BALANCED ALLOCATION STRATEGY IN MULTI-USER OFDM WITH CHANNEL STATE INFORMATION AT THE TRANSMITTER

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ABSTRACT

Multi-user systems based on OFDM are frequently used in powerline or quasi-static wideband wireless channels. Typical scenarios assume that precise channel state information can reasonably be obtained at the transmitter and at the receiver. In downlink, a spectral mask constraint is usually imposed too. In such previous context, we assume two multiple access schemes, MC-DS-CDMA and a simpler OFDMA, and we investigate their achievable-rate regions. In particular, we study the so-called "balanced rate criterion", in order to select a point of the achievable-rate region which guarantees fairness between all the active users. We propose a simplified algorithm to calculate an approximate balanced rate solution for the OFDMA case. The loss of the OFDMA solution with respect to the MC-DS-CDMA solution is shown to be acceptable. Comparisons with other OFDMA allocation algorithms have also been reported.

1. INTRODUCTION

Different techniques are supposed to provide reliable and high-speed connections to home consumers ("last-mile problem"), and the recently proposed PowerLine Communications (PLC) generate a certain appeal in the research and industrial community. Powerline's characteristics are similar to the ones of the cable used in the digital subscriber line (xDSL). For this reason, Orthogonal Frequency Division Multiplexing (OFDM) has been retained as a good modulation able to assure high data rates in a frequency selective medium.

The powerline channel impulse response can be considered quasi-static over frame time-scale, as in other wired systems or in wireless scenarios with no mobility and static environment. Under this assumption, channel state information (CSI) can be made available at the transmitter and at the receiver. It is well known that in point-to-point transmissions with perfect CSI at both ends, water-filling with bit-loading can approach capacity. In the case of multipoint-to-point transmission, the capacity region for wideband Gaussian channels has been addressed in [1], for instance. When a suboptimal transmitter and receiver structure is imposed by practical constraints, we prefer to use the term "achievable region" instead of "capacity region". There exist many contributions in this area, and results strongly depends on the modem structure and on the transmitted signal constraints [2], [3].

In this paper we deal with a multiuser OFDM-based system, with perfect CSI at both sides. We focus on the study of the achievable-rate region for the downlink under spectral mask constraint. This setting is motivated by the following PLC context: a unique cell with K modems is connected to a repeater through the powerline. The medium access is done by assigning to the users different spreading orthogonal codes on every subcarrier (MC-DS-CDMA). This choice can be motivated by the fact that time spreading helps to cope with inter-cell interference. When all the spreading codes allocated in a subcarrier are assigned to one given user, the MC-DS-CDMA boils down to a simpler OFDM access technique (OFDMA). It will be interesting to study then the achievable-rate loss due to the simplification in the access technique (from MC-DS-CDMA to OFDMA). Spectral mask constraint is used in downlink in order to satisfy the ElectroMagnetic Compatiblity (EMC) constraints of the PLC system. We recall that, although introduced in PLC context, this model is sufficiently general to include also wideband wireless multi-user systems with quasi-static channel. Once introduced the model parameters and equations, in Section 3 we investigate the problem of describing the achievable region for the MC-DS-CDMA. In Section 4, we restrict our study to OFDMA. To assure fairness between different users, we would like to assign different priorities to the users in the following way: the "worse" the user is, the more important the priority is. We actually focus on the balanced rate criterion introduced in [4], which aims at providing to each user the same proportion of its maximal rate. We propose an algorithm which calculates an approximate OFDMA solution of the balanced rate problem. In Section 5, we provide comparisons with other policies, especially the min-max one introduced in [5], which guarantees a minimal rate to each user. A conclusion is drawn in Section 6.

2. SYSTEM MODEL

Let B be the total bandwidth and N the number of OFDM subcarriers. With an appropriate guard interval, the selective channel of the k-th user is equivalent to N parallel flat fading channels $\{H_{k,n}\}$, where $H_{k,n}$ is the channel frequency response evaluated at the Fourier frequency corresponding to the n-th subcarrier. Under our hypothesis, $H_{k,n}$ is a complex scalar known at the transmitter and at the receiver. The equivalent channel gains are defined as

$$G_{k,n} = \frac{|H_{k,n}|^2}{\Gamma_k \sigma_{k,n}^2} \tag{1}$$

where $\sigma_{k,n}^2$ is the noise variance experienced by the k-th user on the n-th subcarrier and Γ_k its a power penalty which measures the gap between the Shannon capacity and the current coding scheme adopted by the user k. Let $\gamma_{k,n}$ be the ratio of the spreading codes allocated to the user k on the n-th subcarrier. The orthogonality of the spreading codes assures that

$$\sum_{k=1}^{K} \gamma_{k,n} = 1\,, \qquad \forall n. \tag{2}$$

where K is the total number of users in the cell. Besides, we assume that $\gamma_{k,n}$ for all n, k takes on continuous values between 0 and 1.

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This hypothesis is useful for solving the optimization problem, but in practice $\gamma_{k,n}$ takes on quantized values depending on the number of spreading codes. We suppose also the spectral mask and all signal spectra to be constant inside each OFDM subcarrier. Then, the sum power spectral density constraint boils down to a sum power constraint M_n per sub-carrier

$$\sum_{k=1}^{K} \gamma_{k,n} P_{k,n} = \sum_{k=1}^{K} Q_{k,n} \leq M_n\,, \qquad \forall n. \tag{3}$$

where $P_{k,n}$ and $Q_{k,n}$ are respectively the transmit power per symbol and the total transmit power of user k on the n-th sub-carrier. Under these conditions and the orthogonality constraints imposed by the MC-DS-CDMA scheme, it is possible to show that the achievable-rate $R_{k,n}$ of the user k on the sub-carrier n is [6]

$$R_{k,n} = \frac{B}{N}\gamma_{k,n}\log_2(1+P_{k,n}G_{k,n}) = \frac{B}{N}\gamma_{k,n}\log_2\left(1+\frac{Q_{k,n}}{\gamma_{k,n}}G_{k,n}\right) \tag{4}$$

which is measured in bit per second. We stress that the previous model describes also all access methods which make use of orthogonality conditions in time (TDMA) or in frequency (FDMA) to discriminate between different users [7].

Let $\alpha_k \geq 0$ denote the priority associated with user k and let α be the vector which collects users' priorities. We want to solve the following weighted sum-rate maximization problem, *given a certain* α

$$\max_{Q_{k,n},\gamma_{k,n}} \sum_{k=1}^{K} \alpha_k R_k = \max_{Q_{k,n},\gamma_{k,n}} \sum_{k=1}^{K} \alpha_k \sum_{n=1}^{N} R_{k,n} \tag{5}$$

where $R_{k,n}$ is given in (4) and under the constraints (2) and (3), which we recall hereafter

$$Q_{k,n} \geq 0 \quad \forall k,n; \qquad \gamma_{k,n} \geq 0 \quad \forall k,n$$
$$\sum_{k=1}^{K} Q_{k,n} \leq M_n \quad \forall n; \qquad \sum_{k=1}^{K} \gamma_{k,n} = 1 \quad \forall n.$$

In [2] the same mathematical problem is solved with the noticeable difference that a per-user power constraint (sum over the subcarriers) is applied instead of a per-subcarrier power constraint as in (3). This changes the physical meaning of the problem[1] and its solution.

3. ACHIEVABLE REGIONS

Since per-subcarrier constraints are given and the priorities[2] are fixed, problem (5) is actually equivalent to N separate maximization problems, one for each sub-carrier.

3.1 Solution for $N = 1$

Let us focus on one given sub-carrier and let us drop the sub-carrier index n in all the expressions for simplicity's sake. Maximization (5) can be rewritten as

$$\max_{P_k,\gamma_k} \sum_{k=1}^{K} \alpha_k \gamma_k \log_2(1+P_k G_k)\,, \qquad \sum_{k=1}^{K} P_k\gamma_k \leq M,\ \sum_{k=1}^{K} \gamma_k = 1. \tag{6}$$

with $P_k, \gamma_k \geq 0$ (we have omitted for simplicity the constant factor B/N). This problem has already been addressed by Li and Goldsmith in [8], where it has been introduced in the context of TDMA systems with average power constraint. Problem (6) can be solved by use of the constraint's structure combined with the convexity of the functions $f_k(P) = \alpha_k \log_2(1+PG_k)$. An algorithm is given in

[1] A multiple access problem is investigated in [2], while we consider a broadcast setting.

[2] We will impose also that $\sum_k \alpha_k = 1$. This is not a real constraint, since the maximization problem (5) is defined up to a common scalar factor multiplied to all the priorities.

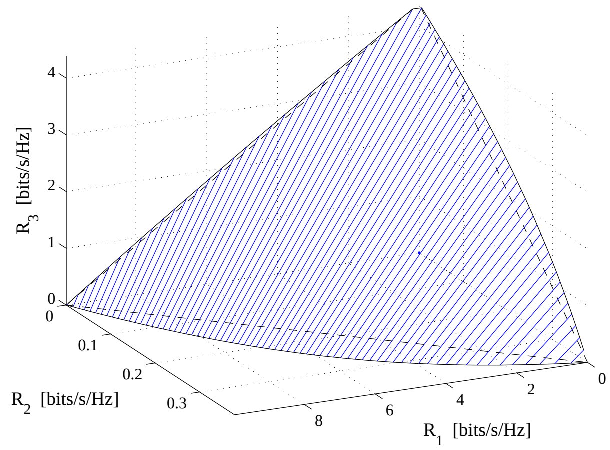

Figure 1: achievable-rate region: $K = 3$, $N = 1$ and $G_1 > G_3 > G_2$.

[8], which provides the active users and the optimal values P_k^* and γ_k^*, starting from a number K of users, their corresponding priorities α_k and the equivalent channels G_k. Surprisingly, it turns out that for every K, the maximum number of active users is only 2. In the particular case of equal priorities, if $k^* = \arg\max_k\{G_k\}$, then $f_{k^*}(P) > f_k(P)$, $k \neq k^*$ for all $P > 0$. This means that (6) is maximized by $f_{k^*}(P)$ for every P and hence the optimal solution is to assign all the resources to user k^*, the one with the best channel.

Let us suppose that $K > 2$ and $N = 1$, but we search for the solutions with $R_k > 0$ for all k. Suppose to fix the priority values (α_1, α_2) so that two users are active. Then, one can find $(\alpha_3, \cdots, \alpha_K)$ such that problem (6) associated with priority vector $(\alpha_1, \cdots, \alpha_K)$ actually admits more than one equivalent solution. In other words, there are different couples of users, with their respective code and power allocations, which solve the same problem (we do not report the proof here for lack of space). Then, if we want to allocate a positive rate to everyone, the unique optimal solution is to *time-share* the resources between the couples of users which solves problem (6) for the same priorities.

In the case $K = 3$, we also succeeded in showing that the achievable region is formed by segments, whose points represent the time-shared solution between two different couples of users (see Fig. 1). Let us consider a segment whose ends are $\mathbf{R}_A^* = (R_{1,A}^*, R_{2,A}^*, 0)$ and $\mathbf{R}_B^* = (R_{1,B}^*, 0, R_{3,B}^*)$: the couples of users $A = (1,2)$ and $B = (1,3)$ with the corresponding optimal resources $(\gamma_A^*, \mathbf{P}_A^*)$ and $(\gamma_B^*, \mathbf{P}_B^*)$ are two different solutions of (6) for a same value of priorities α'. In fact, α' is the vector orthogonal to the tangent plane at the achievable region which contains the considered segment. The generic point of the segment $\tau\mathbf{R}_A^* + (1-\tau)\mathbf{R}_B^*$, $0 \leq \tau \leq 1$, is a solution of problem (6). It is achieved by applying the allocation $(\gamma_A^*, \mathbf{P}_A^*)$ for a time τ (user 1 and 2 are active), and the allocation $(\gamma_B^*, \mathbf{P}_B^*)$ for the rest of the time (users 1 and 3 are active). This kind of solution is called hereafter *time-sharing of the allocation policy*.

For $K = 3$ it is possible to show that there exists one and only one set of priorities which corresponds to a given segment, and hence there are at most two couples of users which shares the resources in time. For K greater than 3, there are much more possibilities, but the time-sharing is always made among couples of users.

To conclude, in the case of one subcarrier, all users can transmit at non-zero rates. However this is possible only if we admit to split the transmission time into several slots and to use different allocation policies for each slot, with increased system complexity.

3.2 Solution for $N > 1$

As previously said, given a set of priorities α, the maximization (6) can be separately solved for each subcarrier.

If the priorities are equal, the optimal solution is to allocate each subcarrier to the user with the strongest channel. It is apparent that the allocation algorithm is trivial in this case. This solution is OFDMA (one user per subcarrier), which simplifies the system structure. It has also the very interesting property of maximizing the cumulated rate of the cell $\sum_k R_k$, which is a fundamental parameter of a cellular (wired or wireless) system.

In general, however, some points of the achievable-rate region for $N > 1$ will be achieved only by allocation policy time-sharing, as in the case $N = 1$. So, fixed a set of priorities, it is possible to have optimal OFDMA solutions, or MC-DS-CDMA solutions or time-shared MC-DS-CDMA solutions, according to the particular realization of the channels and the mask constraint.

We decided to consider, in this work, only (not necessarily optimal) MC-DS-CDMA and OFDMA solutions. Time-sharing, in fact, not only increases the structural complexity of the system, but also requires a far more complicate allocation algorithm than the other solutions.

4. BALANCED STRATEGY AND ALGORITHM

In modern communications systems, the concept of fairness is essential. Ideally, we would like all the users to transmit at an acceptable rate and at the same time not to sacrifice the overall performance of the system, measured as the sum rate of all active users. In the sequel, we focus on OFDMA based systems.

One possible choice is to assign equal priorities to all users, which maximizes the sum rate at low computational complexity but favours only users with good channels. A max-min approach was proposed in [5], which maximizes the minimum rate. This algorithm favours users with bad channels at the expense of overall performance. We propose an allocation algorithm that is based on the balanced rate approach introduced in [4] which consists in choosing the point $\mathbf{R}_{eq}$ belonging to the achievable region frontier which satisfies

$$\frac{R_{eq,1}}{R_1^{max}} = \frac{R_{eq,2}}{R_2^{max}} = \ldots = \frac{R_{eq,K}}{R_K^{max}} \qquad (7)$$

where R_k^{max} is the maximum rate at which user k would transmit if all N subcarrier were assigned only to him. The point $\mathbf{R}_{eq}$ is called balanced rate point, since each user transmits at the same percentage of its maximal possibilities. As we will see, the balanced rate criterion is between the max-min and the sum-rate criteria. **enlever cette dernière frase?**

4.1 An approximate OFDMA balanced point algorithm

We propose an algorithm which calculates an approximate OFDMA solution of the balanced rate (7). The entries of the algorithm are the set of constraints $\{M_{k,n}\}$ and the equivalent channels $\{G_{k,n}\}$

1. Initialisation
   - 1.1. $R_k^{max} = \sum_{k=1}^{N} \log_2(1 + M_{k,n} G_{k,n}), \forall k$.
   - 1.2. Run the algorithm max-min [5] and stock in $\mathbf{a}^{(0)} = [a_1^{(0)}, \cdots, a_N^{(0)}]$ the user to which each subcarrier is assigned ($a_n^{(0)} \in \{1, \ldots, K\}, \forall n$).
   - 1.3. Let $\mathbf{R}^{(0)}$ be a $K \times N$ matrix, whose entry $R_{k,n}^{(0)} = \log_2(1 + M_{k,n} G_{k,n})$ if $a_n^{(0)} = k$ or zero otherwise.
   - 1.4. Calculate the relative rates $r_k^{(0)} = [\sum_{n=1}^{N} R_{k,n}^{(0)}]/R_k^{max}\ \forall k$, and stock them in $\mathbf{r}^{(0)}$.
   - 1.5. Calculate $e^{(0)} = \sigma_{\mathbf{r}^{(0)}}/m_{\mathbf{r}^{(0)}}$ where

$$m_{\mathbf{r}^{(0)}} = \frac{1}{K}\sum_{k=1}^{K} r_k^{(0)}, \quad \sigma_{\mathbf{r}^{(0)}} = \sqrt{\frac{1}{K}\sum_{k=1}^{K}(r_k^{(0)} - m_{\mathbf{r}^{(0)}})^2}$$

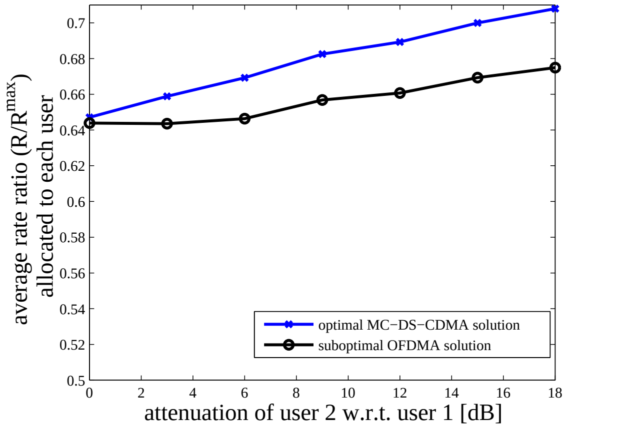

Figure 2: Relative rates as a function of relative attenuation of user 2 with respect to user 1.

2. Let $j = 1$, while $j \leq K$ {
   while $1 > 0$ (endless loop) {
   - (a) Sort $\mathbf{r}^{(0)}$ in decreasing order and stock it in $\mathbf{r}^{(d)}$, select user k_M corresponding to $r_j^{(d)}$.
   - (b) Find the subcarrier of user k_M which has minimum rate: $n_M = \arg\min_{n=1,\ldots,N} R_{k_M,n}^{(0)}$
   - (c) Calculate $r_k^{aux} = \log_2(1 + M_{k,n_M} G_{k,n_M})/R_k^{max}\ \forall k$.
   - (d) For $k = 1$ to K except k_M {
     - $\cdot$ $\mathbf{r}^{(1)} := \mathbf{r}^{(0)}$,
     - $\cdot$ $r_k^{(1)} := r_k^{(0)} + r_k^{aux}$,
     - $\cdot$ $r_{k_M}^{(1)} := r_{k_M}^{(0)} - r_{k_M}^{aux}$,
     - $\cdot$ $e_k^{(1)} = \sigma_{\mathbf{r}^{(1)}}/m_{\mathbf{r}^{(1)}}$.

     } end loop for
   - (e) $k_m = \arg\min\{e_k^{(1)}, k \in \{1, \ldots, K\} \setminus \{k_M\}\}$
   - (f) if $e_{k_m}^{(1)} < e^{(0)}$, then
     - $\cdot$ Assign the n_M-the subcarrier to user k_m; update $\mathbf{R}^{(0)}$, $\mathbf{r}^{(0)}$, $e^{(0)} = e_{k_m}^{(1)}$ and set $j = 1$
     - $\cdot$ else $j = j + 1$ and break (quit loop)

   } }

The algorithm selects the most unbalanced users, i.e. the ones which have allocated respectively the greatest and lowest relative rates, and exchanges between them the subcarrier which gives the more balanced temporary solution. The algorithm stops when it is not possible to balance further the relative rates. By construction, the algorithm converges to a solution more balanced than the starting one. We stress also that the algorithm can be initialized with a generic subcarrier assignment, other than the max-min solution. However, the solution depends on the initialization and so it does not necessarily coincide with the best OFDMA approximation of the balanced rate point.

If the number of users K is greater than $N/2$, then the algorithm does not perform well and returns the max-min solution, with which it is initialized. This behaviour is due to the lack of degrees of freedom: when $K \simeq N$ each user has a very limited choice of subcarrier to exchange. On the contrary, when $K \ll N$, several subcarriers are generally allocated to each user, whose rate is given by the sum of many different contributions. Hence there are more possible exchanges between users, and the algorithm converges to a balanced solution with a finer precision. We stress that this is the

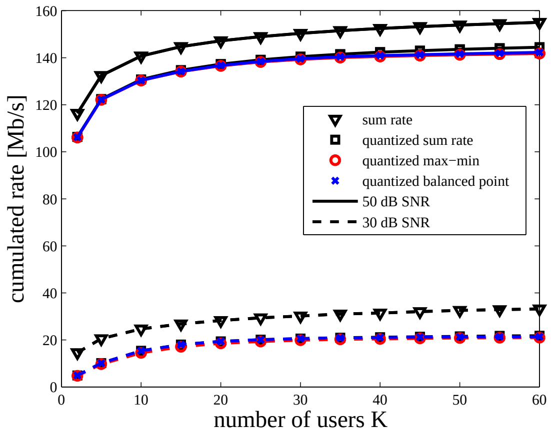

Figure 3: Cumulated rates provided by different algorithm for average SNR of 50 dB and 30 dB. Rates are averaged over at least 100 channel realizations.

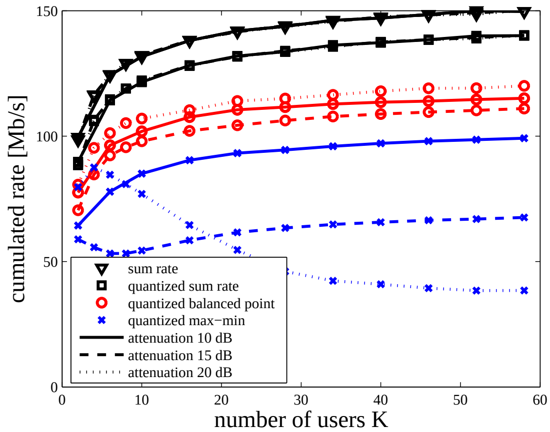

Figure 4: Cumulated rates when half of the users are attenuated with respect to the others. The strongest group has 50 dB SNR. Rates are averaged over at least 10 channel realizations.

case of realistic wired systems, where N can be greater than 1000 and K is usually less than 100: in this context, the balanced solution can not be found by exhaustive search, and algorithms such as ours or different versions of the max-min one (see [5], [9]) become relevant.

4.2 Validation of the approximate algorithm for $K = 2$

The approximated OFDMA solution of the balanced rate problem has been compared to the optimal solution in the case $K = 2$, in order to verify the loss of our suboptimal algorithm. For this setting the optimal solution is MC-DS-CDMA. A simple (but computationally expensive) algorithm has been found, which starts from a given priority vector $[\alpha^{(0)}\ 1-\alpha^{(0)}]$ and converges by binary search to the priority vector $[\alpha_{eq}\ 1-\alpha_{eq}]$ associated with the balanced solution.

In Fig. 2 we plotted the ratio $\beta_{OFDMA} = R_1/R_1^{max} = R_2/R_2^{max}$ and $\beta_{eq} = R_{eq,1}/R_1^{max} = R_{eq,2}/R_2^{max}$ corresponding respectively to the approximate OFDMA solution and the optimal one. The simulation envirommnent is inspired by the physical model of the PLC system described in [9], we have set $N = 2048$, $B = 20$ MHz. We have assumed a uniform channel power delay profile with 80 taps following a Rayleigh law and maximum time spread of 2 μs. The channels of the two users are assumed to be statistically independent. The ratios plotted in Fig. 2 are obtained averaging over 100 channel realizations. The OFDMA solution is more suboptimal at increasing relative attenuation because in this case the optimal balanced solution requires an increasing number of shared subcarriers. On the contrary, in the case of no relative attenuation the optimal solution is generally achieved by sharing only one subcarrier. Finally, we remark that the suboptimal OFDMA solution is only a few percentage points lower than the optimal solution in a large range of attenuation. Using balanced OFDMA approach leads only to a small loss in performance.

5. SIMULATION RESULTS

In this section we present results for $N = 2048$, $B = 20$ MHz and $K \geq 2$ under different channel configurations. The spectral mask constraint is assumed to be flat. We make also the more realistic assumption that bits are loaded on each subcarrier, which means that only integer values of spectral efficiencies are admitted. In our algorithm, this condition amounts to substitute the $\log_2(\cdot)$ operations with $\lfloor \log_2(\cdot) \rfloor$, where $\lfloor x \rfloor$ returns the greatest integer lower than or equal to x. Moreover, since the Rhee-Cioffi max-min algorithm [5] does not operate on bits, we will make comparisons with the quantized max-min algorithm presented in [9]. The latter algorithm is not a simple quantized version of the Rhee-Cioffi max-min, but it contains a mechanism to deal with subcarriers which can support only a fraction of a bit. Finally, without loss of generality we fix $\Gamma_k = 1$ for all the simulations.

5.1 Users with independent channels and equal attenuation

In the first scenario we consider a group of K users in a mono-cellular context. The users' channels are independently generated according to a uniform power delay profile with 80 taps following a Rayleigh law and maximum time spread of 2 μs. In Fig. 3 we have drawn the cumulated rates, i.e. the sum of the users' rates, associated with different allocation algorithms. All the users have the same average SNR equal to 50 dB (solid curves) and to 30 dB (dashed curves). The cumulated rates corresponding to the sum-rate capacity *without* quantization are given only as an upper bound. As can be seen in Fig. 3, the three allocation algorithms approximately give the same cumulated rates for all SNR values. This is not surprising since all the users have independent channels with the same attenuation. Then, there is no good or bad user: everyone has a set of subcarriers over which he is the best user, in particular for $K \ll N$. Consequently the subcarrier repartition generated by the three algorithms is similar. Hence, in this case, the best algorithm is the quantized sum-rate one, due to its optimality and low complexity.

5.2 Users with independent channels and unequal attenuation

In the second scenario, we consider the same statistical channel model of the previous subsection. This time, the users are divided in two groups: one half has an average SNR of 50 dB, while the other half is attenuated by 10, 15 and 20 dB (respectively solid, dashed and dotted lines in Fig. 4). Due to the statistical independence of the channels, inside each group there is no best user in average. In a PLC context this setting corresponds to two distinct groups of users, the first one closer to the transformer than the second one [3]. An analogous situation could be found in a wireless scenario in which users are fixed and form groups at different distances from the base station.

In this case the algorithms behave differently, both from a cumulated rate and a subcarrier repartition perspective. The unquantized sum-rate solution is given as a reference upper bound. As we can see in Fig. 4, the quantized (and the unquantized) sum-rate algorithm gives the same cumulated rates independently of the attenuation of the second group. This is due to the fact that, as previously stated, the sum-rate algorithm favours the strongest users,

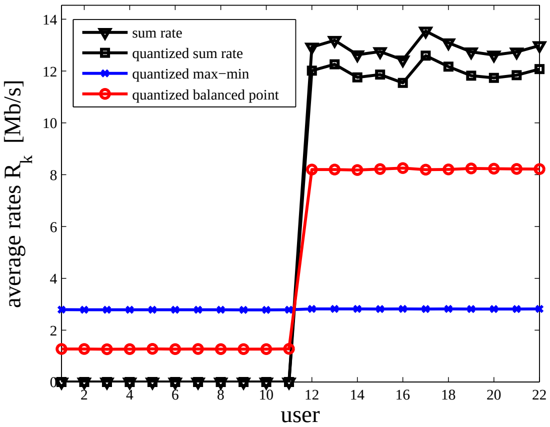

Figure 5: Average absolute rates per user when half of the users experience 15 dB attenuation. Rates are averaged over 10 channel realizations.

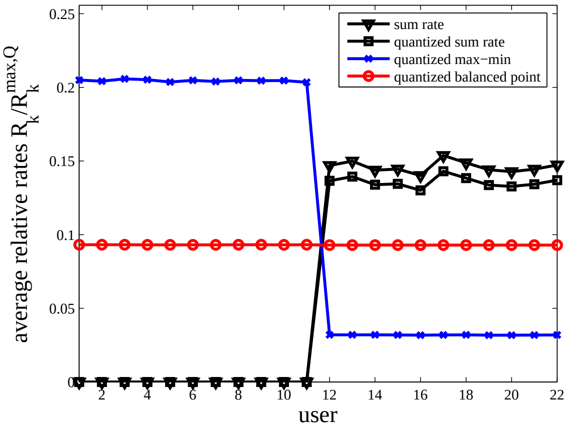

Figure 6: Average relative rates per user when half of the users experience 15 dB attenuation. Rates are averaged over 10 channel realizations.

which are always the ones of the first groups. This behaviour is apparent in Fig. 5, where we plotted the average rates of 22 users (users from 1 to 11 undergo an attenuation of 15 dB, while users from 11 to 22 are never attenuated). Since the users of a given group experience the same average channel condition, no user is preferred.

Coming back to Fig. 4, for low attenuations the quantized max-min algorithm manages to guarantee the increasing cumulated rates for increasing number of users K, as the quantized sum-rate solution. However, for high attenuations, the algorithm performance degrades with increasing K, since the number of bad users increases as well and the algorithm assigns most of the resources to these users. Moreover, for all considered attenuations, the cumulated rates of the max-min algorithm are sensibly lower than the one of the quantized sum-rate algorithm due to the priority given to bad users.

The balanced point algorithm does not suffer from the drawbacks of the max-min algorithm. It is able to increase the cumulated rates for increasing K at all considered attenuation levels. In particular, when the attenuation is 20 dB, the balanced point algorithm assures even a better rate than in the case of attenuation equal to 10 dB. As a matter of fact, the balanced point algorithm is able to assure to each user the same percentage of its total rate, so even when many bad users are active, they do not penalize global performance. This can be understood also observing the users' absolute and relative rates respectively in Fig. 5 and Fig. 6. Each user can transmit at about 9% of its maximal rate, while the max-min algorithm allocates to good users only about 3% of their maximal rate. In this configuration with two groups, the balanced point algorithm is able to guarantee an almost constant cumulated rate under a wide interval of attenuations and of number of users.

Finally, we have also tested the algorithms in the following extreme case: each new user is more attenuated than the previous ones. In a PLC context, this case corresponds to a group of users that are plugged on the same powerline, but at increasing distances from the repeater. We do not report the simulation results for lack of space. However, the gain of the balanced point algorithm with respect to the max-min one in term of cumulated rate versus the number K of users is even greater than in the case of the two groups treated above.

6. CONCLUSION

In this paper we have studied the shape of the achievable region of a MC-DS-CDMA downlink system with spectral mask constraint. Based on the balanced point (or rate) criterion which selects a system operation point on the frontier of the achievable-rate region, we propose a balanced point algorithm for and OFDMA systems. We have checked that the perfomance of the proposed algorithm is not far from the performance of an optimal balanced point algorithm for MC-DS-CDMA systems, in the case $K = 2$. In the OFDMA case, we have compared the proposed algorithm with other propositions of the literature. It has been pointed out that our approximate balanced rate algorithm can assure considerable gains in configurations with users affected by different attenuations. The cumulated rate is substantially increased, while fairness is still reasonably guaranteed.

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