14th European Signal Processing Conference (EUSIPCO 2006), Florence, Italy, September 4-8, 2006, copyright by EURASIP

A PERFORMANCE COMPARISON OF TWO TIME DIVERSITY SYSTEMS USING CMLD-CFAR DETECTION FOR PARTIALLY CORRELATED CHI-SQUARE TARGETS AND MULTIPLE TARGET SITUATIONS

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ABSTRACT

In radar systems, detection performance is always related to target models and background environments. In time diversity systems, the probability of detection is shown to be sensitive to the degree of correlation among the target echoes. In this paper, we derive exact expressions for the probabilities of false alarm and detection of a pulse-to-pulse partially correlated target with 2K degrees of freedom for the Censored Mean Level Detector Constant False Alarm Rate (CMLD-CFAR). The analysis is carried out for the "non conventional time diversity system" (NCTDS). The obtained results are compared with the "conventional time diversity system" (CTDS) in both single and multiple target situations.

1. INTRODUCTION

According to Swerling's models, if only one pulse per scan hits a target, we cannot distinguish between cases I and II and cases III and IV. However, if multiple pulses are transmitted per antenna scan, the problem of detecting slow fluctuating targets and fast fluctuating targets can be easily overcome. Nevertheless, we should take into consideration the partial correlation of the target signal, otherwise the processor fails to predict the actual system performance. In other words, the more we know about the statistics of the target signal, the better the detection is.

In the literature of CFAR detection, the echoed signals of the transmitted pulses are processed non coherently within the same receiver. Dealing with either uncorrelated or partially correlated data samples, we often seek to improve detection while maintaining a constant false alarm rate. Several authors have considered different applications of the non coherent integration. Here, we only list very few of them [1-3]. In [1], Kanter has studied the detection performance of a noncoherent integration detector accumulating M-correlated pulses from a Rayleigh target with two degrees of freedom. The noise was assumed to be uncorrelated. Wiener [2] extended the work in [1] by deriving exact expressions for the probabilities of detection for partially correlated chi-square targets with four degrees of freedom. The work done in [1, 2] used a fixed threshold detection. It is known that radar detectors with fixed threshold can not maintain a CFAR, and thus adaptive threshold detection is considered. Hou [3] used the method of residues to evaluate exact formulas for the detection performance for the chi-square family with 2K degrees of freedom.

The idea of processing independently the received target pulses to yield preliminary decisions in distributed CFAR detection, was first suggested by Himonas and Barkat [4, 5]. They studied the case of partially correlated target returns with different architectures of time diversity and distributed CFAR detectors to minimize the effect of the correlation factor among the received target pulses. They called it "time diversity systems" referring to multiple-pulse systems. El Mashade [6, 7] has thoroughly developed this idea by considering the integration of all the individual noise level estimates. More precisely, as shown in figure 1, the reference samples of the individual pulse returns are ranked in an ascending order. Then, each ordered window is processed by the suited one-pulse order-statistic algorithm. Finally, the obtained noise level estimates are added to get the overall background level. Consequently, for the sake of comparison, we shall adopt in this paper the terminology "conventional time diversity system" (CTDS) to refer to the non coherent integration accumulating many pulses and processing them as an entity to form the noise level estimate [1-3]. and "non conventional time diversity system" (NCTDS) to refer to the technique used in [6, 7].

In summary, we observe that the work using the NCTDS did not show a comparison of the CMLD-CFAR detector with its corresponding detector for the CTDS in neither single nor multiple target situations. Moreover, the two systems did not consider the general case of a pulse-to-pulse partially correlated chi-square target with 2K degrees of freedom. To complete the study, we introduce a detailed detection analysis for a mathematical model representing the case of detecting a pulse-to-pulse chi-square partially correlated target with 2K degrees of freedom embedded in a pulse-to-pulse Rayleigh and uncorrelated thermal noise.

The paper is organized as follows. In Section 2, we formulate the statistical model. In Section 3, we derive the exact false alarm probability (P_{fa}). Then, in Section 4, we give the moment generating function (mgf) of the test cell under hypothesis H_1 in terms of K and use it to derive the

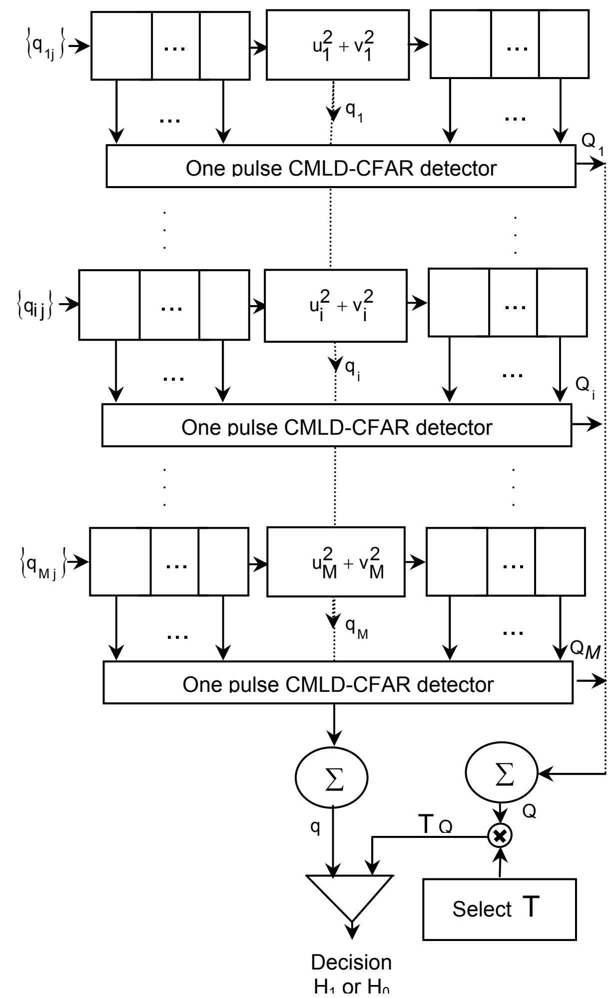

Figure 1 - Decision Element: NCTDS CMLD-CFAR detector

exact detection probability (P_d). Next, in Section 5, by deriving detection curves, we show the performances of the detector. A conclusion is given in Section 6.

2. STATISTICAL MODEL

The received signal is processed by the in-phase and quadrature phase channels. Assuming a correlated chi-square target with 2K degrees of freedom embedded in uncorrelated noise, the in-phase and quadrature phase samples $\{a_{ij}\}$ and $\{b_{ij}\}$ at pulse i and range cell j, respectively, i=1, 2, 3, …, M and j=1, 2, 3, …, N, are observations from Gaussian random variables. M and N are the number of radar processed pulses and the number of reference range cells, respectively. Assuming that the total noise power is normalized to unity, the output of the (i, j) $^{\text{th}}$ cell, is

$$q_{ij} = \frac{1}{2}\left\{a_{ij}^2 + b_{ij}^2\right\} \tag{1}$$

The thermal noise samples are assumed to be independent and identically distributed (IID) random variables with zero mean and variance σ_n^2 ($\sigma_n^2 = 1$) from pulse-to-pulse and from cell-to-cell.

The detection performance is based upon the statistics of q, which is given by

$$q = \frac{1}{2}\sum_{i=1}^{M}\left\{u_i^2 + v_i^2\right\} \tag{2}$$

Extending the target model introduced in [2] to 2K degrees of freedom, we have

$$u_i = x_i + a_i \text{ and } v_i = y_i + b_i\,,\quad \text{i=1, 2,…, M} \tag{3}$$

where $x_i = \left(\sum_{k=1}^{K} x_{ik}^2\right)^{1/2}$ and $y_i = \left(\sum_{k=1}^{K} y_{ik}^2\right)^{1/2}$ (i=1, 2, ..., M) are, respectively, the magnitudes of the in-phase and the quadrature phase components of the complex target signal at pulse i, present in the test cell q. x_i^2 and y_i^2 represents each the sum of the squares of K real Gaussian variables. K is called either the fluctuation parameter or the number of degrees of freedom. a_i and b_i represent the in-phase and quadrature phase samples of the uncorrelated thermal noise. The target signal is assumed to be independent from the thermal noise signal. The in-phase samples are assumed to be independent of the quadrature phase samples.

A useful representation of the target signal vector, which will be used later in this paper, is

$$\mathbf{X} = \left[\mathbf{X}_1^T,\ \mathbf{X}_2^T, ..., \mathbf{X}_K^T\right]^T \text{ and } \mathbf{Y} = \left[\mathbf{Y}_1^T,\ \mathbf{Y}_2^T, ..., \mathbf{Y}_K^T\right]^T \tag{4}$$

where, $\mathbf{X}_k = \left[x_{1k}, \cdots, x_{Mk}\right]^T$, $\mathbf{Y}_k = \left[y_{1k}, \cdots, y_{Mk}\right]^T$ and k=1, 2,…, K. The M x 1 target in-phase vectors $\mathbf{X_1}$, $\mathbf{X_2}$,…, $\mathbf{X_k}$ and quadrature phase vectors $\mathbf{Y_1}$, $\mathbf{Y_2}$, …, $\mathbf{Yk}$ are independent from each other but their respective components are correlated pulse-to-pulse. The K-block in-phase and quadrature phase target vectors $\mathbf{X}$ and $\mathbf{Y}$ of the correlated chi-square target model with 2K degrees of freedom are uncorrelated, but their respective components are correlated with a known correlation matrix $\mathbf{\Lambda_t}$.The random variables x_{ik} and y_{ik} (i=1, 2, …) representing the target samples are assumed to be first-order Markov processes with zero mean and variance σ_t^2. Hence, the $(i, j)^{th}$ element of the covariance matrix of the target process $\mathbf{\Lambda_t}$ can be expressed as

$$\left[\mathbf{\Lambda}_t\right]_{i,j} = \begin{cases} \sigma_t^2\, \rho_t^{|i-j|} & 0 \langle \rho_t \langle 1 \\ \sigma_t^2\, \delta_{ij} & \rho_t = 0 \\ \sigma_t^2 \quad \forall i,j & \rho_t = 1 \end{cases} \tag{5}$$

According to Edrington's measurements, if M pulses hit a target, the return echoes from commonly encountered models are exponentially correlated. $\rho_t = \exp(-T_R\omega_t)$ is the correlation coefficient between the pulse-to-pulse received target samples for a given k, where $f_t = \frac{\omega_t}{2\pi}$ is the mean Doppler frequency of the target signal.

Our model assumes that any value of $K \geq 1$ is realizable. The four Swerling cases (I, II) and (III, IV) correspond to K=1 and K=2, respectively. In this manner, we can model the partial correlation between pulses as in [1, 2]. Thus, for example, to model Swerling cases I and II, we should set K=1, but $\rho_t = 1$ and $\rho_t = 0$, respectively.

The test cell q is then compared to the adaptive threshold TQ to make a decision H_1 or H_0, according to the following hypothesis test

$$q \underset{H_0}{\overset{H_1}{\gtrless}} TQ \tag{6}$$

Q denotes the estimated background level, H_0 denotes the absence of a target while H_1 denotes the presence of a target. The probabilities of false alarm and detection of a CFAR detector can be obtained by using the contour integral, which can also be expressed in terms of the residue theorem as in [3] to yield

$$P_{fa} = -\sum_{i_0} \operatorname{res}\left[s^{-1}\Phi_{q|H_0}(s)\Phi_Q(-Ts), s_{i_0}\right] \tag{7}$$

$$P_d = -\sum_{i_1} \operatorname{res}\left[s^{-1}\Phi_{q|H_1}(s)\Phi_Q(-Ts), s_{i_1}\right] \tag{8}$$

where res [.] denotes the residue. s_{i_0} (i_0 = 1, 2, …) and s_{i_1} (i_1 = 1, 2, …) are, respectively, the poles of the moment generating function (mgf) $\Phi_{q|H_0}(s)$ of the noise and the mgf $\Phi_{q|H_1}(s)$ of the target plus noise, lying in the left-hand of the complex s-plane. $\Phi_Q(-Ts)$ is the mgf of the estimated background level evaluated at s= -Ts.

3. EVALUATION OF THE FALSE ALARM PROBABILITY

In order to derive the exact expression for the P_{fa}, we must evaluate the mgf of the test cell q in the absence of a target and the mgf $\Phi_Q(s)$.

The mgf of q in the absence of a target is defined, in terms of the noise vectors, as

$$\Phi_{q|H_0}(s) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} p(\mathbf{A},\mathbf{B}) \exp\left[-\frac{1}{2}s\left(|\mathbf{A}|^2 + |\mathbf{B}|^2\right)\right] d\mathbf{A}\, d\mathbf{B} \tag{9}$$

As the in-phase and quadrature phase samples of the noise are IID, therefore the joint probability density function (pdf) of **A** and **B** satisfies $p(\mathbf{A},\mathbf{B}) = p(\mathbf{A})p(\mathbf{B})$ and $p(\mathbf{A}) = p(\mathbf{B})$. $p(\mathbf{A})$ is the joint Gaussian pdf of the in-phase noise vector with zero mean and identity covariance matrix which generates uncorrelated thermal noise samples [1]

$$p(\mathbf{A}) = \frac{1}{(2\pi)^{M/2}} \exp\left(-\frac{1}{2}|\mathbf{A}|^2\right) \tag{10}$$

Combining equations (9) and (10) and after some mathematical manipulations, the mgf of the cell under test can be expressed as

$$\Phi_{q|H_0}(s) = (1+s)^{-M} \tag{11}$$

The poles of the mgf of Q under hypothesis H_0 are a simple pole at $s = -1$ of multiplicity M lying in the left-hand s-plane.

The overall background noise level q is estimated by taking the average over the M pulses as follows

$$Q = \sum_{i=1}^{M} Q_i \tag{12}$$

The background noise level Q_i of the CMLD-CFAR is estimated by the average of the lowest N-L cells taken among the N ordered cells [8]. L is the number of the largest cells and is assumed to be the same for all processed pulses. Hence, Q_i is defined as

$$Q_i = \sum_{j=1}^{N-L} q_{i(j)} \qquad i=1, 2, \ldots, M \tag{13}$$

Assuming that Q_i, i=1, 2, …, M are IID from pulse-to-pulse, we obtain the mgf of Q as [9]

$$\Phi_Q(s) = \binom{N}{N-L}^M \prod_{j=1}^{N-L}\left(s + a_j\right)^{-M} \tag{14}$$

where, $a_j = (N-j+1)\,(N-L-j+1)^{-1}$

The P_{fa} of the CMLD-CFAR detector can be evaluated by substituting equations (11) and (14) for s = -T s into equation (7). Then by using the partial-fraction expansion, we obtain

$$P_{fa} = -\binom{N}{N-L}^M \frac{1}{\Gamma(M)\,(-T)^{M(N-L)}} \frac{d^{M-1}}{ds^{M-1}}\left[s^{-1}\left(\sum_{l=1}^{N-L}\frac{k_l}{s-s_l}\right)^M\right]_{s=-1} \tag{15}$$

where, $k_l = T^{N-L-1}\prod_{\substack{i=1\\ i\neq l}}^{N-L}\left(a_l - a_i\right)^{-1}$ and $s_l = T/a_l$

4. EVALUATION OF THE DETECTION PROBABILITY

Now let us derive the exact expression for the detection probability. In doing this, we study the effect of the correlation coefficient ρ_t of the target returns on the detection performance. The determination of the P_d given by equation (8) requires the knowledge of the mgf of the test statistic q under H_1 and the mgf of the background noise level Q of the CMLD-CFAR detector.

Taking into account that $\mathbf{X}$ and $\mathbf{Y}$ are independent, **A** and **B** are independent and that the noise signal is Gaussian, stationary and independent of the target signal, we can write the mgf of q in the presence of the target for any target model [2, as Eq. (7)], as

$$\Phi_{q|H_1}(s) = \frac{1}{(1+s)^M}\left[\int_{-\infty}^{+\infty} p(\mathbf{X})\exp\left(-\frac{s|\mathbf{X}|^2}{2(1+s)}\right)d\mathbf{X}\right]^2 \quad (16)$$

The target K-bloc vector $\mathbf{X}$ as defined in (4), has a multivariate Gaussian pdf, defined by [10]

$$p(\mathbf{X}) = \frac{1}{(2\pi)^{KM/2}\,|\mathbf{\Lambda}|^{1/2}}\exp\left(-\frac{1}{2}\mathbf{X}^T\mathbf{\Lambda}^{-1}\mathbf{X}\right) \quad (17)$$

The K x K block diagonal covariance matrix $\mathbf{\Lambda}$ is defined as

$$\mathbf{\Lambda} = \text{diag}\,(\mathbf{\Lambda}_t, ..., \mathbf{\Lambda}_t) \quad (18)$$

Note that since $\mathbf{\Lambda}_t^{-1}$ exists then, $\mathbf{\Lambda}^{-1}$ also exists.

In order to find the mgf of the test statistic under H_1, we need to evaluate equation (16). That is, we first define the pdf of the vector X_k, k=1, 2, …, K, as in [2].

$$p(\mathbf{X_k}) = \frac{1}{(2\pi)^{M/2}|\mathbf{\Lambda}_t|^{1/2}}\exp\left(-\frac{1}{2}\mathbf{X}_k^T\mathbf{\Lambda}_t^{-1}\mathbf{X}_k\right) \quad (19)$$

Since the M x 1 target in-phase vectors X_1, X_2,…, X_k and quadrature phase vectors Y_1, Y_2, …,Y_k are independent from each other, we introduce the pdf of the K-dimensional vector $\mathbf{X}$ as a generalization of [2, as Eq. (10)]

$$p(\mathbf{X})\,d\mathbf{X} = \prod_{k=1}^{K} p(\mathbf{X}_k)\; d\mathbf{X}_k \quad (20)$$

From equation (4), we define also the norm of the vector $\mathbf{X}$, i.e. $|\mathbf{X}|^2$ as

$$|\mathbf{X}|^2 = \sum_{k=1}^{K}\mathbf{X}_k^T\mathbf{X}_k \quad (21)$$

Inserting equation (17) into (16) and integrating over the X_k's, we may write, by using equations (18) to (21)

$$\Phi_{q|H_1}(s) = \frac{(1+s)^{M(K-1)}}{|\,\mathbf{I} + (\mathbf{I}+\mathbf{\Lambda_t})s\,|^K} \quad (22)$$

If the target signal is assumed to be a stationary process, then $\mathbf{\Lambda}_t$ is a symmetric Toeplitz matrix with M distinct positive real eigenvalues denoted by β_i, i=1, 2, …, M. Therefore, the determinant which appears in (22) may be expressed as the product of its eigenvalues.

$$\Phi_{q_0|H_1}(s) = \frac{(1+s)^{M(K-1)}}{\prod_{i=1}^{M}\left[1+(1+\beta_i)s\right]^K} \qquad \text{K=1, 2, 3,} \quad (23)$$

Note that the mgf of [3] given for chi-square targets with 2K degrees of freedom, is a special case of the mgf given by equation (23) for partially correlated chi-square target with 2K degrees of freedom. The poles of $\Phi_{q/H_1}(s)$ are M poles at $s_i = -(1+\beta_i)^{-1}$ for i=1, 2…, M, of multiplicity K lying in the left-hand s-plane.

The P_d of the CMLD-CFAR detector can be found by inserting equations (14) for s = $-T^{OS}$ s and (23) into equation (8). Then by using the partial-fraction expansion, we get

$$P_d^{CMLD} = -\alpha\sum_{i=1}^{M}\frac{d^{K-1}}{ds^{K-1}}\left[\frac{(1+s)^{M(K-1)}}{s}\left(\sum_{\substack{j=1\\j\neq i}}^{M}\frac{k_j^{CMLD}}{s-s_j^{CMLD}}\right)^K\left(\sum_{l=1}^{N-L}\frac{k_l}{s-s_l}\right)^M\right]_{s=s_i} \quad (24)$$

where, $\alpha = \frac{1}{\Gamma(K)}\binom{N}{N-L}^M\left(-T^{M(N-L)}\right)^{-1}\prod_{m=1}^{M}(1+\beta_m)^{-K}$ and

$$k_j^{CMLD} = \prod_{\substack{n=1\\n\neq j\\n\neq i}}^{M}\left(\frac{1}{1+\beta_n}-\frac{1}{1+\beta_j}\right)^{-1}$$

5. SIMULATION RESULTS

To evaluate the detection performance and the false alarm properties of the proposed model, we assume a reference window size of N=16 and design $P_{fa}=10^{-4}$. First, we compute the threshold multiplier T of the CMLD-CFAR (L=4) for the NCTDS using equation (15) to achieve the prescribed P_{fa}. Then, for the same assigned P_{fa}, we obtain by simulation the threshold multiplier of the corresponding detector for the CTDS. The P_d for the NCTDS is computed using equation (24) while the P_d for the CTDS is obtained by simulation. In both cases, the detection performance of the detector for a design P_{fa}, depends upon several parameters. Our attention is focused on the signal-to-noise ratio (SNR), the target correlation coefficient ρ_t, the number of processed pulses M and the number of degrees of freedom K with an emphasis on the problem of multiple target situations. Figure 2 shows the effect of four interferers present in the range cells added on a pulse by pulse basis, on the probability of detection against the SNR. We assume that the secondary targets are of the same nature as the primary ones. Inspection of this figure reveals that the CMLD-CFAR is more sensitive to the presence of interfering targets for the NCTDS than for the CTDS. That is, contrary to the NCTDS, in the CTDS, spikes or interferences present in the individual pulses may not be considered as such when added to flats (presence of noise only) having the same range cell index. Also, note that a target correlation going from $\rho_t = 1$ to $\rho_t = 0$ helps the detection and that the greater the K, the more insensitive to ρ_t the detection becomes. Finally, the CFAR loss between the two systems becomes more significant when K increases.

Denoting by INR the interference-to-noise ratio and assuming that INR=SNR, we now examine the effect of the number of interferers (NI), on the detectability of correlated chi-square targets with 2K degrees of freedom at

SNR=INR=5dB. We observe in figure 3 that in the absence of interferers, the CMLD-CFAR for the CTDS performs slightly better than the CMLD-CFAR for the NCTDS. However, the presence of interferers affects more the detector for the NCTDS than the corresponding detector for the CTDS. That is, the CTDS takes full advantage of the compensation mechanism as NI increases. As expected, the CMLD-CFAR for the two systems becomes vulnerable when NI exceeds four. Note though that when M is sufficiently large, the detection improves significantly for a moderate NI.

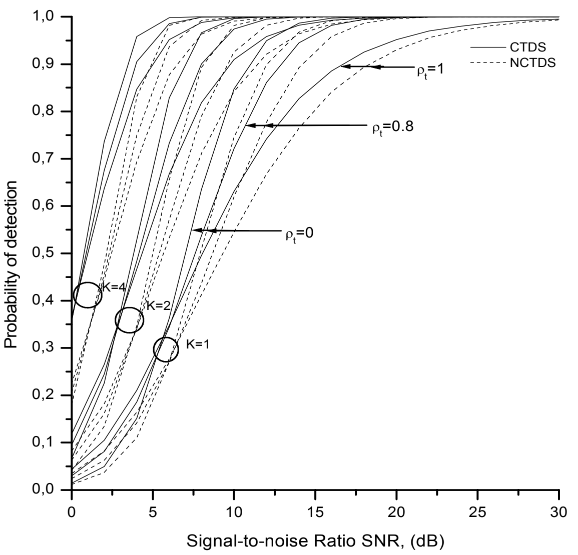

Figure 2 - Probability of detection of the CTDS and the NCTDS CMLD-CFAR detector against SNR in the presence of four interferers, for N=16, ρ_t =0, 0.8 and 1, M=4 and $P_{fa}=10^{-4}$.

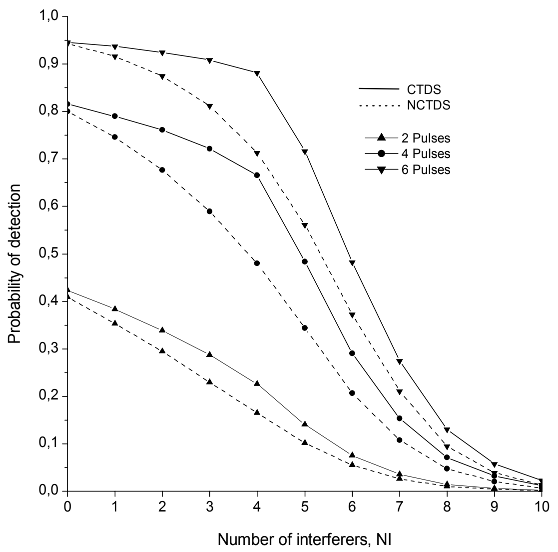

Figure 3 - Probability of detection of the CTDS and the NCTDS CMLD-CFAR detector against Number of interferers at SNR=5dB for K=2, N=16, ρ_t =0.5, INR=SNR and $P_{fa}=10^{-4}$.

6. CONCLUSION

In this paper, we analyzed and compared the performance of the CMLD-CFAR detector using two different noncoherent integration systems for the detection of a pulse-to-pulse partially correlated target with 2K degrees of freedom immersed in a pulse-to-pulse Rayleigh uncorrelated noise and multiple target situations. The obtained results showed that their performance are similar in the absence of interfering targets, in which case the simple mathematics induced by the NCTDS makes it a good alternative since the problem of large processing time required can be easily overcome by using new generation high speed processors. However, in the presence of interfering targets and due to its compensation mechanism, the CTDS is more robust than the NCTDS.

Acknowledgement

The authors are grateful to the reviewers for their constructive comments.

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