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A NEW VARIABLE FORGETTING FACTOR QR-BASED RECURSIVE LEAST *M*-ESTIMATE ALGORITHM FOR ROBUST ADAPTIVE FILTERING IN IMPULSIVE NOISE ENVIRONMENT

Y. Zhou, S. C. Chan, and K. L. Ho

Department of Electrical and Electronic Engineering, The University of Hong Kong
Pokfulam Road, Hong Kong
email: {yizhou, scchan, klho}@eee.hku.hk

ABSTRACT

This paper proposes a new variable forgetting factor QR-based recursive least M-estimate (VFF-QRRLM) adaptive filtering algorithm for impulsive noise environment. The new algorithm is a QR-based implementation of the RLM algorithm, which offers better numerical stability and a similar robustness to impulsive noise. A new VFF control scheme based on the approximated derivatives of the filter coefficients is also proposed to improve its tracking performance. Simulation results show that the proposed algorithm not only offers improved robustness in impulsive noise environment but also possesses fast transient converging and tracking behaviours.

1. INTRODUCTION

Adaptive filters are widely used in communications, control, and many other systems in which the statistical characteristics of the signals to be filtered are either unknown a priori or, in some cases, slowly time varying [1]. These algorithms can broadly be classified into two main families: the least mean squares (LMS) algorithm and recursive least squares (RLS) algorithm. In many applications, the additive noise is impulsive in nature and not Gaussian-distributed. In this situation, the performance of conventional LS-based algorithms will deteriorate significantly. A more robust adaptive filtering algorithm is the recursive least *M*-estimate (RLM) adaptive algorithm [2,3] which is based on the sum of weighted *M*-estimate error (SWME) cost function. It can be regarded as the generation of the conventional RLS algorithm with an excellent ability to suppress impulses in the input and desired signals of the filter. The purpose of the instructions collected in this manuscript is to specify the format and style of admissible EUSIPCO 2006 papers and to facilitate paper submission and review. First of all, here is some general information about the paper preparation, submission, and review process.

Like RLS-type algorithm, the RLM also suffers from problems such as numerical stability problem in finite word-length implementation and slow convergence in sudden system change due to the use of a constant forgetting factor. The former problem can be solved by seeking for other more stable implementations such as the QR-decomposition, whereas the latter one usually requires the use of adaptive forgetting factors. To address the latter, many algorithms have been introduced which include the data weighting approach [4], the self-perturbation RLS (SPRLS) [5] algorithm and the variable forgetting factor (VFF)-based, GN-VFF-RLS [6] and GVFF-RLS [7] algorithms. The VFF technique is very appealing for its simple implementation and good performance. However, the VFF control design of present algorithms mainly relies on the estimation error ([6] and [7] are respectively based on the error and the gradient of the MSE).

In this paper, we proposed a new VFF control scheme based on the approximated derivatives of the filter coefficients. This approach was first employed by Hoshuyama et al to control the stepsize in their Proportionate Affine Projection Algorithm (GP-APA) [8]. The main idea is to measure the convergence behavior of the adaptive filter from the variations of the weight vector. Near steady state, the weight vector exhibits much less variations and a smaller stepsize can be used. Similarly, when the weight vector exhibits considerable variation, a larger stepsize can be chosen. We notice that this approach is also applicable to the RLS algorithm and a new VFF control scheme for the RLM is proposed in this paper. Moreover, to improve the numerical stability of the RLM algorithm, a new QR-based implementation of the RLM algorithm is proposed. This resulting algorithm is called variable forgetting factor QR-based RLM (VFF-QRRLM) algorithm. Simulation results show that the proposed algorithm not only offers improved robustness in impulsive noise environment but also possesses fast transient converging and tracking behaviours.

This paper is organized as follows: the RLM algorithm is briefly reviewed in section 2. The proposed VFF-QRRLM algorithm is presented in section 3. Experimental results and comparisons are given in section 4. Finally, conclusions are drawn in section 5.

2. THE RLM ALGORITHM

Since the conventional RLS algorithm is based on the LS criterion, its performance will deteriorate considerably when the desired or the input signal is corrupted by impulsive noise. Nonlinear techniques are usually employed to reduce the hostile effects of impulsive noise on LS-based algorithms. In [3], a robust adaptive transversal filtering algorithm called RLM algorithm based on an *M*-estimate cost

function is developed. More specifically, in the system identification problem depicted in Fig. 1, $d(n)$ is the desired signal, $\boldsymbol{x}(n) = [x_1(n) \quad x_2(n) \quad \cdots \quad x_L(n)]^T$ is the input vector (L is the filter length), $\boldsymbol{w}(n) = [w_1(n) \quad w_2(n) \quad \cdots \quad w_L(n)]^T$ is the adaptive filter weight or coefficient vector and $e(n) = d(n) - \boldsymbol{w}^T(n-1)\boldsymbol{x}(n)$ is the a prior error signal. $\eta_0(n)$ and $\eta_S(n)$ are the additive interferences and/or noises. Instead of the LS estimator, a new cost function, defined as the sum of exponentially weighted *M*-estimate errors (SWME), is proposed: $J_\rho(n) = \sum_{i=1}^{n} \lambda_i^{(n)} \rho(e(i)) = \sum_{i=1}^{n} \lambda_i^{(n)} \rho(d(i) - \boldsymbol{w}^T(n)\boldsymbol{x}(i))$.
$\rho(e)$ is a robust *M*-estimate function such as the Hampel's three parts re-descending function shown in equation (1) and Fig. 2.

$$\rho(e) = \begin{cases} e^2/2, & 0 \le |e| < \xi \\ \xi|e| - \xi^2/2, & \xi \le |e| < \Delta_1 \\ \frac{\xi}{2}(\Delta_2 + \Delta_1) - \frac{\xi^2}{2} + \frac{\xi}{2}\frac{(|e| - \Delta_2)^2}{\Delta_1 - \Delta_2}, & \Delta_1 \le |e| < \Delta_2 \\ \frac{\xi}{2}(\Delta_2 + \Delta_1) - \frac{\xi^2}{2}, & \Delta_2 \le |e| \end{cases} \quad (1)$$

It can be seen that $\rho(e)$ is an even real-valued function and ξ, Δ_1 and Δ_2 are the threshold parameters used to control the degree of suppression of the outliers. The contribution of the error e to $\rho(e)$ is reduced when its magnitude is increased beyond these thresholds. Therefore, the smaller the values of ξ, Δ_1 and Δ_2, the greater the suppression will be of the outliers. These threshold parameters are usually estimated continuously, which will be discussed later. $J_\rho(n)$ is therefore capable of smoothing out momentary fluctuation caused by the impulsive noise. By setting the first-order partial derivatives of $J_\rho(n)$, with respect to $\boldsymbol{w}(n)$, to zero, it was shown in [3] that optimal weight vector should satisfy the *M*-estimate normal equation:

$$\boldsymbol{R}_\rho \boldsymbol{w}_\rho^* = \boldsymbol{P}_\rho \quad (2)$$

where

$$\boldsymbol{R}_\rho = \sum_{i=1}^{n} \lambda_i^{(n)} q(e(i))\boldsymbol{x}(i)\boldsymbol{x}^T(i), \ \boldsymbol{P}_\rho = \sum_{i=1}^{n} \lambda_i^{(n)} q(e(i))d(i)\boldsymbol{x}(i)$$

are the *M*-estimate autocorrelation matrix of $\boldsymbol{x}(n)$ and the *M*-estimate cross-correlation vector of $d(n)$ and $\boldsymbol{x}(n)$, respectively, and $\rho'(e) = \partial\rho(e)/\partial e = q(e) \cdot e$. Considering $\boldsymbol{R}_\rho$ and $\boldsymbol{P}_\rho$ will become smaller and smaller due to λ when a series of impulses appear at $x(n)$ or $d(n)$, a better choice for $\lambda_i^{(n)}$ is therefore proposed as $\lambda_i^{(n)} = \lambda_e(n)\lambda_i^{(n-1)}$, $\lambda_n^{(n-1)} = \lambda_e^{-1}(n)$, and

$$\lambda_e(n) = \begin{cases} \lambda & |e(n)| < t_\lambda \\ 1 & |e(n)| \ge t_\lambda \end{cases}, \quad (3)$$

where t_λ is a threshold that can be chosen according to the threshold parameters of $\rho(e(i))$ or other useful measures. Accordingly, $\boldsymbol{R}_\rho$ and $\boldsymbol{P}_\rho$ can be updated as

$$\boldsymbol{R}_\rho(n) = \sum_{i=1}^{n} \lambda_i^{(n)} q(e(i))\boldsymbol{x}(i)\boldsymbol{x}^T(i) = \lambda_e(n)\boldsymbol{R}_\rho(n-1) + q(e(n))\boldsymbol{x}(n)\boldsymbol{x}^T(n) \quad (4)$$

$$\boldsymbol{P}_\rho(n) = \sum_{i=1}^{n} \lambda_i^{(n)} q(e(i))d(i)\boldsymbol{x}(i) = \lambda_e(n)\boldsymbol{P}_\rho(n-1) + q(e(n))d(n)\boldsymbol{x}(n) \quad (5)$$

Thus, with a similar derivation of the conventional RLS algorithm, the RLM algorithm can be obtained as:

$$\boldsymbol{V}(n) = \lambda_e^{-1}(n)(\boldsymbol{I} - \boldsymbol{K}(n)\boldsymbol{x}^T(n))\boldsymbol{V}(n-1) \quad (6)$$

$$\boldsymbol{K}(n) = \frac{q(e(n))\boldsymbol{V}(n-1)\boldsymbol{x}(n)}{\lambda_e(n) + q(e(n))\boldsymbol{x}^T(n)\boldsymbol{V}(n-1)\boldsymbol{x}(n)} \quad (7)$$

$$\boldsymbol{w}(n) = \boldsymbol{w}(n-1) + \boldsymbol{K}(n)[d(n) - \boldsymbol{w}^T(n-1)\boldsymbol{x}(n)]. \quad (8)$$

Eqns. (6)-(8) represent one step of the Newton method in solving the nonlinear equation in (2). If more iteration is used, it becomes an iteratively re-weighted LS algorithm with higher arithmetic complexity. Other algorithms can also be used. The threshold parameters ξ, Δ_1 and Δ_2 can be determined by the method proposed in [3]. Here we refer the readers to this paper for the details and only summarize the main procedures into the following formulas.

$$A_e(n) = \{e^2(n), \ \cdots, \ e^2(n - N_w + 1)\} \quad (9)$$

$$c_1 = 1.483(1 + 5/(N_w - 1)) \quad (10)$$

$$\hat{\delta}_e^2(n) = \lambda_e \hat{\delta}_e^2(n-1) + c_1(1 - \lambda_e) med(A_e(n)) \quad (11)$$

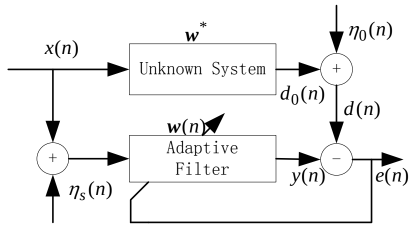


Figure 1 – System identification structure

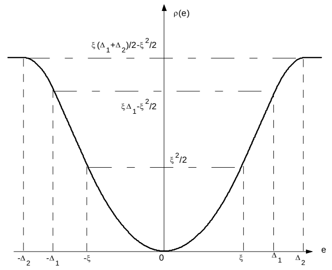


Figure 2 –The Hampel's three parts re-descending M-estimate

$$\xi = 1.96\hat{\delta}_e(n)\,,\ \Delta_1 = 2.24\hat{\delta}_e(n)\,,\ \Delta_2 = 2.576\hat{\delta}_e(n) \quad (12)$$

where N_w is the length of the estimation window, λ_e is a constant close to 1 and *med*(.) is the median operation.

3. THE PROPOSED VFF-QRRLM ALGORITHM

In order to derive the new VFF-QRRLM algorithm, we need to seek for the QR-based implementation of the RLM algorithm as described by (6) ~ (12) and design a new VFF control scheme. The details are shown below:

A. QR-BASED IMPLEMENTATION OF RLM ALGORITHM

The QR-based RLS algorithm, as summarized in Table I, is mathematically equivalent to but has higher numerical stability than the conventional RLS algorithm. Besides, it can be efficiently implemented using a systolic array involving only CORDIC processors [1]. If we modify (T-3) in Table 1 as

$$\boldsymbol{D}(n) = \Lambda(n)\big[\boldsymbol{X}(n) \quad \boldsymbol{d}(n)\big] = \begin{bmatrix} \sqrt{\lambda_e(n)}\boldsymbol{D}(n-1) \\ \psi'^T(n) \end{bmatrix} \quad (13)$$

where $\psi'(n) = \sqrt{q(e(n))}\big[\boldsymbol{x}^T(n) \quad d(n)\big]^T$, $\lambda_e(n)$ and $q(e(n))$ are defined as in RLM algorithm, (T-4) then becomes

$$\boldsymbol{Q}^{(N)}(n)\cdots\boldsymbol{Q}^{(1)}(n)\boldsymbol{Q}'(n)\boldsymbol{D}(n) = \begin{bmatrix} \tilde{\boldsymbol{R}}'_L(n) & \hat{\boldsymbol{d}}'_L(n) \\ 0 & \boldsymbol{c}'_{\boldsymbol{n}} \\ 0^T & d'_{L+1}(n) \end{bmatrix} \quad (14)$$

Consequently, the back-substitution step is modified to

$$\tilde{\boldsymbol{R}}'_L(n)\boldsymbol{w}(n) = \hat{\boldsymbol{d}}'_L(n)\,. \quad (15)$$

From (13) ~ (15) and the relationship between QRRLS and RLS algorithms, the following update formulas can be obtained:

$$\boldsymbol{R}'_\rho(n) = \tilde{\boldsymbol{R}}'^T_L(n)\tilde{\boldsymbol{R}}'_L(n) = \lambda_e(n)\boldsymbol{R}'_\rho(n-1) + q(e(n))\boldsymbol{x}(n)\boldsymbol{x}^T(n) \quad (16)$$

$$\boldsymbol{P}'_\rho(n) = \tilde{\boldsymbol{R}}'^T_L(n)\hat{\boldsymbol{d}}'_L(n) = \lambda_e(n)\boldsymbol{P}'_\rho(n-1) + q(e(n))d(n)\boldsymbol{x}(n) \quad (17)$$

Comparing (16) and (17) with (4) and (5), we can see the above QRRLM algorithm is mathematically equivalent to conventional RLM algorithm but it is more numerically stable for finite precision implementation.

B. DESIGN OF NEW VFF CONTROL SCHEME

Unlike the conventional VFF schemes, the proposed VFF control scheme is based on the approximated derivatives of the filter coefficients. This approach was firstly adopted in the GP-APA algorithm [8] and can be formulated as

$$\left\|\hat{\boldsymbol{c}}(n-1)\right\|_1 = \sum_{i=1}^{L}\left|\hat{c}_i(n-1)\right|,\ \hat{c}_i(n) = w_i(n-1) - \hat{w}_i(n-1) \quad (18)$$

$$\hat{w}_i(n) = \eta\hat{w}_i(n-1) + (1-\eta)w_i(n-1), \quad i = 1,2,\cdots,L\,. \quad (19)$$

1. Given the augmented data matrix

$$\boldsymbol{D}(n-1) = \Lambda(n-1)[\boldsymbol{X}(n-1) \quad \boldsymbol{d}(n-1)] \quad \text{(T-1)}$$

$\Lambda(n) = diag(\sqrt{\lambda^n}, \sqrt{\lambda^{n-1}}, \cdots, \sqrt{\lambda}, 1)$, and its QRD at time (*n*-1):

$$\boldsymbol{D}^*(n-1) = \boldsymbol{Q}(n-1)\boldsymbol{D}(n-1) = \begin{bmatrix} \tilde{\boldsymbol{R}}_L(n-1) & \hat{\boldsymbol{d}}_L(n-1) \\ 0 & \boldsymbol{c}_{\boldsymbol{n}-1} \end{bmatrix} \quad \text{(T-2)}$$

where $\boldsymbol{Q}(n-1)$ and $\tilde{\boldsymbol{R}}_L(n-1)$ are unitary and upper triangular matrices, respectively.

2. (**QRD**) Form the new augmented data matrix

$$\boldsymbol{D}(n) = \Lambda(n)\big[\boldsymbol{X}(n) \quad \boldsymbol{d}(n)\big] = \begin{bmatrix} \sqrt{\lambda}\boldsymbol{D}(n-1) \\ \psi^T(n) \end{bmatrix} \quad \text{(T-3)}$$

where $\psi(n) = \big[\boldsymbol{x}^T(n) \quad d(n)\big]^T$ · $\boldsymbol{X}(n) = [\boldsymbol{x}(0), \boldsymbol{x}(1), \cdots, \boldsymbol{x}(n)]^T$. Get the new QRD by Givens rotations or Householder reflections as

$$\boldsymbol{Q}^{(N)}(n)\cdots\boldsymbol{Q}^{(1)}(n)\boldsymbol{Q}'(n)\boldsymbol{D}(n) = \begin{bmatrix} \tilde{\boldsymbol{R}}_L(n) & \hat{\boldsymbol{d}}_L(n) \\ 0 & \boldsymbol{c}_{\boldsymbol{n}} \\ 0^T & d_{L+1}(n) \end{bmatrix} \quad \text{(T-4)}$$

3. (**Back-solving**) Solve the triangular system $\tilde{\boldsymbol{R}}_L(n)\boldsymbol{w}(n) = \hat{\boldsymbol{d}}_L(n)$ for the *LS* estimate $\boldsymbol{w}(n)$ at time *n* by back-substitution:

$$w_L(n) = [r_{L,L+1}(n)]/r_{L,L}(n) \quad w_i(n) = [r_{i,L+1}(n) - \sum_{j=i+1}^{L} r_{i,j}(n)w_j(n)]/r_{i,i}(n),\ i = L-1,\cdots,1 \quad \text{(T-5)}$$

where $r_{i,j}$ and $r_{i,L+1}$ are the corresponding elements in $\tilde{\boldsymbol{R}}_L(n)$ and $\hat{\boldsymbol{d}}_L(n)$ · $w_i(n)$ is the i-*th* element of $\boldsymbol{w}(n)$.

Table 1– QR-RLS Algorithm

where $w_i(n)$ is the *i*-th filter tap and $\hat{c}_i(n)$ is its approximated time derivative. η is the forgetting factor for calculating the smoothed tap weight $\hat{w}_i(n)$. $\|\cdot\|_1$ denotes the l_1 norm of a vector. The roles of $\hat{c}_i(n)$'s in the prototype algorithm are two folds: First, in time-invariant channels with sparse impulse responses, they allow significant tap weights to be given a larger stepsize and vice versa. This results in a faster initial converging speed. Secondly, since $\hat{c}_i(n)$ tends to reflect the time variations of the filter weights, it yields a faster tracking speed in slowly time-varying channels. We found that the value of $\left\|\hat{\boldsymbol{c}}(n-1)\right\|_1$ will decrease and converge gradually from its initial value to a very small value when the algorithm is about to converge to its steady state. However, this value is rather unstable during tracking the impulse response of time-varying channels. Therefore, we propose a measure to map the convergence status of the adaptive filter through $\left\|\hat{\boldsymbol{c}}(n-1)\right\|_1$ to the expected variance of the VFF $\lambda(n)$. More precisely, we compute the absolute value of the approximate derivative of $\left\|\hat{\boldsymbol{c}}(n-1)\right\|_1$ as

$$G_c(n) = \Big|\left\|\hat{\boldsymbol{c}}(n)\right\|_1 - \left\|\hat{\boldsymbol{c}}(n-1)\right\|_1\Big| \quad (20)$$

and obtain a smoothed version of $G_c(n)$, $\overline{G}_c(n)$, by averaging it over a time window of length *T*. The approximate initial value of $\overline{G}_c(n)$ is obtained by averaging the first *M* data and it is denoted by $\overline{G}_{c0}$. By normalizing $\overline{G}_c(n)$ with $\overline{G}_{c0}$, we get $\overline{G}_N(n)$, which is a more stable convergence measure of the adaptive filter. Denote the lower and upper bounds of

$\lambda(n)$ as λ_L and λ_U, we propose to update $\lambda(n)$ at each iteration as

$$\lambda(n) = \lambda_L + [1 - \overline{G}_N(n)](\lambda_U - \lambda_L)\ . \quad (21)$$

Accordingly, (3) is modified to

$$\lambda_e(n) = \begin{cases} \lambda(n) & |e(n)| < t_\lambda \\ 1 & |e(n)| \geq t_\lambda \end{cases}\ . \quad (22)$$

Hence, the new VFF QRRLM algorithm can be obtained by replacing formula (T-3), (T-4) in QR-RLS algorithm with (13) and (14) and including (18)~(22). The computational complexity of the proposed algorithm is thus similar to the RLM algorithm. It has an $O(L^2)$ complexity together with L+1 more multiplications in (13) and $O(N_w \log N_w)$ operations for computing $\hat{\sigma}_e^2(n)$. Moreover, it needs 3L+3 extra additions for updating $\lambda(n)$ and a little initial computational cost for calculating $\overline{G}_N(n)$ from $G_c(n)$.

4. SIMULATION RESULTS

We now evaluate the performance of the proposed algorithm using computer simulation of the system identification problem in impulsive noise environment (Fig. 1). The unknown system has 30 coefficients which are randomly generated and normalized to unit power. The input signal is an AR process with coefficients [1 -0.65 0.693 -0.22 0.309 -0.177]. The signal-to-noise ratio at the system output is given by $SNR = 10\log 10(\delta_{d_0}^2 / \delta_g^2)$, where $\delta_{d_0}^2$ is the variance of the output of the unknown system to be identified. The interference $\eta_0(n)$ $(or\ \eta_S(n))$ is chosen as $\eta_0(n) = \eta_g(n) + \eta_{im}(n) = \eta_g(n) + b(n)\eta_w(n)$, which is a contaminated Gaussian (CG) noise with $\eta_g(n)$ and $\eta_w(n)$ being i.i.d. zero mean Gaussian processes with variance δ_g^2 and δ_w^2, respectively. $b(n)$ is an independent and identically distributed (i.i.d.) Bernoulli random process assuming a value of either 1 or 0 with occurrence probabilities $P_r(b(n)=1) = p_r$ and $P_r(b(n)=0) = 1 - p_r$. The strength and frequency of the impulsive noise are specified by the ratio $r_{im} = p_r\delta_w^2 / \delta_g^2$. The parameters in our experiment are: $SNR = 30dB$, $p_r = 0.005$, and $r_{im} = 100$. The threshold parameters ξ, Δ_1 and Δ_2 are obtained according to (9)~(12). The forgetting factor λ_e is set to 0.99, and the window length N_w is 14. Five algorithms, the QRRLS, the RLM, the SPRLS, the GVFF RLS and the proposed VFF-QRRLM were compared in three experiments. For the VFF control scheme, $\eta = 0.9999$, T=M=20, $\lambda_L = 0.8$ and $\lambda_U = 0.99$; For the SPRLS algorithm, $\beta = 1$ and $\gamma = 1$; For the GVFF RLS algorithm, $\alpha = 0.3$, $\beta = 0.99$ and $\mu = 0.04$. These parameter settings were chosen so that the steady state MSE of all the algorithms is approximately the same. 50 Monte Carlo experiments were conducted to obtain the resultant curves. **Experiment 1**: Tracking sudden system change. The system parameters were switched to their reverse values at the 1500th iteration. It can be observed from Fig. 3 that both the initial converging speed and the tracking speed after sudden system change of the proposed algorithm is the fastest among all the algorithms. **Experiment 2**: Impulsive noise environment. For illustration purpose, the interference noise $\eta_0(n)$ is an additive Gaussian noise from time $n=1$ to $n=600$ and 2501 to 3000. From $n=601$ to 2500, the CG noise is applied. In order to visualize more clearly the effect of impulses in the desired signal, the locations of impulses are respectively fixed at $n = 800, 1500$ but the amplitudes of impulses are independent variables governed by $\eta_w(n)$. Similarly, for the same reason one impulse is added to the input signal of the filter at $n = 2200$. From Fig. 4 it can be seen the proposed algorithm and the RLM algorithm have nearly the same satisfactory performance in suppressing the impulses in both the desired and the input signals and outperform the other three algorithms. **Experiment 3**: Tracking slowly varying system parameters. The slowly varying system model is defined as $w_i(n+1) = w_i(n) + \varepsilon|w_i(n)|v_i(n)$, $i = 1,2,\cdots,L$, where ε is a small constant equal to 0.01 and $v_i(n)'s$ are a set of independent Gaussian white noise sequences with unit variance. The performance index is the sum of squared coefficient error (MSD). Fig. 5 reveals that the VFF-QRRLM algorithm outperforms the other algorithms.

5. CONCLUSION

A new VFF-QRRLM adaptive filtering algorithm in impulsive noise environment is presented. This new algorithm is a QR-based implementation of the RLM algorithm, which is numerically more stable. A new VFF control scheme based on the approximated derivatives of the filter coefficients is also presented. The improved tracking ability and robustness to impulsive noise of the proposed algorithm is verified by computer simulation.

REFERENCES

[1] S. Haykin, Adaptive Filter Theory, 4th ed. Prentice Hall, 2001.

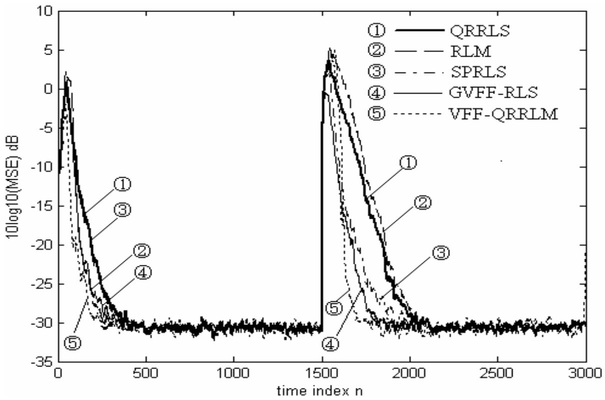
[2] Y. Zou, "Robust Stastitics Based Adaptive Filtering Algorithms for Impulsive Noise Suppression," PhD Dissertation, The University of Hong Kong, 2000.

[3] Y. Zou, S. C. Chan and T. S. Ng, "A recursive least M-estimate (RLM) adaptive filter for robust filtering in impulse noise," *IEEE Signal Processing Letters*, vol. 7, no. 11, 2000.

[4] D. T. M. Slock and T. Kailath, "Fast transversal filters with data sequence weighting," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 33, no. 3, pp. 346-359, Mar. 1989.

[5] D. J. Park and B. E. Jun, "Self-perturbing RLS algorithm with fast tracking capability," *Electron. Lett.*, vol. 28, pp. 558-559, Mar. 1992.

[6] S. Song et al., "Gauss Newton variable forgetting factor recursive least squares for time varying parameter tracking," *Electro. Lett.*, vol. 36, pp. 988-990, May 2000.
[7] S. H. Leung and C. F. So, "Gradient-based variable forgetting factor RLS algorithm in time-varying environments," *IEEE Trans. Signal Processing*, vol. 53, no. 8, pp. 3141–3150, Aug. 2005.
[8] O. Hoshuyama, R. A. Goubran and A. Sugiyama, "A generalized proportionate variable step-size algorithm for fast changing acoustic environments," *IEEE ICASSP-04*, vol. 4, pp. 161-164, May 2004.



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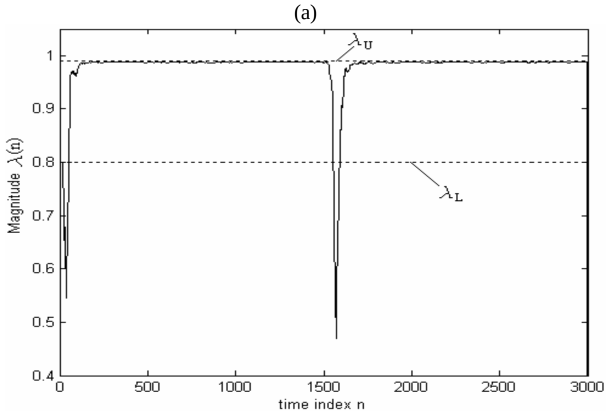
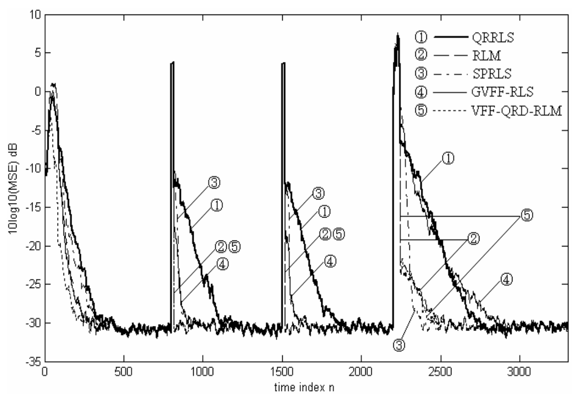


(b)

Figure 3 – (a) Example of sudden system change. (MSE results vs. time *n*) (b) VFF $\lambda(n)$ vs. time *n*



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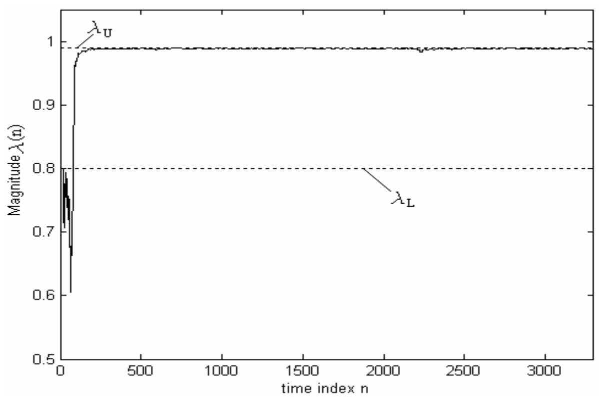
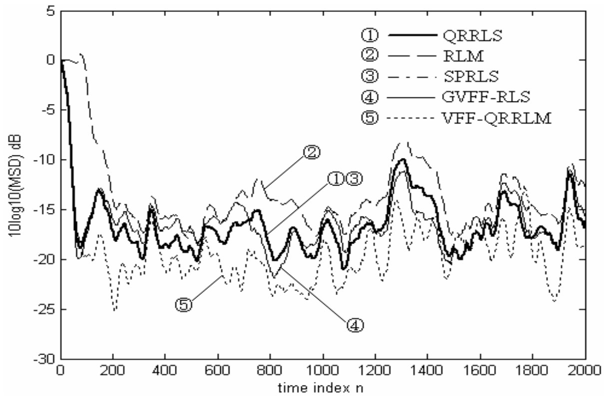


(b)

Figure 4 – (a) Example of impulsive noise environment. (MSE results vs. time *n*) (b) VFF $\lambda(n)$ vs. time *n*



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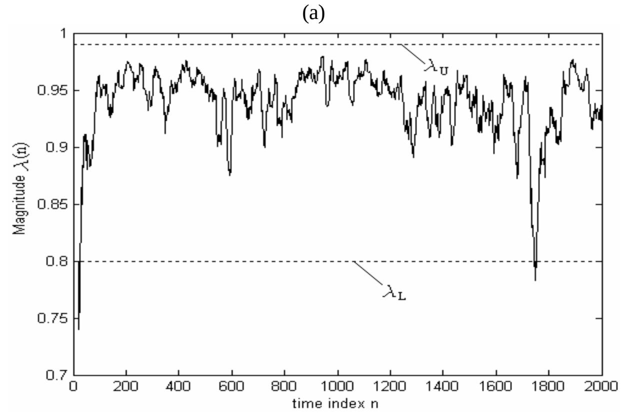

(b)

Figure 5 – (a) Example of tracking slowly varying system parameters. (MSD results vs. time *n*) (b) VFF $\lambda(n)$ vs. time *n*