-3-3).

Table (1) required values of p_N for each n in order to achieve in a P_{fa} equal to 10^{-4}

n	**1**	**2**	**3**	**4**	**5**	**6**	**7**
p	**0.000**	**0.0007**	**0.00781**	**0.0273**	**0.0547**	**0.0898**	**0.1289**
n	**8**	**9**	**10**	**11**	**12**	**13**	**14**
p	**0.168**	**0.2031**	**0.2422**	**0.2773**	**0.3086**	**0.3398**	**0.3672**

At this point we are able to calculate P_d for each n, using equations (3-1-5) and (3-1-6) or (3-2-2) and (3-2-3).

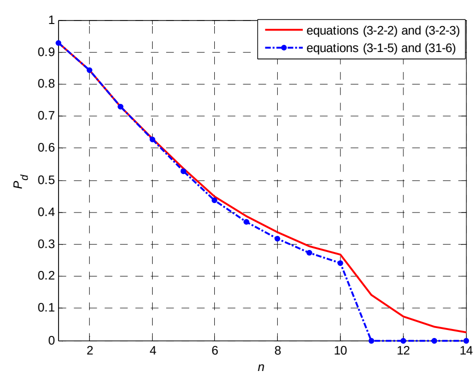


Figure (5) P_d of detector for different pairs of n and τ

Figure 5 shows the results, using both methods. As we see, in many points, there is a little difference between two results. It is apparent from this figure that, the best choice for the problem is to select the highest value for the threshold (τ) and then searching for only a single 1 in the stream of zeros and ones. But it is true only if the probability distribution of both noise and signal are Rayleigh. In other situations, the table 1 is usable yet, but we can not use equation (3-3-3) to calculate the corresponding values of p_S. In figure 6 we have calculated P_d for the case in which the noise distribution is Weibull with a shape parameter of 0.5, but target distribution is Rayleigh. In this figure the SNR value is about 10 dB.

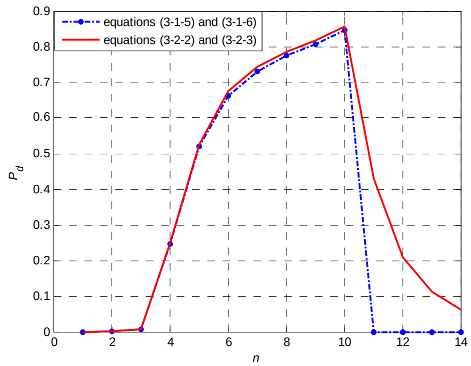


Figure (6) P_d of detector for different pairs of n and τ for a Weibull noise with a shape parameter of 0.5

Here the optimum value of n is equal to 10. Again, in this figure we see that, in many points of the graphs, the difference between two methods is little, therefore it may be reasonable in many cases to use more easier equations (3-1-5) and (3-1-6).

4. SOLUTION FOR GENERAL CASE OF *K* OUT OF *N* DETECTOR

Similar to the case of n out of n detection, we can define P_m as the probability that in the first m bits, we have at least one detection and similar to equation (3-1-1) we can write:

P_m=P(detection in first m samples | detection in first m-1 samples) (4-1)
× P(detection in first m-1 samples)
+ P(detection in first m samples ∩ no detection in first m-1 samples)

$$\Rightarrow P_m = 1 \times P_{m-1} + P(A) \quad (4\text{-}2)$$

A= detection in first m samples ∩ no detection in first m-1 samples

For event A to occur we need followings:

1- A one in m^{th} bit.(event A_1)
2- A zero in $(m\text{-}n)^{th}$ bit.(event A_2)
3- Exactly $(n\text{-}k)$ zeros between $(m\text{-}n\text{+}1)^{th}$ and $(m\text{-}1)^{th}$ bits.(event A_3)
4- No detection in first $(m\text{-}n\text{-}1)$ bits.(event A_4)
5- No detection in any sequence of n bits, beginning between $(m\text{-}2n\text{+}1)^{th}$ and $(m\text{-}n\text{-}1)^{th}$ bit.(event A_5)

This situation is shown in figure (7). Between above events, cases A_1, A_2, and A_3, are independent of the others. And the calculation of their probability is easy. But unfortunately cases A_4 and A_5 are not independent; as a result it is not easy to calculate the joint probability of these two events. We will show that in practical situations it is possible to use some approximations to be able to calculate corresponding probabilities

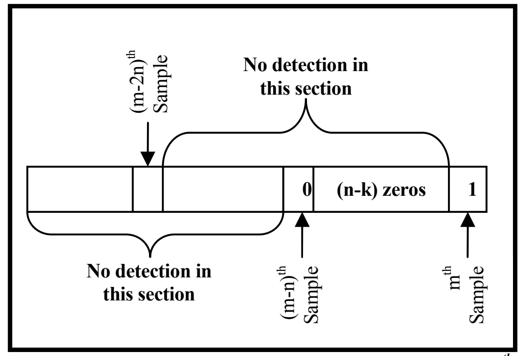


Figure (7) the situation needed for first detection to occur in m^{th} bit

The following equation can be used for calculation of the probability of event A:

$$P(A) = P(A_1)P(A_2)P(A_3)P(A_4 \cap A_5) \quad (4\text{-}3)$$

$$= p \times q \times \binom{n}{k} p^k q^{(n-k)} \times P(A_4 \cap A_5)$$

In order to find the probability of A_5, we can do as follows. If we assume that we have a total number of J zeros in first m bits and name the place of i^{th} zero as L_i, then for A_5 to occur we need that:

$$\begin{aligned} L_J - m &\geq L_{(J-k-1)} - (m-n-1) \quad (4\text{-}4)\\ L_{J-1} - m &\geq L_{(J-k-2)} - (m-n-1)\\ &\cdots\cdots\\ L_{J-k+1} - m &\geq L_{(J-2k)} - (m-n-1) \end{aligned}$$

We can calculate the probability of this event for any k and n, but it is obvious that the calculation is not simple when the k is many times less than n. The remaining problem is dependence of A_4 and A_5. In general it is not easy to calculate the joint probability of these two events, but when m is less than $2n$, the probability of A_4 is equal to 1 and the joint probability of A_4 and A_5 is equal to probability of A_5. In other case, when the k is near to n, and the n is large enough, then the assumption that A_4 and A_5 are independent is almost realistic. Therefore we can write:

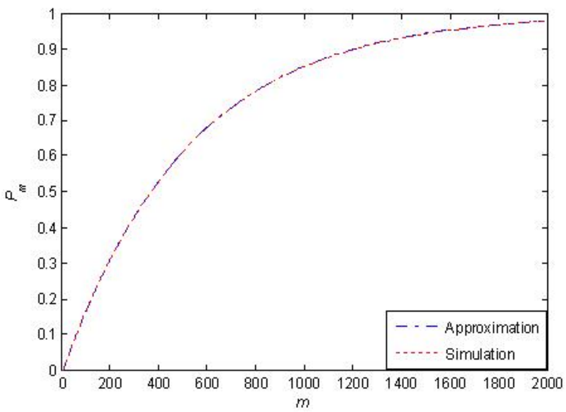
$$P(A) \cong p \times q \times \binom{n-1}{n-k} p^{k-1} q^{(n-k)} \quad (4\text{-}5)$$

$$\times (1 - P_{m-n-1}) \times P(A_5)$$

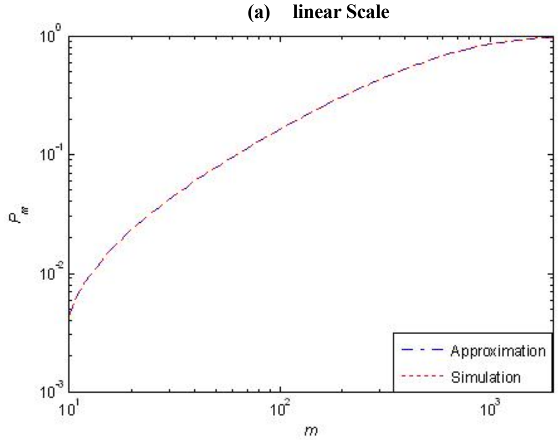
In this equation, $P(A_5)$ depends on k and n and it can be calculated directly from (4-4). For example for k equal to n-1 we have:

$$P(A_5) = \frac{1}{n-1}\sum_{i=1}^{n-1}(1-p^i) \qquad k = n-1 \quad (4\text{-}6)$$

Similar equations can be calculated for any other case of n and k. Now if we assume the approximate equation of (4-5) as a true equation, again we have a difference equation for probability calculation. In figure 8 we have shown the difference between the result of above equation and the result of the simulations. Here we want to find at least 9 ones in a sequence of 10 bits. In these simulations, p_N is equal to 0.45 and q_N is 0.55 and P_m is calculated for all values of m between 10 and 2000. The simulation is based on 1,000,000 samples of 2000 bits. As it is shown, the results of the equation have almost complete coincidence with the simulation results, and it shows the suggested approximation is good. But we should remember that the approximation is good only if the value of k is near to the value of n. otherwise, such an approximation may deviate greatly for true solution.



(a) linear Scale



(b) logarithmic Scale

Figure (8) Probability of detection of at least 9 ones in 10 successive bits, as a function of the length of the string (m)- for more precise comparison the results are shown both in linear and log scales (P(bit = 1) = 0.45 and the simulation is based on 10^6 samples of 2000 bits)

5. CONCLUSION:

In this paper we derived some simple equations to calculate the performance of n out of n detector, when the time of arrival (TOA) of the signal is unknown. These equations are linear difference equations and some times they are time varying. But in any case we can use a simple computer program to solve the equation. In the next part, we illustrate the use of these equations in order to design a powerful detector.

Then we derived a general solution for k out of n detector, with unknown time of arrival. In this case the equations are based on some approximations, but, as it was shown, in the case of k near to n, the result of approximation has almost complete coincidence with the true result.

The calculations of this paper may be used in the problem of binary integration in radar detection or detection of gated noise jammers.

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