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Nonlinear Equalization Structure for High-Speed ADSL in Ideal and Non Ideal Conditions

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Abstract–**The ADSL G.DMT technology, based on discrete multi-tone (DMT) modulation, employs an equalization structure consisting of a time-domain equalizer (TEQ) and a frequency-domain equalizer (FEQ). These two complementary systems allow good immunity against inter-symbol interference (ISI), to improve the bit error rate while optimizing the throughput. In literature, some of these equalization structures are proposed, but they are usually tested in ideal conditions; with perfect knowledge of channel characteristics and with linear channels. In this paper, we present a TEQ and a FEQ designed for non ideal transmission conditions, while keeping with the technology requirements. A nonlinear TEQ based on neural network structure is proposed to attenuate the interference due to the non ideal conditions.**

I. INTRODUCTION

In order to combat inter-symbol interference (ISI), discrete multi-tone (DMT) modulation [1]-[3] technique, also known as orthogonal frequency division multiplexing (OFDM), showed more efficiency in robustness and complexity than total equalization techniques for high-speed digital transmission [2]. In practice, one solution to counterbalance the negative effect of ISI is to add cyclic prefix (CP) (e.g. [2] and [5]) along with the use of an equalization structure consisting of a TEQ and a FEQ.

In the literature, usually, a one-tap (1-tap) FEQ method employing a single weight per subchannel is used. While main research was especially conducted on the TEQ's design. So, several methods were derived namely; minimum mean square error (MMSE) [7], minimum shortening SNR (MSSNR) [8], maximum geometric SNR (MGSNR) [9], maximum bit rate (MBR) [6] and minimum ISI (Min-ISI) algorithms [6]. MMSE and MSSNR methods, if different, try only to accomplish the first objective of a TEQ, i.e., eliminating the ISI through channel reduction. Both TEQs work on the medium impulse response [2], [4]. MGSNR, MBR and Min-ISI techniques, on the other hand, have been designed to satisfy the second TEQ expectations, i.e., to improve the bit rate too. Since it is based on stronger approximations [6], MGSNR method does not allow approaching the optimal rate. MBR presented by Arlan and Evans [6] is an almost optimal method for channel reduction and the obtained bit rate. However, this one is not practical in real time due to its high implementation complexity level. The same authors worked on this method in order to make it practical, while preserving same performances. The derived method is the Min-ISI [6] which can be considered as the most powerful method [11]. While analyzing Min-ISI, two points triggered our attention: (i) it requires, during its processing, the knowledge of the channel impulse response. It thus implies the use of a channel estimator. (ii) the performances of this method have been given for ideal simulation conditions, i.e., linear channel with a perfect estimates. These simulation hypotheses are arguable. Indeed, first of all, there are always errors during estimation. Other phenomenon to be mentioned; the DMT nature can imply the appearance of nonlinearities during the transmission [12].

This paper evaluates an equalization structure based on neural network taking into account non ideal conditions, the presence of estimation errors on channel coefficients and nonlinearities. The progress allowing reaching such a result led us to realize a TEQ and a FEQ. Section II presents our realized TEQ, named TEQ-NL. In Section III, we will introduce the proposed FEQ. The simulation results, presented in Section IV, compare the Min-ISI and TEQ-NL methods for various conditions of simulation. The conclusion will be established in Section V.

II. NONLINEAR TEQ BASED ON NEURAL NETWORK

To immunize the TEQ method against channel estimation errors, we opt for a "direct" adaptive method with the same structure than the direct MMSE method [7], Fig. 1. This technique consists of a filter, TEQ block, in cascade with the channel, represented by the impulse response $h(k)$ and the additive noise $\eta(k)$, and of a parallel branch established for a delay Δ and a FIR filter, the target impulse response, block TIR; with k the integer indicating the sampling moment. When the error, $e(k)$, approaches zero for any input signals in the channel, the equalized channel impulse response SIR (*Shortening Impulse Response*) will be equal to the version of the TIR delayed by Δ. Of course this linear TEQ can not be robust against a nonlinear channel. So, in this paper, we replaced the linear filter structure of the TEQ by a multi-layers neuronal network (MNN), Fig. 2. Indeed, neuronal networks showed their ability to take into account and to fight channel's nonlinear effects [11]-[12].

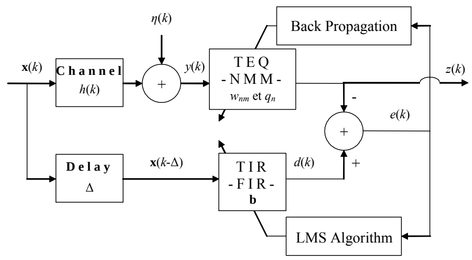

Fig. 1 Structure of the TEQ method

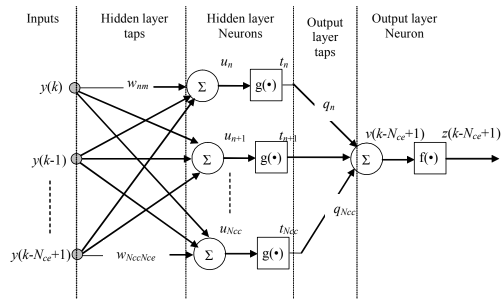

Fig. 2 MNN structure used for the TEQ by the TEQ-NL method

The following equations describe the forward propagation in the MNN:

$$u_n = \sum_{m=1}^{N_{ce}} w_{nm}\, y(k-m+1) \tag{1}$$

$$t_n = \mathrm{g}(u_n) \tag{2}$$

$$v(k) = \sum_{n=1}^{N_{cc}} q_n t_n \tag{3}$$

$$\mathrm{z}(k) = \mathrm{f}(v(k)) \tag{4}$$

$$d(k) = \mathbf{b}(k)\, \mathbf{x}(k-\Delta) \tag{5}$$

where the signals u_n and t_n represent the sum and the output of the n^{th} neuron of hidden layer respectively. N_{ce} and N_{cc} are the number of inputs and hidden layers. Equation (5) corresponds to the propagation of the signal $x(k)$ delayed by Δ in the TIR; the vector $\mathbf{b}(k)=[b_1(k),b_2(k),\cdots,b_{Nb}(k)]$ represents the weights of the TIR, with N_b being the size of the FIR of the TIR.

The adaptation of the taps w_{nm} and q_n is made from the error between the output network, $z(k)$, and the desired signal, $d(k)$. This error, $e(k)$, propagates then from the network output towards its inputs to adjust all the taps of the structure. Weight adaptation is done using gradient backward propagation described by the following equations:

$$e(k) = d(k) - z(k) \tag{6}$$

$$q_n(k+1) = q_n(k) + \mu_{\mathrm{w}}\, e(k)\, t_n(k) \tag{7}$$

$$w_{nm}(k+1) = w_{nm}(k) + \mu_{\mathrm{w}}\, e(k)\, q_n(k)\, \mathrm{g}'(u_n(k))\, y_m(k) \tag{8}$$

$$\mathbf{b}(k+1) = \mathbf{b}(k) - \mu_{\mathrm{b}}\, e(k)\, \mathbf{x}(k-\Delta) \tag{9}$$

Equation (9) identifies the weights adaptation of the TIR by an LMS method using the error $e(k)$. The constants μ_{w} and μ_{b} represent the step-sizes used to adapt TEQ and TIR structures. During the learning of the TIR, it is necessary to avoid the trivial solution. A constraint has thus been added in the TIR algorithm using unit-energy constraint (UEC) formulation [7]. This constraint imposes the norm of all the weights of the TIR to 1 at each iteration.

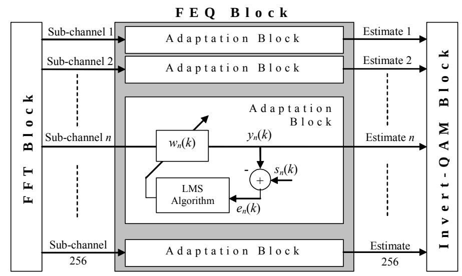

Fig. 3 FEQ adaptation blocks by subchannel.

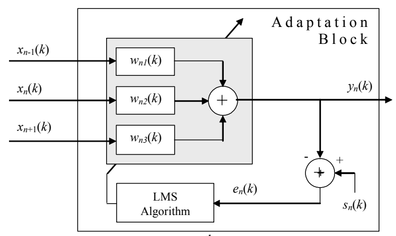

Fig. 4 Adaptation block of the n^{th} subchannel of the 3-taps FEQ

III. FEQ ALGORITHM

Usually, the FEQ method based on one-tap [2] can be generalized on n-taps such as shown in Fig. 3. Every input enters in one adaptation block. For the n^{th} subchannel, with $1 \le n \le 256$, the value of this subchannel is multiplied by the n^{th} taps vector $\mathbf{w}_n(k)=w_{n1}(k)$. This tap is adapted in order to minimize $e_n(k)=s_n(k)-y_n(k)$, with $s_n(k)$ being the desired signal. This process is carried independently but simultaneously for all 256 subchannels.

We can easily understand that the FEQ efficiency depends on the TEQ used. The one-tap FEQ was sufficient because until now the TEQ consists of adapting the taps of a simple FIR filter. But the TEQ-NL is a more complex structure. It is thus likely that the one-tap FEQ will not be able to invert perfectly the frequency response of the channel equalized by a NMM structure. For the n^{th} subchannel, Fig. 4, we use the information of two neighboring subchannels, which are those of index n-1 and n+1, in order to improve estimation of the subchannel n. The input vector of the n^{th} subchannel becomes, in the case of our FEQ, $\mathbf{x}_n(k)=[x_{n-1}(k)\ x_n(k)\ x_{n+1}(k)]^{\mathrm{T}}$ and the taps vector $\mathbf{w}_n(k)=[w_{n1}(k)\ w_{n2}(k)\ w_{n3}(k)]$. We thus named this FEQ, a 3-taps FEQ, and this one is described by the following equations:

$$y_n(k) = \mathbf{w}_n(k)\, \mathbf{x}_n(k) \tag{10}$$

$$e_n(k) = \mathrm{s}_n(k) - y_n(k) \tag{11}$$

$$\mathbf{w}_n(k+1) = \mathbf{w}_n(k) + \mu_n\, e_n(k)\, \mathbf{x}_n(k)^* \tag{12}$$

$$\mu_n(k) = \frac{e_n(k)'\, e_n(k)}{e_n(k)'\, x_n(k)'\, x_n(k)\, e_n(k)} \tag{13}$$

Respectively, equations (10), (11) and (12) represent the propagation of the inputs $\mathbf{x}_n(k)$ into the FIR of three taps, the calculation of the error and the adaptation of the taps vector using LMS algorithm. The expression (13) makes the step-size μ_n dynamic according to the steepest descent method [10] using the error $e_n(k)$. Without this equation, it would have been necessary to determine the 256's μ_n of the 256 adaptation blocks. Note that x' and x^* are respectively transposed conjugate and conjugate of x.

IV. Performance Evaluations

In this section, TEQ-NL and Min-ISI associated to the 1-tap and 3-taps FEQ will be tested for various simulation conditions.

A. Simulation conditions

The channel used in our simulations is the carrier serving area (CSA) standard, namely the CSAloop#4 [2],[4]. The impulse response of this channel constitutes of 512 samples taken with a frequency of 2.208 MHz. A highpass Chebyshev filter of the fifth order with a cut frequency of 5.4 KHz has been added to this CSA channel to take into account the effect of the shaping filter in the modem [6]. Furthermore, subchannels from 1 to 5 on 256 are not taken into account, because they are in the frequency band used by the plain old telephone system (POTS) [1]. The noise of the canal is modeled by an additive white Gaussian noise (AWGN) with a power of 140 dBm/Hz distributed on all the bandwidth. The transmitted sequence power is 23 dBm distributed fairly in all the used subchannels and the size of the FFT is of N=512. The SNR Gap, Γ, equal to 11.6 dB, is calculated with a system margin of 6dB and a coding gain of 4.2 dB. There is no algorithm for bit loading in this system; all the bit rates presented in simulation are calculated by a distribution of the SNR once the TEQ is adapted. The allocation of the bits is supposed to be constant. Finally the Reed Solomon decoder is not considered in the ADSL receiver in our simulations.

B. Perfect knowledge of the channel

We are going to present here the performances in term of channel reduction, bit rate and raw bit error rate (BER), which is the BER at the decoder input. In order to follow the standard of the technology and immunize against ISI, the TIR filter size has been put at N_b=33, N_b=υ+1, for both TEQ. Values of the parameters of both studied methods bringing the best results, for Min-ISI the optimal delay chosen from the set {15, 35} and the size of the TEQ is N_w=16. Our learning sequences are established by 800 frames of 512 data. The TEQ-NL the parameters are Δ=26, N_{ce}=12, N_{cc}=16, μ_w=0.71 and μ_b=0.041.

One of the ways to calculate the channel reduction consists of calculating the shortening SNR (SSNR) [8]. The larger the SSNR the more reduced the energy out of a target window. A value of SSNR superior to 20dB can be considered as satisfactory in term of channel reduction. Fig. 5a depicts the results in SSNR obtained for both TEQ methods as a function of the additive noise. Fig. 5b shows the bit rate performances given according to the equation presented in [9] for both TEQ

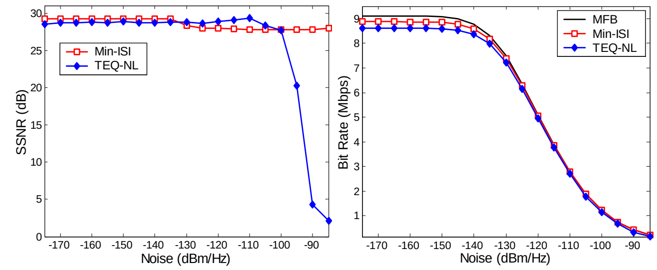

Fig. 5 Average plots on 20 iterations of the SSNR (a) and bit rate (b) for Min-ISI and TEQ-NL according to the additive noise.

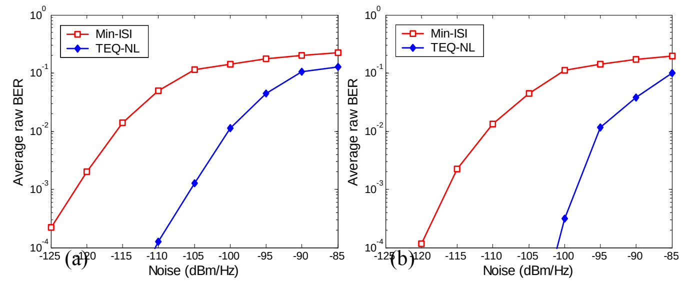

Fig. 6 Average plots on 20 iterations of the raw BER with the 1-tap (a) and the 3-taps (b) according to the additive noise for Min-ISI and TEQ-NL.

studies compared the matched filter bound (MFB) [2],[6] used as reference method. From Fig. 5b, we observed that:

- Min-ISI immunizes the system against the ISI, even with the strongest power of injected noise, -85dBm/Hz. Indeed, the obtained SSNR remains superior to 20 dB. TEQ-NL reduces sufficiently the channel until a noise of -95dBm/Hz is reached.
- For the obtained bit rate, Min-ISI is more powerful than TEQ-NL, its bit rates according to the noise are more close to the curve of the maximum reachable bit rate. From a power of noise of -100dBm/Hz, TEQ-NL gives bit rates more close to Min-ISI and to the MFB.

The Fig. 6 shows our results obtained for raw average BER. We compared the TEQ with FEQ 1-taps and 3-taps. This raw BER arises from an average dependent on the number of iterations indicated on every curve from a sequence of 200 000 data. According to Fig. 6, we observed the noise level to target a raw BER of 10^{-3} :

- Coupled with the 1-tap FEQ, Min-ISI allows obtaining the targeted BER until -123dBm/Hz noise level. The TEQ-NL/1-tap structure returns finally better results than the Min-ISI/1-tap structure with satisfactory values until a noise level of -107dBm/Hz.
- The 10^{-3} value BER obtained with the Min-ISI/3-taps structure are valid until a noise level of -117dBm/Hz. The use of the 3-taps FEQ permits to improve the BER performances of the Min-ISI/FEQ structure. However, the TEQ-NL with FEQ 3-taps stays the most robust in front of noise up to -98dBm/Hz.

C. Non-ideal conditions

In order to have the non-ideal simulation conditions, we introduced nonlinearities due to the amplifier of the transmitter [12]. The expression representing the addition of

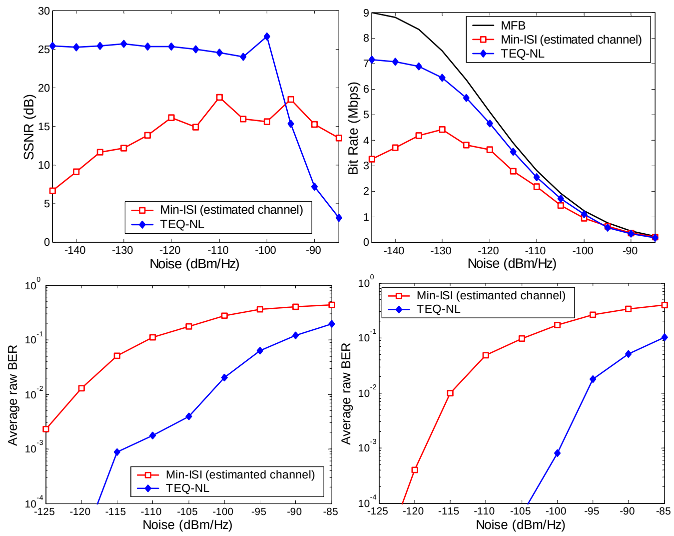

Fig. 7 Average plots on 30 iterations of the SSNR (a), bit rate (b) and raw BER coupled with the FEQ 1-tap (c) and with the 3-taps (d) according to the Min-ISI with channel estimation [13] and the TEQ-NL.

these nonlinearities is given by x_{ampl}=20/(1+e$^{0.2x}$) where x is the signal stemming from the modulation DMT and x_{ampl} the output signal of transmitter's amplifier. The estimation of the coefficients of the channel is based on the method in [13], over 400 samples. So here channel estimation for Min-ISI is not perfect. Thus, it is necessary to search for new parameters for TEQ-NL: N_{ce}=12, N_{cc}=45, μ_w=0.7 and μ_b=0.06. Fig. 7 presents the results in term of SSNR, bit rates and raw BER, we have observed:

- Min-ISI does not reach any more to immunize the system against the ISI. Indeed, the maximal values of SSNR do not exceed 20 dB. TEQ-NL gives satisfactory values of SSNR at even noise powers of -97dBm/Hz.
- In term of bit rate, Min-ISI does not allow in these conditions to follow the curve of the optimal MBF bit rate. Its maximum attainable bit rate is 4.4Mbps for -130dBm/Hz, while TEQ-NL and MFB returns respectively 6.5Mbps and 7.5Mbps to the same noise. For TEQ-NL, its fight against nonlinearities made decrease its obtained bit rate in the ideal conditions. However, bit rates reached with the TEQ-NL follow relatively those of the MFB. It is not any more the case of the bit rate reached by Min-ISI.
- The Min-ISI/1-tap structure returns raw BER lower than 10^{-3} until a noise of -132dBm/Hz. The same structure for ideal conditions was functional until -123dBm/Hz. For TEQ-NL associated with the 1-tap, the obtained raw BER is correct up to -114dBm/Hz, against -107dBm/Hz in ideal conditions.
- In the same way, Min-ISI/3-taps reaches 10^{-3} raw BER up to -119dBm/Hz and until -100dBm/Hz for the TEQ-NL/3-taps. These values of noise, for both structures, remain very close that those obtained for ideal conditions. It explains by the good faculty which has the 3-taps to invert the equalized frequency response of the channel.

V. Conclusion

The introduction of the MNN to replace the FIR structure in the basic MMSE structure allowed obtaining, in ideal conditions, performances close to those of the Min-ISI at the expense of good parameters' search. Coupled with the 1-tap FEQ, the TEQ-NL gives better results in term of BER than Min-ISI. This equalization structure, TEQ-NL / 3-taps, proves to be the most robust against noise. We observe that this structure satisfies the technology requirements, even in presence of estimation errors and nonlinearities. The Min-ISI/1-tap structure provides the best performances in SSNR and bit rate in ideal conditions but is less robust against noise as compared to TEQ-NL / FEQ structure. With the addition of nonlinearities and estimation errors, the Min-ISI results are not valid any more. Finally we notice the contribution of the 3-taps FEQ. This one allows a considerable improvement of the results in terms of raw BER as well as a stronger robustness against additive noises.

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