

STRATEGIES OF ADAPTIVE NONLINEAR NOISE CANCELLING VOLTERRA-WIENER FILTER STRUCTURE SELECTION

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ABSTRACT

In this article a nonlinear orthogonal noise cancelling filter parameters selection strategy is proposed to maximize noise cancelling quality to computation complexity ratio. Presented approach leads to the orthogonal realization of the nonlinear noise cancelling filter of Volterra-Wiener class which structure changes due to higher-order statistics of the filtered signals.

1. INTRODUCTION

Using the results presented in [1][2], during the noise reduction filtering the following multi-dimensional Fourier series expansion of the original signal (x_0) is obtained

$$\begin{aligned} \hat{x}_0 &= \sum_{i_1=0}^N x \rho^{i_1} r_0^{i_1} + \sum_{i_1=0}^N \sum_{i_2=i_1}^N x \rho^{i_1, i_2} r_0^{i_1, i_2} + \dots \\ &\dots + \sum_{i_1=0}^N \sum_{i_2=i_1}^N \dots \sum_{i_M=i_{M-1}}^N x \rho^{i_1, \dots, i_M} r_0^{i_1, \dots, i_M} \end{aligned} \quad (1)$$

where

$$\begin{aligned} x \rho^{i_1} &= (x_0, r_0^{i_1}) \\ x \rho^{i_1, i_2} &= (x_0, r_0^{i_1, i_2}) \\ &\dots \\ x \rho^{i_1, \dots, i_M} &= (x_0, r_0^{i_1, \dots, i_M}) \end{aligned} \quad (2)$$

are the generalized (multi-dimensional) Fourier [3][4] (i.e. Schur-type) coefficients. The coefficients (2) can be interpreted as the orthogonal representation of the random variable x_0 in the subspace S spanned by the orthonormal elements $\{r_0^{i_1}, \dots, r_0^{i_1, i_2}, \dots, r_0^{i_1, i_2, i_3}, \dots, r_0^{i_1, \dots, i_M}\}$ (where $i_1 = 0, \dots, N$, $i_2 = i_1, \dots, N$, $i_3 = i_2, \dots, N$, $i_M = i_{M-1}, \dots, N$) on one hand, on the other - as the coefficients in the orthogonal realization of the multi-dimensional nonlinear noise-cancelling filter [2] (figure 1, figure 2).

The least-square error of the original signal estimation is equal

$$\begin{aligned} x R_{N,0}^{\{M\}} &= (x \varepsilon_{N,0}^{\{M\}}, x \varepsilon_{N,0}^{\{M\}}) = \dots = \\ &= \|x_0\|_{\Omega}^2 - \sum_{j_1=0}^N |x \rho^{j_1}|^2 - \dots - \\ &+ \sum_{j_1=0}^N \dots \sum_{j_M=j_{M-1}}^N |x \rho^{j_1, \dots, j_M}|^2 \end{aligned} \quad (3)$$

This error depends only on the coefficients $x \rho^{j_1, \dots, j_q}$

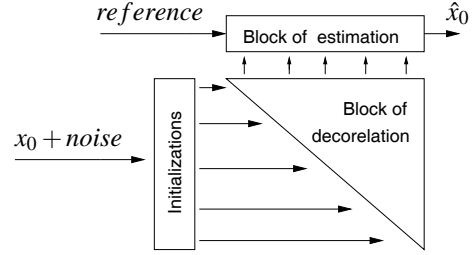


Figure 1: General structure of the nonlinear orthogonal noise-cancelling filter.

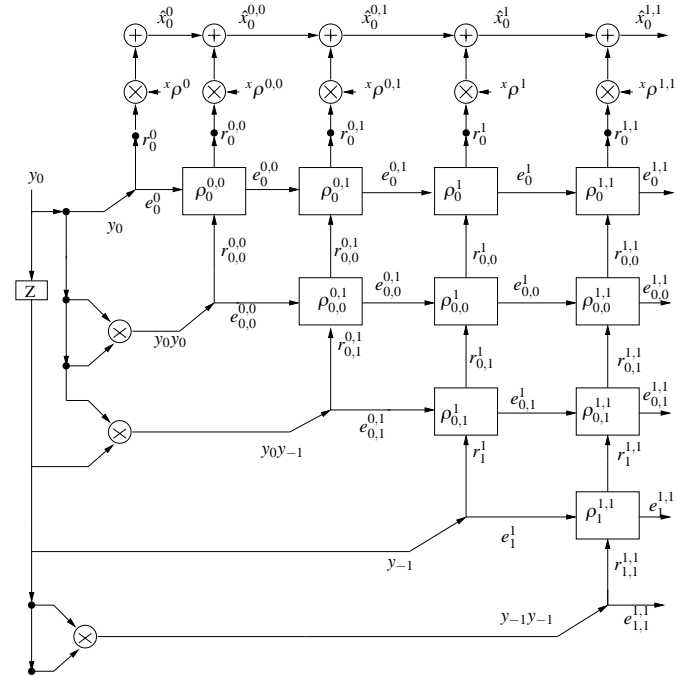


Figure 2: The nonlinear orthogonal noise-cancelling filter ($N = 1, M = 2$).

Considering (1) it is easy to notice, there are two parameters which present filter complexity: the filter order (filter memory) and degree of the filter nonlinearity. Increasing one of them entails quick growth a computing complexity needed to estimate (\hat{x}_0). The number of the elementary section in the

estimation block of the noise cancelling filter is

$$K_{N,M} = \sum_{m=1}^M \frac{(N+m-1)!}{m!(N-1)!} \quad (4)$$

Figure 3 presents how quick rises the value of (4) when N or M is changing. The number of the elementary sections

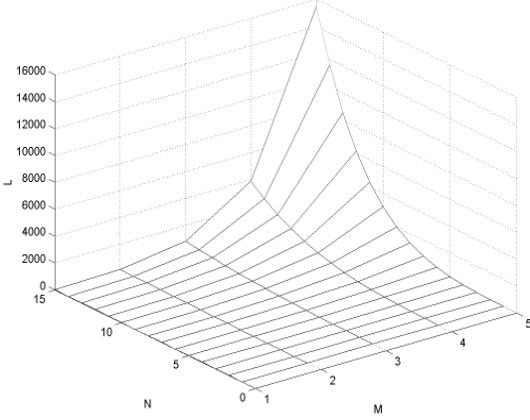


Figure 3: The number of elementary sections $K_{N,M}$ in the estimation block

in the decorelation block of the noise cancelling filter is

$$L_{N,M}^{ns} = \frac{(1 + K_{N,M})K_{N,M}}{2} \quad (5)$$

The realization of the decorelation block is more laborious. The key is to select such the values of N and M , that the minimal number of computed ON basis elements allow to determine the 'good' estimate (achieving the desire value of the error (3)).

It is possible to change N or M , or both of them at one time. Real system should be efficient. This aim can be achieve by adaptive filter parameters selection using some strategy.

2. THE CRITERION OF THE NOISE-CANCELLING FILTER PARAMETERS SELECTION STRATEGY

The aim of increasing degree of filter nonlinearity or its order is a improvement of the noise cancelling quality (minimizing (3)).

To judge the proper values of N and M the objective measure is needed, which shows the relative improvement of the noise cancelling quality. It should describe a change of the estimation error causes by extension of filter structure (increasing the number of elementary sections in the decorelation block). The following cost function is defined

$$FK(N_1, M_1; N_2, M_2) \triangleq - \frac{\frac{L_{N_2, M_2}^{ns} - L_{N_1, M_1}^{ns}}{L_{N_1, M_1}^{ns}}}{\frac{xR_{N_2}^{M_2} - xR_{N_1}^{M_1}}{xR_{N_1}^{M_1}}} \quad (6)$$

where N_1, N_2 are the filter orders, M_1, M_2 are the filter nonlinearity degrees. The value $L_{N,M}^{ns}$ (4) describes the number of elementary sections in the decorelation block for filter of order N and nonlinearity degree of M . xR is defined in (3). The

equation (6) is interpreted as a relative change of the number of elementary sections in the decorelation block to relative improvement of a noise cancelling quality (change value of the estimation error) when the parameters N, M are changing. Because the denominator is always negative the sign minus is used. This cost function allows to judge efficiency of the selected filter structure and the filter complexity needed to improved the estimate \hat{x}_0 .

3. THE STRATEGY OF ESTIMATION IMPROVEMENT BY SELECTION OF FILTER PARAMETERS

The pattern of one loop of filtering process is presented in figure 4.

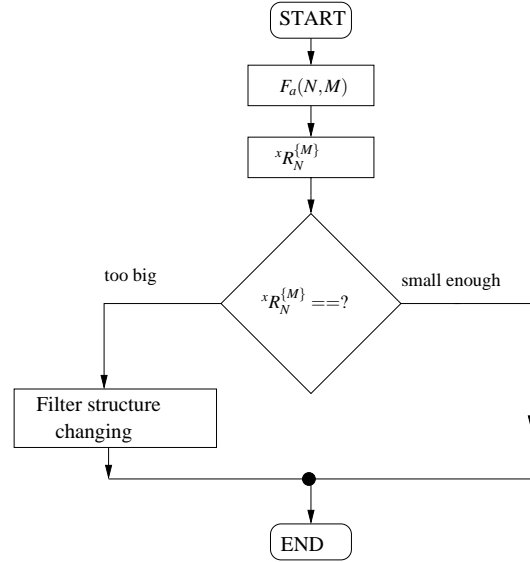


Figure 4: One loop of the filtering process

After filtering by a filter $F_a(N, M)$, the estimation error is computed xR_N^{M} . If its value is small enough the filter structure is unchanged. If the error is too big the filter parameters are changing according to some strategy. The block 'Filter structure changing' represents proposed in this article strategy of selecting filter parameters.

Assuming value of the cost function (6) determines the possible values of the parameters N and M .

The dominator of (6) is the control element, which depends on the Schur coefficients $x\rho$. Adding a new non-zero valued coefficient decreases the value of (3). It is equivalent to changing the filter order or/and its nonlinearity degree. It causes the increasing of filter complexity (especially the decorelation block).

When one or few coefficients $x\rho$ are added it is not necessary to compute the whole ON basis anew, but only its new element. The filter $F_a(N_1, M_1)$ has $L_{N_1, M_1}^{ns} = K_{N_1, M_1} (K_{N_1, M_1} + 1)/2$ elements in the decorelation block, where $K_{N,M}$ is described in (5). The value of cost function (6) after adding a new coefficient $x\rho_{new}$ (it means adding $K_{N_1, M_1} + 1$ elementary

sections to the decorelation block) is

$$FK(N_1, M_1; N_2, M_2) = \frac{2}{K_{N_1, M_1}} \cdot \frac{xR_{N_1}^{\{M_1\}}}{|x\rho_{new}|^2} \quad (7)$$

Using (7) and considering presented discussion the following strategy for filter structure selection is proposed

Strategy 3.1 If K_{N_1, M_1} is the number the coefficients $x\rho$ for the filter $F_a(N_1, M_1)$, then increasing K_{N_1, M_1} by one (adding the new coefficient $x\rho_{new}$), which is effective in a new filter $F_a(N_2, M_2)$, follows when and only when

$$F_a(N_1, M_1) \rightarrow F_a(N_2, M_2) \Leftrightarrow FK(N_1, M_1; N_2, M_2) = \frac{2}{K_{N_1, M_1}} \cdot \frac{xR_{N_1}^{\{M_1\}}}{|x\rho_{new}|^2} < \delta \quad (7)$$

'The partial actualization' by a coefficient $x\rho_{new}$ can be done by changing N or/and M . This allows to check the different combinations of N and M . The choice for testing the new Schur coefficient depends on the maximal value of $K_{N, M}$. Using proposed strategy 3.1 the following rules are proposed:

- for determining M increase the parameter N up to reach maximal value $K_{N, M}$; it means the linear, bi-linear, tri-linear,... elements of ON basis are checked, and choosing is one for which $x\rho_{new}$ meets (3.1)

Selection the values of N i M is stopped when the desire value of (3) is reached.

Figure 5 presents the process of adding a new coefficient $x\rho_{new}$ to the filter structure.

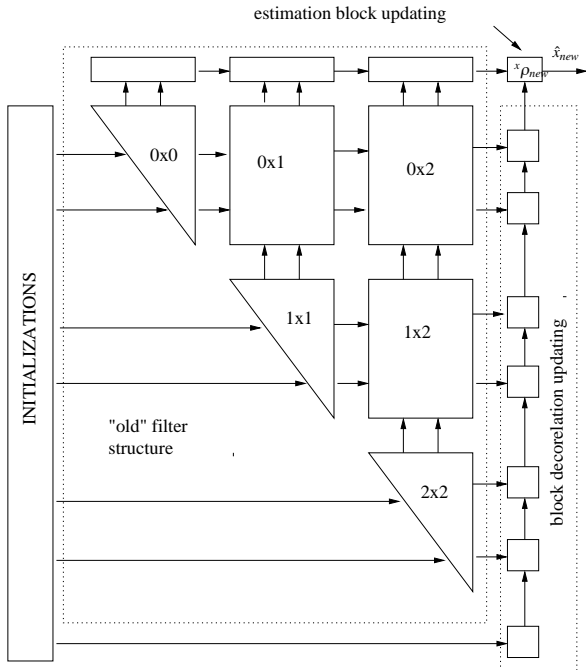


Figure 5: Process of updating the filter structure

Using proposed strategy it is possible to determine the $x\rho$ to eliminate these which only insignificant minimize the

value of (3). It is impossible to determine one way of the parameters selection. Every configuration corrupted signal - reference signal is specific and requires the individual decision process.

It is easy to notice it is not important if the next Schur coefficient $x\rho$ is higher order or higher degree. In both cases the new column of elementary sections is added to the decorelation block.

It should be notice, the new coefficient $x\rho_{new}$ can not be the neighbor of already existing one [1]. If the last determined $x\rho$ came from the filter eg: $N = 2, M = 3$ the new $x\rho_{new}$ can come from the filter eg: $N = 5, M = 10$.

4. SIMULATIONS

To present proposed strategy in first simulation a telecommunication signal (MSK modulation) was used (figure 6). Corrupting signal had nonsymmetric probability density

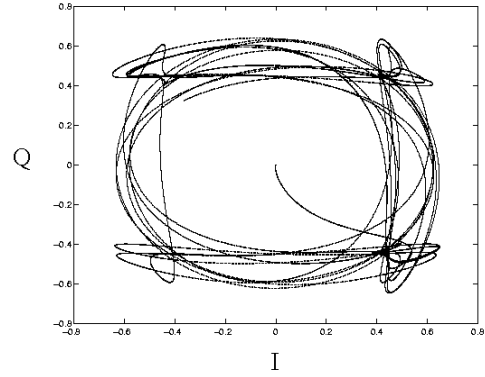


Figure 6: IQ graph of original MSK signal

function, and was lowpass. Bandwidths of both signals overlap. Figure 7 presents the IQ graph of corrupted signal. It is impossible to demodulate the original signal because of noise. Figure 8 presents demodulated signal after filtering.

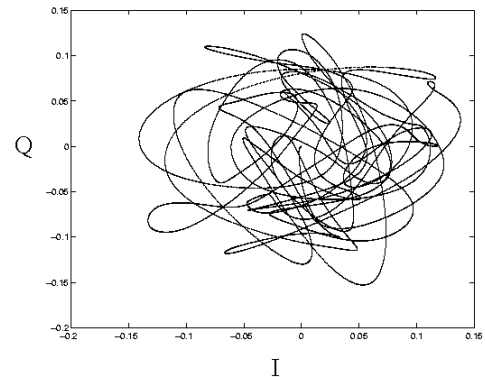


Figure 7: IQ graph of corrupted MSK signal

The filter of parameters $M = 3$ and $N = 10$ was used. Improvement of S/N ratio is about 14dB.

The decorelation block had 40755 elementary sections and the estimation block had 284 elementary sections. Using criterion (3.1) and choosing $\delta = 100$, for every instant of time the number of the estimation block elementary sections was established, presented in figure 9. It can be seen how the

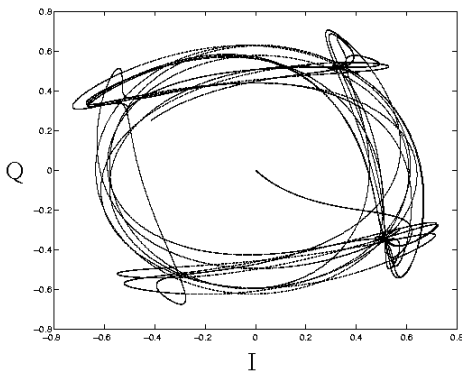


Figure 8: IQ graph of MSK signal after filtering

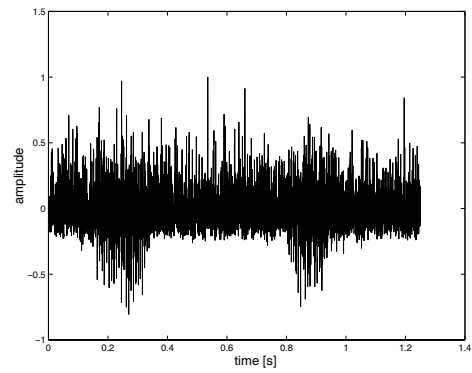


Figure 11: Corrupted speech signal

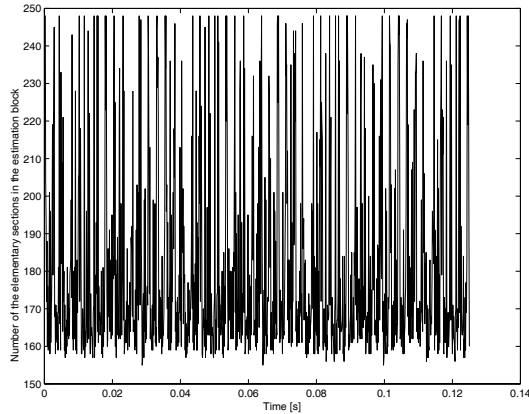


Figure 9: The number of the estimation block elementary sections for strategy 3.1

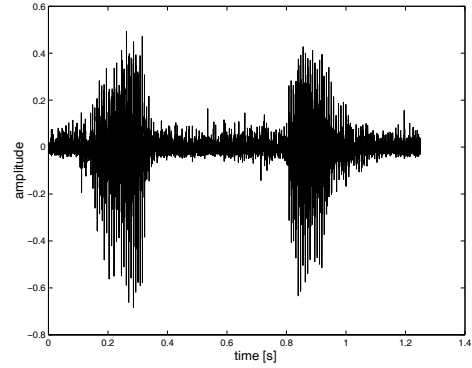


Figure 12: Speech signal after filtering

structure of the filter is changing during the time.

In the second simulation speech signal was used 10.

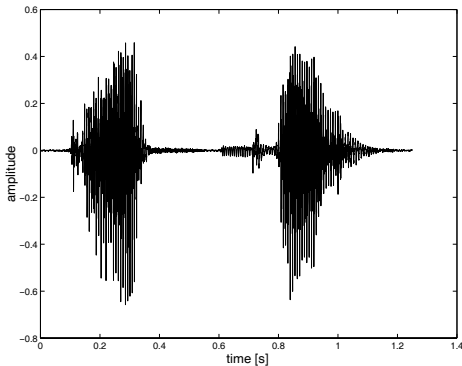


Figure 10: Original speech signal

In this case calculated improvement of the S/N ratio is about $MN_Q = 12dB$ (see 11 and 12). Optimized filter is worse than the original one but only for about 2dB. Presented simulation required 11634×10^3 hyperbolic rotations for original filter and 837×10^3 for optimized filter. It means that proposed selection of the filters parameters allowed to decrease the computing complexity 13.9 times and only 2dB of quality improvement was lost.

5. CONCLUSION

Presented results allows to draw the following conclusions:

- The structure of presented filters makes possible their rebuilding very easy. Using proposed strategy of filter parameter selection the filter complexity can be adopted to filtered signals.
- The value of the estimation error can be controlled by selecting filter structure during filtering process.

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