14th European Signal Processing Conference (EUSIPCO 2006), Florence, Italy, September 4-8, 2006, copyright by EURASIP

An Efficient Feedback Scheme with Adaptive Bit Allocation for Dispersive MISO Channels

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***Abstract*— In this paper, we focus on a cellular system with M transmit antennas at the base station (BTS) and one receive antenna at the mobile (i.e. an M-input/single-output (MISO) channel), where the BTS commands each mobile to transmit its channel state information back to the BTS. Our main result is a specific feedback scheme with adaptive bit allocation, where a binary tree-structured vector quantizer is used to separately quantize each channel tap at a different level of quantization. We show that the proposed feedback scheme allows us to exploit the different statistics of the channel taps and results in a performance very close (within 1dB) to the performance that can be obtained with perfect channel knowledge at the BTS.**

I. Introduction

The focus of this paper is communication over dispersive multiple-input/single-output (MISO) channels where the receiver informs the transmitter of the state of these downlink channels, and these channels are block fading (i.e. the fading coefficients are constant during each coding block, but they change independently from block to block).

The problem of feeding back detailed channel information has been studied by a number of authors. In [1]- [3] a codebook design criterion has been proposed for quantizing the transmit beamforming vectors in flat MIMO channels. It has been shown that for a flat channel the near-optimal beamformer codebooks can be constructed by minimizing the maximum inner product between any two beamforming vectors in the codebook. This design criterion is similar to the design criterion of signal constellations for Gaussian channels, which maximizes the minimum Euclidean distance between any two signal points in the constellation. However, the design criterion does not apply to dispersive MISO channels.

The feedback method that can work in dispersive MISO channels has been proposed in [4]. This feedback method is particularly well-suited for wireless packet data cellular systems on the downlink where the entire downlink channel is allocated to one downlink user at a time. Meanwhile, the feedback scheme of [4] consumes a large proportion of uplink resources to achieve high level of performance. For example, it was shown in [4] that for a 4-tap channel, the mobile should send 160 bits to the BTS to get reasonable performance.

In this paper, we present a more effective feedback scheme with adaptive bit allocation, where a binary tree-structured vector quantizer (TSVQ) is used to separately quantize each channel tap at a different level of quantization. We show that such approach allows us to exploit the different statistics of the channel taps in order to reduce the amount of feedback. This results in a performance very close to the performance that is obtained with perfect channel knowledge at the transmitter.

The outline of the rest of this paper is as follows. In section II, a model for the MISO channel is presented along with a transmitter structure. In Section III, we propose the feedback scheme with adaptive bit allocation. Then, we show that the feedback scheme proposed in [4] can be viewed as an instance of a conventional TSVQ. In Section IV, Monte-Carlo simulations are used to show that our feedback method results in a performance (in terms of maximum data rate that can be reliably transmitted to each mobile) very close to the performance that can be obtained with perfect channel knowledge at the BTS. Finally, Section V concludes the paper.

II. System Model

Our system model is presented in Fig.1. In this figure, $b[n]$'s are the information bits at the transmitter which are coded and modulated to get the analog, complex, baseband signal $s(t)$. The base station (BTS) transmitter has M transmit antennas, and on the m-th antenna the signal $s(t)$ is passed through a pre-filter with impulse response $h(t,m)$. The impulse response of the downlink channel from the m-th transmit antenna to the single receive antenna at the mobile is assumed to be linear time invariant and is denoted by $g(t,m)$. Therefore, the received baseband signal at the receiver can be expressed by:

$$r(t) \;=\; \sum_{m=1}^{M} h(t,m) \star g(t,m) \star s \;+\; v(t), \tag{1}$$

where "$\star$" denotes convolution, and $v(t)$ is the baseband noise.

For closed-loop MISO antenna systems (cf. [5]), the pre-filters $h(t,m)$ in Fig.1 are just the scaled versions of the filters matched to the forward channels $g(t,m)$, i.e.,

$$h(t,m) = \alpha \; g^*(-t,m), \tag{2}$$

where α is a real, positive scaling factor used to ensure that the total transmit power σ_X^2 is constant, regardless of the actual channel realization.

One can see that the closed-loop MISO scheme requires that the forward link channel knowledge be fed back explicitly from the receiver to the transmitter..

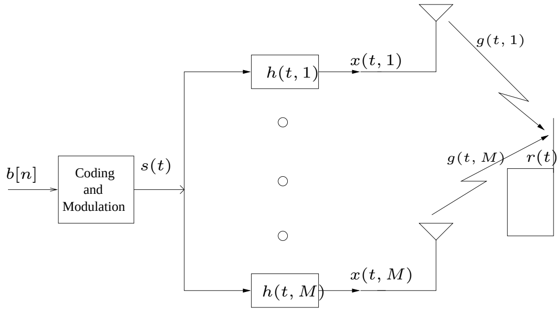

Fig. 1. MISO System with Transmit Diversity

In the next section, we propose an effective feedback scheme with adaptive bit allocation. We will show that when our proposed method for feedback of channel state information from the mobiles to the BTS is used in conjunction with the transmitter structure of Fig.1, information can be reliably transmitted to each mobile at rates very close to the capacity of the MISO channel seen by each mobile.

III. Feedback Scheme with Adaptive Bit Allocation

Referring to Fig.1, let us model the m-th downlink channel as:

$$g(t,m) = \sum_{k=1}^{K} a_{m,k}\ \delta(t-\tau_k), \qquad m \in [1,M]. \quad (3)$$

Each mobile can form an estimate of its $g(t,m)$ according to:

$$\hat{g}(t,m) \leftrightarrow \hat{g}_m[n] = \sum_{k=1}^{Q} \hat{a}_{m,k}\ \delta[n-k], \quad (4)$$

where $\hat{a}_{m,k}$ are the estimates of the channel taps $a_{m,k}$.

In this section, we propose an efficient method for feeding back the estimates $\hat{a}_{m,k}$ using a conventional tree-structure vector quantizer (TSVQ) [6] with adaptive bit allocation. Then, we show that the feedback scheme proposed in [4] can be viewed as an instance of a conventional TSVQ.

A. Tree-Structured Vector Quantization

We first briefly review the encoding and decoding operations of a conventional TSVQ. A K-dimensional TSVQ is characterized by the use of a tree of hyperplanes in $\Re^K$ in the encoder to facilitate the encoding process. Here we only consider fixed-rate quantization so that the encoding tree is assumed to be balanced with the maximum depth $(D-1)$. Each node of the tree other than the root node is labeled by a specific bit sequence $\mathbf{B}_n = (b[1], b[2], \cdots, b[n]) \in \{0,1\}^n$, where $n \in [1, D-1]$ denotes the depth on the tree at which the node is located. At the root node, a multi-dimensional hyperplane is stored in terms of its normal vector $\mathbf{p} \in \Re^K$ and an associated offset η. Similarly, at each node $\mathbf{B}_n$ at depth $n \in [1, D-1]$, a hyperplane is stored in terms of its normal vector $\mathbf{p}_{\mathbf{B}_n}$ and its offset $\eta_{\mathbf{B}_n}$. Given a real-valued vector $\mathbf{x} \in \Re^K$, the quantization encoder begins at the root node of the tree with the corresponding hyperplane $(\mathbf{p}, \eta)$ and computes

$$b[1] = \text{sign}(\mathbf{p}^T\mathbf{x} - \eta). \quad (5)$$

At the next level, the encoder computes

$$b[2] = \text{sign}(\mathbf{p}_{\mathbf{B}_1}^T\mathbf{x} - \eta_{\mathbf{B}_1}) \quad (6)$$

using the hyperplane $(\mathbf{p}_{\mathbf{B}_1}, \eta_{\mathbf{B}_1})$ that corresponds to the value of $\mathbf{B}_1 = b[1]$. The encoder repeats this process at subsequent levels to compute

$$b[n] = \text{sign}(\mathbf{p}_{\mathbf{B}_{n-1}}^T\mathbf{x} - \eta_{\mathbf{B}_{n-1}}) \quad (7)$$

until either the maximum depth D of the tree or the number of bits $R \leq D$, allocated for quantizing $\mathbf{x}$ is reached. At this time, the encoder outputs the R bit sequence $\mathbf{B}_R = (b[1], b[2], \cdots, b[R])$.

Upon receipt of the encoded bit sequence $\mathbf{B}_R$, the decoder of TSVQ generates the quantized vector $Q(\mathbf{x})$ by tracing through a decoding tree of depth D whose nodes at each level contain a pre-computed estimate of $\mathbf{x}$ with the corresponding level of accuracy. The estimate stored at the node that corresponds to the bit sequence $\mathbf{B}_R$ are outputed as $Q(\mathbf{x})$.

The described approach can be used to quantize the estimates $\hat{a}_{m,k}$ of the channel taps. Let $\hat{a}_{m,k} = \hat{a}^R_{m,k} + j\hat{a}^I_{m,k}$ for all $m \in [1,M]$ and $k \in [1,Q]$. Also let $\hat{\mathbf{a}}^R \equiv (\hat{\mathbf{a}}^R_1, \hat{\mathbf{a}}^R_2, \cdots, \hat{\mathbf{a}}^R_Q)^T$ denote the vector of the real parts of all channel taps across all M antennas, where $\hat{\mathbf{a}}^R_k \equiv (\hat{a}^R_{1,k}, \hat{a}^R_{2,k}, \cdots, \hat{a}^R_{M,k})$, for $k \in [1,Q]$. Similarly, let $\hat{\mathbf{a}}^I \equiv (\hat{\mathbf{a}}^I_1, \hat{\mathbf{a}}^I_2, \cdots, \hat{\mathbf{a}}^I_Q)^T$ denote the vector of the imaginary parts of all channel taps, where $\hat{\mathbf{a}}^I_k \equiv (\hat{a}^I_{1,k}, \hat{a}^I_{2,k}, \cdots, \hat{a}^I_{M,k})$, for $k \in [1,Q]$.

Suppose that two TSVQs are used to separately quantize $\hat{\mathbf{a}}^R$ and $\hat{\mathbf{a}}^I$, where the dimension of these quantizers is $K = MQ$. From (7), we see that the bit sequences generated by the encoders of the two quantizers are given by

$$b_R[n] = \text{sign}(\mathbf{p}_{\mathbf{B}_{n-1}}^T\ \hat{\mathbf{a}}^R - \eta_{\mathbf{B}_{n-1}}), \qquad n \in [1,R], \quad (8)$$

$$b_I[n] = \text{sign}(\mathbf{p}_{\mathbf{B}_{n-1}}^T\ \hat{\mathbf{a}}^I - \eta_{\mathbf{B}_{n-1}}), \qquad n \in [1,R]., \quad (9)$$

B. Feedback Scheme with Random Excitation

In this subsection, we show that the feedback scheme described in [4] can be viewed as an instance of a conventional TSVQ.

It has been shown in [4], that to communicate $\{\hat{g}_m[n])\}_{m=1}^M$ to the BTS the mobile should form a scalar-valued sequence $s[n]$ from $\hat{g}_m[n]$'s according to:

$$s[n] = \sum_{m=1}^{M}\sum_{k=1}^{Q} \hat{a}_{m,k} \cdot p_m[n-k], \qquad n \in [1,P], \quad (10)$$

where $p_m[n]$'s are independent, real-valued, pre-determined, pseudo-noise sequences (e.g. i.i.d. samples uniformly distributed between [-1,+1]).

Next, the mobile generates a binary representation of $s[n]$ by quantizing each $s[n]$. As a specific example, the mobile can quantize the real part of $s[n]$ (denoted by $s_R[n]$) and the imaginary part of $s[n]$ (denoted by $s_I[n]$) to one bit each, i.e. the mobile forms two binary sequences $\bar{b}_R[n]$ and $\bar{b}_I[n]$ according to:

$$\bar{b}_R[n] = \text{sign}(s_R[n]), \quad n \in [1, P], \tag{11}$$

$$\bar{b}_I[n] = \text{sign}(s_I[n]), \quad n \in [1, P]. \tag{12}$$

The two sequences $\left\{\bar{b}_R[n]\right\}_{n=1}^{P}$ and $\left\{\bar{b}_I[n]\right\}_{n=1}^{P}$ constitute a binary representation of the $M \times Q$ non-zero coefficients and the Q delays of the estimated downlink channels (estimated at the mobile). These $2 \times P$ bits are then transmitted by the mobile on the uplink channel (using a particular spreading factor and a particular transmit power). At the BTS, these $2 \times P$ bits are detected. Given these detected bits and given the knowledge of $p_m[n]$'s, the BTS can estimate $\hat{g}_m[n]$ from these detected bits.

Now we will show that the algoritm (11)-(12) is actually equivalent to a binary TSVQ [6] with (pseudo) random encoding hyperplanes. By breaking (10) into real and imaginary parts, Eqs.(11)-(12) can be expressed as

$$\bar{b}_R[n] = \text{sign}(\mathbf{p}^T[n] \; \hat{\mathbf{a}}^R), \tag{13}$$

$$\bar{b}_I[n] = \text{sign}(\mathbf{p}^T[n] \; \hat{\mathbf{a}}^I), \tag{14}$$

where

$$\mathbf{p}[n] \equiv (\mathbf{p}_1[n], \mathbf{p}_2[n], \cdots, \mathbf{p}_Q[n])^T,$$

$$\mathbf{p}_k[n] \equiv (p_1[n-k], p_2[n-k], \cdots, p_M[n-k]),$$

for $n \in [1, P]$.

Comparing the equations (8)-(9) and (13)-(14), we see that the encoding operations performed in the feedback scheme with random excitation is similar to that performed in a conventional TSVQ. The main difference is that the hyperplanes used in a conventional TSVQ are carefully designed to match the statistical distribution of the channel taps using conventional design algorithms [6]- [7], instead of randomly chosen as suggested in [4]. Moreover, unlike the hyperplanes of the conventional TSVQ, the randomly chosen hyperplanes always pass through the origin of the underlying Euclidean space since their offsets are zero. In addition, the hyperplane $\mathbf{p}_{\mathbf{B}_{n-1}}$ used in algorithm (8)-(9) at each level n depends on the output bits $\mathbf{B}_{n-1}$ computed at the previous levels, while the hyperplanes $\mathbf{p}[n]$ in algorithm (13)-(14) are chosen independently of each other. Consequently, using a conventional TSVQ to quantize the channel taps can lead to substantial improvement over the random-excitation feedback method, as shown later.

C. Adaptive Bit Allocation

Another reason for the inefficiency of the feedback scheme of [4] is due to the strategy of collective quantization of all channel taps using a fixed source coding scheme. Although quantizing all taps collectively may provide some shaping gain in high dimensions, this strategy makes it difficult to exploit the statistics of the channel taps in an adaptive manner. The potential gain in matching the coding scheme with the statistics of the channel taps can easily outweight the potential shaping gain in high dimensions. In this paper, we propose coding each channel tap $\hat{\mathbf{a}}_k \equiv \left(\hat{\mathbf{a}}_k^R, \hat{\mathbf{a}}_k^I\right)$ separately to exploit the different statistics of the channel taps.

Our proposed feedback scheme is presented in Fig.2. One can see this scheme consists of two logical feedback channels, one at lower rate (i.e. feeding back information less often) than the other. Information regarding the number of bits allocated for quantization of each channel tap is periodically sent back to BTS through the low-rate feedback channel. Such information is adaptively computed using certain long-term statistics of the channel response (such as the relative powers or variances of the channel taps) collected at the terminals, so that a certain distortion measure of the resulting quantized channel response is minimized for the total number of available bits. On the other hand, the information regarding the quantized version (according to the current bit allocation) of the estimate of each specific channel realization, is sent back periodically through the high-rate feedback channel.

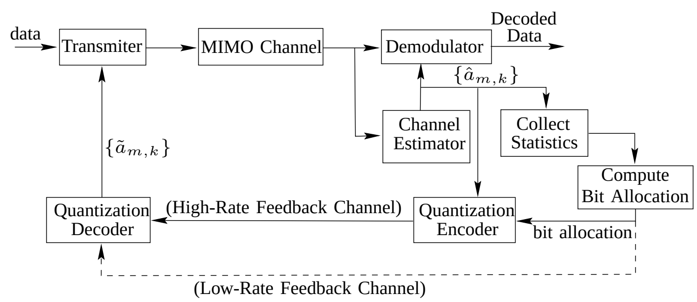

Fig. 2. Block Diagram of Adaptive Feedback Scheme

Figure 3 shows a variant of the proposed feedback scheme where the low-rate feedback channel is eliminated. In this case, the statistics of the channel taps, based on which the number of allocated bits for each channel tap is computed, are collected after quantization so that the same statistics can be generated at the transmitter.

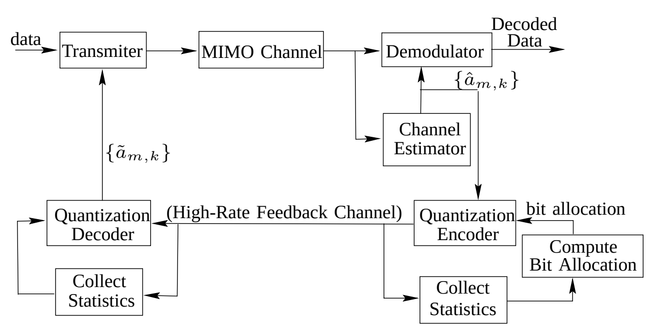

Fig. 3. Simplifed Adaptive Feedback Scheme

The bit allocation for each channel tap may be computed so that the mean-squared difference between the estimated channel response and its quantized version is minimized. Let

$Q_k(\cdot)$ denote a real-valued vector quantizer of dimension $2M$ with N_k quantization points used to quantize $\hat{\mathbf{a}}_k \equiv (\hat{\mathbf{a}}_k^R, \hat{\mathbf{a}}_k^I)$. The source coding rate of $Q_k(\cdot)$ is defined as $R_k = (2M)^{-1}\log_2 N_k$, which denotes the number of bits allocated to quantize each real-valued element of $\hat{\mathbf{a}}_k$. The goal is to find the optimal bit allocation vector $\mathbf{R} = (R_1, R_2, \cdots, R_Q)$ such that the sum of mean-squared distortions of all taps given by

$$D(\mathbf{R}) = \sum_{k=1}^{Q} D(R_k) \tag{15}$$

is minimized, where

$$D(R_k) = E\|\hat{\mathbf{a}}_k - Q_k(\hat{\mathbf{a}}_k)\|^2. \tag{16}$$

It is well known in source coding [6] that a good approximate solution can be derived for $\mathbf{R}$ using the high-rate approximate formula for $D(R_k)$ [8]- [10]:

$$D(R_k) \approx 4^{-R_k}\sigma_k^2\gamma_k, \qquad k \in [1, Q], \tag{17}$$

where σ_k^2 is the variance of vector channel tap $\hat{\mathbf{a}}_k$ and γ_k is a quantity that depends on the joint probability density $p_k(\cdot)$ of $\hat{\mathbf{a}}_k$ and some design characteristics of the quantizer $Q_k(\cdot)$. Substituting (17) in (15), it is easy to show that the components of the optimal vector $\mathbf{R}$ that minimizes $D(\mathbf{R})$ are given by

$$R_k = R + \frac{1}{2}\log\frac{\sigma_k^2\gamma_k}{\left(\prod_{j=1}^{Q}\sigma_j^2\gamma_j\right)^{1/Q}}, \qquad k \in [1, Q], \tag{18}$$

where $R = Q^{-1}\sum_{k=1}^{Q} R_k$ denotes the average number of bits allocated per vector channel tap.

Assuming that $\hat{\mathbf{a}}_k$ are identically distributed for all k except for their variances (i.e. for each k $p_k(\mathbf{x}) = \sigma_k^{-2M}p(\mathbf{x}/\sigma_k)$) and that the quantizers $Q_k(\cdot)$ have the same design characteristics, then γ_k are identical for all k. It follows that (18) simplifies to

$$R_k = R + \log\frac{\sigma_k}{\left(\prod_{j=1}^{Q}\sigma_j\right)^{1/Q}}, \qquad k = \in [1, Q]. \tag{19}$$

Note that in order to quantize the channel taps at different rates according to their variances, the receiver and transmitter must store, respectively, the encoders and decoders of multiple quantizers of different source coding rates so that different levels of quantization can be provided according to its measured statistics. Storing multiple quantizers may not be feasible in practice. The use of TSVQ provides a simple solution to this problem since a single TSVQ can be used to quantize each channel tap at a different level of quantization by simply terminating the encoding and decoding process before reaching the maximum depth of the trees.

Since the rates computed using (19) may not exactly match the available rates, certain rounding operation may be performed when computing the rates R_k. To ensure that the overall rates after rounding will not exceed the capacity of the feedback channel, one can compute (19) sequentially as

$$R_k = \frac{QR - \sum_{j=1}^{k-1}\hat{R}_j}{Q - k + 1} + \log\frac{\sigma_k}{\left(\prod_{j=k}^{Q}\sigma_j\right)^{1/(Q-k+1)}} \tag{20}$$

for $k \in [1, Q]$, where $\hat{R}_j$ denotes an approximation of R_j due to rounding. It is easy to show that if $\hat{R}_j = R_j$ for all $j = 1, 2, \cdots, k-1$, then R_k computed by (20) is the same as that computed by (19). To ensure good performance, it is preferable to compute the rates in the descending order of the corresponding channel-tap variances and to use rounding-up operation so that dominant channel taps are assured of adequate number of bits.

Note that in a practical communication systems, since the channel taps often vary slowly from one feedback time instant to another, differential quantization of the channel taps may be also used. In this case, the proposed feedback scheme can operate in conjunction with any differential quantization scheme to quantize the changes of the channel taps.

IV. Simulated Performance Results

We have performed Monte-Carlo simulations to quantify the performance of the proposed feedback scheme. We consider a system with 4 transmit antennas and one receive antenna, and we assume that all downlink channels are mutually independent, the noise at the receiver is AWGN, and SNR is defined as ratio of the total transmitted power to the variance of the noise (after the receive filter) at the receiver, i.e. $SNR = \sigma_X^2/\sigma_v^2$, where σ_v^2 is the variance of $v(t)$.

We model the medium part of each downlink channel with four paths separated by one chip, relative strengths of 0, -3, -6, and -9 dB. In the UMTS standardization, this so-called Case-3 channel model is often used to approximate each of the downlink channels of the WCDMA system in a typical urban environment. Each downlink channel is normalized to have unit power gain on average.

To evaluate the performance of the proposed feedback scheme the following link-level (single-user, single-cell) simulation is performed. First, a large number of channel realizations are generated. For each realization, the instantaneous throughput is obtained by mapping the SNRs at the output of the mobile receiver to a data rate using a MCS look-up table [11]. The SNR switch points in the MCS table correspond to the 10% block error rate points obtained through simulation of the WCDMA turbo code performance in AWGN. In this table, the MCSs with rates below about 5 Mbps use QPSK modulation, and the higher-rate MCSs use 16-QAM modulation. The FEC coding rates range from 0-0.9974, resulting in a peak per-stream data rate of approximately 11.5 Mbps.

The performance measure used for evaluation is average data rate, obtained by averaging the instantaneous data rates over many channel realizations. This type of single-user, single-cell link simulation implicitly assumes perfect link adaptation and a continuous traffic model, i.e., the single user has data in queue all of the time. In Fig. 4 the average data rate is plotted versus SNR. The solid curve on the top corresponds to the MISO system with 4 transmit antennas when the BTS knows the downlink channels perfectly. The curve with stars, ”-*-”, corresponds to the MISO system with channel knowledge at the transmitter obtained using proposed feedback scheme sending back to the BTS 60 bits. The curve with ”-o-” corresponds to the MISO system with feedback scheme with random excitation sending back 160 bits.

From Fig. 4, we see that the proposed feedback scheme allows to achieve significant performance gain compared to the feedback scheme of [4]. Indeed, to achieve data rate of 10 [MBits/sec], the 4×1 MISO system using the feedback method proposed in this paper requires approximately 10 [dB] less SNR than the system with the feedback scheme of [4]. Furthermore, sending back just 60 bits the feedback method proposed in this paper results in a loss of less than 1 dB compared with ideal MISO system with perfect channel knowledge at the BTS. Thus, the proposed scheme can yield significant performance gains while saving significantly on uplink bandwidth.

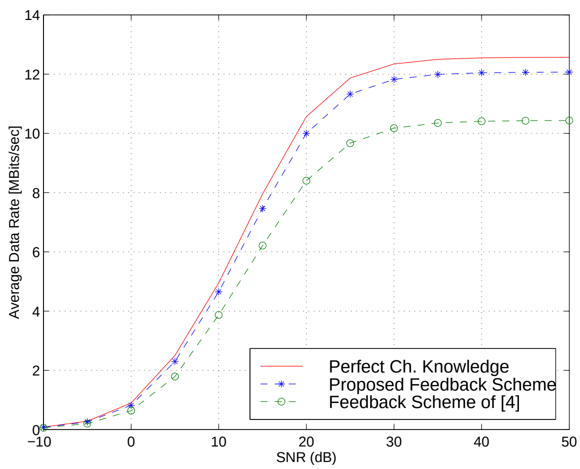

Fig. 4. Average Data Rate vs. SNR

V. Conclusion

For wireless packet data cellular systems whose uplink channel and downlink channel occupy different frequency bands (e.g. IS-95 or WCDMA), the main contribution of this paper is a specific feedback scheme with adaptive bit allocation, where a binary tree-structured vector quantizer is used to separately quantize each channel tap at a different level of quantization. With a small portion of uplink channel resources devoted to feedback of channel state information per site, the proposed feedback method results in a performance very close to the performance that is obtained with perfect channel knowledge at the transmitter. In other words, with the BTS obtaining the state of the downlink channels using the proposed feedback scheme, the data rate that can be reliably transmitted to each mobile is very close to the channel capacity of the MISO channel seen by each mobile.

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