14th European Signal Processing Conference (EUSIPCO 2006), Florence, Italy, September 4-8, 2006, copyright by EURASIP

A SIMPLE ADAPTIVE FILTER METHOD FOR CANCELLTAION OF COUPLING WAVE IN OFDM SIGNALS AT SFN RELAY STATION

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ABSTRACT

In recent years digital terrestrial broadcasting systems have been developed where OFDM signals are used for data transmission with single frequency network (SFN). But in a SFN relay station the effect of the coupling wave from the transmitter to the receiving antenna is significant and needs to be cancelled. In this paper a simple adaptive filter method is applied to this problem. The stationary point of the conventional LMS algorithm is first derived and its local stability is examined by using the averaging method. It is found that this algorithm has a bias. Then a modified algorithm is proposed to remove this bias. Simulation results show the validity of the theoretical findings.

1. INTRODUCTION

Recently there have been some works concerning the cancellation of the coupling wave at a relay station in single frequency network for digital terrestrial broadcasting systems. In [1][2], rather complicated methods utilizing pilot signals in OFDM system have been proposed for cancellation of this effect. This problem is similar to that of hearing aids [3][4]. In hearing aids there is an acoustic feedback path from the speaker to the microphone and this causes annoying effects such as whistling and howling. An adaptive filter is used to model this acoustic feedback path and cancel its effect. In [4] the stationary point of the conventional LMS algorithm and its local stability condition were derived by first expressing the algorithm in frequency domain and then applying the averaging method to it.

In this paper, a simple adaptive filter method is applied for continuous cancellation of the effect based on the results obtained for the problem of hearing aids. First by a method that is more direct than that in [4] but is still considering the causality constraint, an explicit expression of the stationary point is derived. Also, the stability near this point is examined. Similarly to the situation in hearing aids, the stationary point of the adaptive filter contains a bias from the transfer function of the path of the coupling wave. Then, a method to remove this bias is also proposed. Finally, simulation results are presented to see the validities of the theoretical findings.

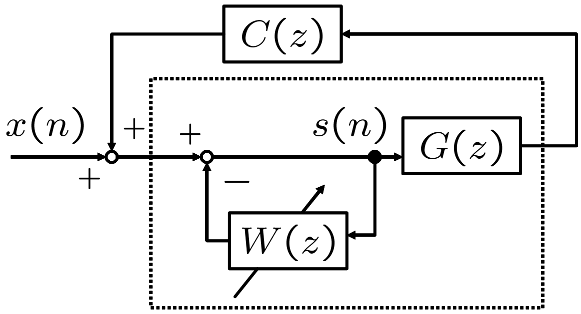

Figure 1: Block diagram of the coupling wave canceller

2. DERIVATION OF THE STATIONARY POINT

Figure 1 shows the block diagram of the coupling wave canceller with the conventional adaptive filter where $x(n)$ is the OFDM signal transmitted through a multipath channel which is modeled as a zero-mean cyclostationary process [5] and $C(z)$, $G(z)$ denote the transfer functions of the coupling wave path and the amplifier characteristics of the transmitter in the relay station, respectively. Though $G(z)$ is fixed and known, $C(z)$ is unknown and may be slowly time-varying. To cancel this effect continuously, an adaptive filter denoted by $W(z)$ with the conventional LMS algorithm is used. Although this seems to be a typical adaptive filter problem for noise canceling, actually it is not so. Usual adaptive filter problems treat cases where the transfer function to be cancelled is in the feedforward path. But here the adaptive filter tries to cancel the effect of the coupling wave in the feedback path. There seem no systematic treatments for this case in the literature. The signal $s(n)$ in Fig. 1 is expressed as

$$s(n) = x(n) + C(z)G(z)s(n) - W(z)s(n). \tag{1}$$

Hence,

$$s(n) = Q(z)x(n) \tag{2}$$

with

$$Q(z) = \frac{1}{1-(C(z)G(z)-W(z))} \tag{3}$$

where the adaptive filter is treated as time-invariant for the moment , z^{-1} denotes the unit time delay operator and $Q(z)$ is assumed to be stable, i.e., the zeros of $1-(C(z)G(z)-W(z))$ are all inside the unit circle. Otherwise, the system is unstable and we can not treat the problem properly. The conventional LMS algorithm for cancellation of the effect of the feedback path is

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu \mathbf{s}(n) s^*(n) \tag{4}$$

with

$$\mathbf{w}(n) = [w_0(n)\ w_1(n)\ \ldots\ w_N(n)]^T \tag{5}$$

$$\mathbf{s}(n) = [s(n)\ s(n-1)\ \ldots\ s(n-N)]^T \tag{6}$$

$$W(z) = w_0^* + w_1^* z^{-1} + \ldots + w_N^* z^{-N} \tag{7}$$

where $w_i(n)$ is the i-th weight of the adaptive filter, μ is the positive step size and $''*''$ denotes the complex conjugate. Also, the number of the tap weights N is to be selected appropriately. In (4) the error signal $s(n)$ is also used for the input signal. This is a quite unusual situation in the adaptive filtering literature.

By the averaging method in [6], the stationary point of the adaptive algorithm in (4) is determined by

$$\mathrm{E}[s(n-i)s^*(n)] = 0 \qquad (i = 0, 1, \ldots, N). \tag{8}$$

We should exclude the condition for $i = 0$ in (8), otherwise $s(n) = 0$ is followed. So , we set in (5) and (7) that

$$w_0 = 0 \quad (w_0(n) = 0). \tag{9}$$

Also, in (8) the expectation operation is interpreted as the time average as well as the ensemble average, since $x(n)$ is a cyclostationary process and the expected value about this process is periodically time-varying. So we can treat $x(n)$ as if it is a stationary process with the spectral density $P(e^{j\omega})$ which is equal to the cyclic spectrum of cycle frequency 0 [5][7]. Hence, from (2), (8) can be expressed as

$$\frac{1}{2\pi}\int_0^{2\pi} e^{-ji\omega} Q(e^{j\omega}) P(e^{j\omega}) Q^*(e^{j\omega}) d\omega = 0 \qquad (i = 1, \ldots, N). \tag{10}$$

By the change of variables $z = e^{-j\omega}$, (10) can be rewritten as

$$\frac{1}{2\pi}\int_0^{2\pi} z^{i-1} \times z Q(z^{-1}) \tilde{Q}(z) P(z^{-1}) d\omega = 0. \tag{11}$$

where

$$\tilde{Q}(z) = \frac{1}{1-(\tilde{C}(z)\tilde{G}(z) - \tilde{W}(z))}$$

with

$$\tilde{W}(z) = w_1 z^{-1} + \cdots + w_N z^{-N}$$

and similarly for $\tilde{C}(z)$, $\tilde{G}(z)$. Although (10) holds for $i = 1, \ldots, N$, if N is large enough, it is reasonable to find a solution that satisfies (11) for all $i-1 \geq 0$. For this, the integrand after z^{i-1} in (10) must be a series of positive powers of z. Hence, we have

$$[zQ(z^{-1})\tilde{Q}(z)P(z^{-1})]_+ = 0 \tag{12}$$

where $[.]_+$ denotes the extraction of the causality part, that is, the constant term and negative powers of z. Let the spectral factorization of $P(z)$ be

$$P(z) = R(z)\tilde{R}(z^{-1})\gamma^2 \tag{13}$$

where $R(z)$ is of minimum phase and its constant term is 1. Since from (3), $1/Q(z)$ is a stable polynomial and $1/R(z)$ is also of minimum phase, $1/(R(z^{-1})Q(z^{-1}))$ is expanded in nonnegative powers of z and so $R(z^{-1})Q(z^{-1})$ can be factored out from $[.]_+$ in (12). So we have

$$[z\tilde{R}(z)\tilde{Q}(z)]_+ = \left[\frac{z\tilde{R}(z)}{1-(\tilde{C}(z)\tilde{G}(z) - \tilde{W}(z))}\right]_+ = 0. \tag{14}$$

To obtain $R(z)$ in (14) we first consider the case where $x(n)$ is itself an OFDM signal with the data length M, the length of the cyclic prefix L and the total length of one block $T = M + L$. Let $x_k(n) = x(nT + k)$ with

$$x_k(n) = \sum_{i=0}^{M-1} s_i(n) e^{j2\pi i(k-L)/M} \quad (k = 0, \ldots, T-1) \tag{15}$$

where $s_i(n)$ is an uncorrelated data sequence with zero mean and variance σ^2. Then,

$$\mathrm{E}[x_{k+\tau}(n)x_k^*(n)] = \sigma^2 \sum_{i=0}^{M-1} e^{j2\pi i\tau/M} \times \sum_{r=0}^{T-1} \delta(r-(k+\tau)) \tag{16}$$

where $\delta(r) = 0$ for $r \neq 0$ and 1 for $r = 0$ [5]. The cyclic correlation function with cycle frequency 0 is just the average of (16) with respect to k from 0 to $T-1$ and is given by

$$\begin{aligned} p(0;\tau) &= \frac{\sigma^2}{T}(T-|\tau|)\sum_{i=0}^{M-1} e^{j2\pi i\tau/M} \quad (|\tau| \leq T) \\ &= 0 \qquad\qquad (|\tau| > T) \end{aligned} \tag{17}$$

So we have

$$p(0;\tau) = \frac{\sigma^2}{T}(T\delta(\tau) + L\delta(\tau - M) + L\delta(\tau + M))$$

and

$$P(z) = \frac{LM\sigma^2}{T}\left(\frac{T}{L} + z^M + z^{-M}\right) \tag{18}$$

with the spectral factor

$$R(z) = 1 + \alpha z^{-M} \tag{19}$$

where $\alpha + 1/\alpha = T/L$ and $\gamma = \sigma\sqrt{LM/\alpha T}$. Since in general $T/L > 2$, we can find α such that $0 < \alpha < 1$. Now we obtain an explicit solution of (14). Since $\tilde{R}(z)\tilde{Q}(z)$ is causal, from (14) it must be some constant β^*. Hence, the stationary point $W_0(z)$ satisfies

$$1 - (C(z)G(z) - W_0(z)) = \frac{1}{\beta}R(z). \tag{20}$$

From (7),(9) and (19), if the delay z^{-1} is included in $G(z)$, i.e.,

$$G(z) = z^{-1}G_0(z) \tag{21}$$

then $\beta = 1$ and

$$W_0(z) = C(z)G(z) + \alpha z^{-M}. \tag{22}$$

That is, there is a bias term αz^{-M}. Before presenting a modified algorithm to remove this bias, in the next section the local stability near the stationary point is discussed.

3. THE LOCAL STABILITY PROPERTY

Here we use the averaging or ODE (ordinary differential equation) method to examine the local stability of the stationary point (22). The ODE describing the average behavior of the adaptive algorithm (4) is

$$\dot{\mathbf{w}} = \mathbf{f}(\mathbf{w})$$

where $\mathbf{f}(\mathbf{w}) = (f_1(\mathbf{w}) \ldots f_N(\mathbf{w}))^T$ with

$$f_i(\mathbf{w}) = \mathrm{E}[s(n-i)s^*(n)].$$

The linearized ODE near the stationary point $\mathbf{w}_0$ corresponding to $W_0(z)$ in (22) is described as

$$\dot{\mathbf{w}} = -\Phi(\mathbf{w} - \mathbf{w}_0)$$

where from (8) and (10) the (i,k)th elemet of Φ is given by

$$\begin{aligned}\Phi_{ik} &= -\frac{\partial}{\partial w_k}\frac{1}{2\pi}\int_0^{2\pi} z^i Q(z^{-1})\tilde{Q}(z)P(z^{-1})d\omega|_{\mathbf{w}=\mathbf{w}_0} \\ &= \frac{1}{2\pi}\int_0^{2\pi} z^i Q(z^{-1})z^{-k}\tilde{Q}^2(z)P(z^{-1})d\omega|_{\mathbf{w}=\mathbf{w}_0}.\end{aligned} \tag{23}$$

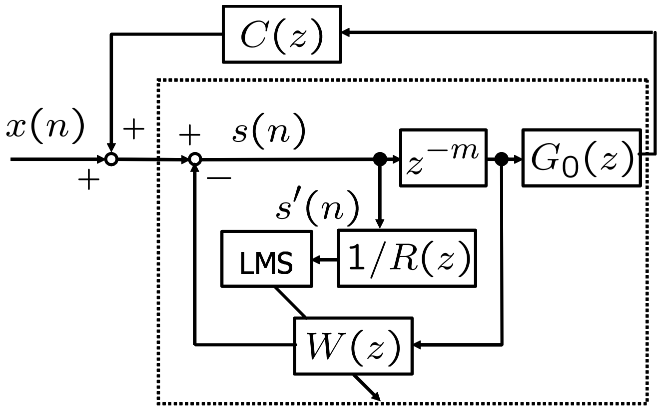

Figure 2: Block diagram of the unbiased coupling wave canceller for a multpath channel

In (23) we use the differential rule $\partial w_k^*/\partial w_k = 0$. If $\Phi + \Phi^H$ is positive definite, the Lyapunov function $V(\mathbf{w}) = ||\mathbf{w} - \mathbf{w}_0||^2$ is decreasing, since $\dot{V}(\mathbf{w}) \le 0$ where the equality holds only at $\mathbf{w} = \mathbf{w}_0$. Hence, $\mathbf{w}_0$ is a locally stable stationary point. But from (23) for any vector $\xi = [\xi_1 \ldots \xi_N]^T$

$$\begin{aligned}&\xi^H(\Phi + \Phi^H)\xi \\ =\ & \frac{1}{2\pi}\int_0^{2\pi} |\sum_i \xi_i z^{-i}|^2 |\tilde{Q}(z)|^2 \gamma\alpha \mathrm{Re}(1 + \alpha z^{-M})d\omega \\ >\ & 0, No.A\end{aligned} \tag{24}$$

so the local stability is guaranteed.

4. UNBIASED IDENTIFICATION OF THE FEEDBACK PATH

A simple modification to remove the bias in (22) is presented. Instead of using $s(n)$ directly in (4), the following filtered signal

$$s'(n) = \frac{1}{R(z)}s(n) \tag{25}$$

is used. This is a whitening operation if at the stationary point $x(n) = s(n)$. That is, (25) is written as

$$s'(n) = -\alpha s'(n-M) + s(n). \tag{26}$$

Then, (12) is replaced by

$$\left[\frac{1}{R(z^{-1})\tilde{R}(z)} zQ(z^{-1})\tilde{Q}(z)P(z^{-1})\right]_+ = 0, \tag{27}$$

so that from (13) the stationary point in this case satisfies

$$[z\tilde{Q}(z)]_+ = \left[\frac{z}{1 - (\tilde{C}(z)\tilde{G}(z) - \tilde{W}_0(z))}\right]_+ = 0. \tag{28}$$

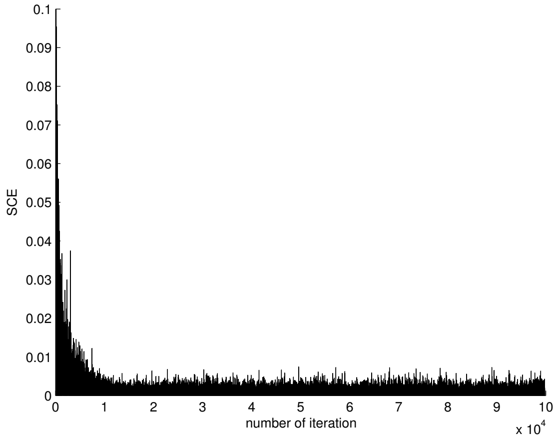

Figure 3: Plots of the cancellation errors using (4)

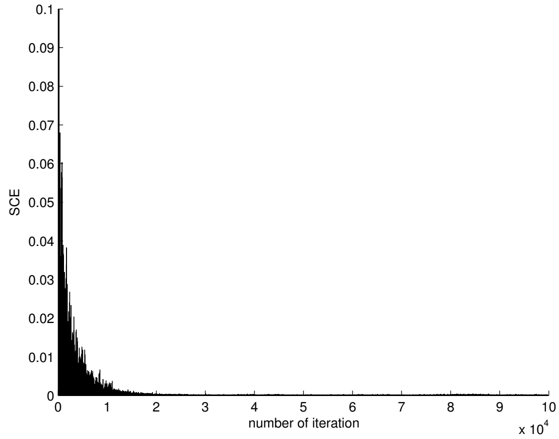

Figure 4: Plots of the cancellation errors using the algorithm with filtered signals in (25)

From this $\tilde{Q}(z)$ must be 1 , so we have

$$W_0(z) = C(z)G(z). \tag{29}$$

This means that unbiased identification of the feedback path and continuous cancellation of the effect of the coupling wave are attained. The local stability near this point is also established, since corresponding to (24) we have

$$\xi^H(\Phi + \Phi^H)\xi = \frac{1}{2\pi}\int_0^{2\pi} |\sum_i \xi_i z^{-i}|^2 P(z^{-1})d\omega > 0.$$

Next we consider the case that the OFDM signal is transmitted through a multipath channel whose transfer function $B(z)$ is of FIR type. Hence, $P(z)$ in this case is given by

$$P(z) = B_0(z)\tilde{B}_0(z^{-1})R(z)\tilde{R}(z^{-1})\gamma^2 \tag{30}$$

where $B_0(z)$ is a stable polynomial satisfying $B_0(z)\tilde{B}_0(z^{-1}) = B(z)\tilde{B}(z^{-1})$. The filtered signal $s'(n)$ in (25) is also used in the adaptive filter with the delay constraint

$$w_0(n) = \ldots = w_{m-1}(n) = 0, \tag{31}$$

then (8) holds for $i \geq m$ and corresponding to (14) we have

$$\left[\frac{z^m\tilde{B}_0(z)}{1-(\tilde{C}(z)\tilde{G}(z)-\tilde{W}(z))}\right]_+ = 0. \tag{32}$$

So if m is taken to be larger than the order of $B(z)$, the numerator inside $[.]_+$ in (32) is non-causal. Also, if we set

$$G(z) = z^{-m}G_0(z), \tag{33}$$

$\tilde{Q}(z)$ is expanded as $1 + z^{-m}q_m + \ldots$. Hence, the stationary point in this case is also given by (29) and it is locally stable. Fig. 2 shows the block diagram of the unbiased coupling wave canceller for a multipath channel.

In actual situations some tones in the OFDM signal are not used. For example, if“ higher ”tones $i = M/2 - \Delta, \ldots, M/2 + \Delta$ are not used [2], this interval is excluded in the summation in (17). But in this case we cannot obtain a closed form expression of the spectral factor like (19) and we need to use a numerical procedure for spectral factorization , for example, the algorithm in [8] to obtain the stable polynomial $R(z)$ of order T.

5. SIMULATION RESULTS

To see the validities of the above theoretical findings, some preliminary simulation results are presented. The OFDM signal is generated with BPSK data of amplitude 1 and $M = 64$, $L = 16$ $(T = 80)$. In this case $\alpha = 0.2087$. Also, we set $C(z) = 0.4z^{-1}$, $G(z) = 2$ and the number of the tap weights $N = 64$. Fig. 3 shows the plots of the squared cancellation error (SCE) $|x(n) - s(n)|^2$ versus the iteration number n when (4) is used with $\mu = 0.01$. But due to the bias it does not converge to zero as shown. Fig. 4 shows the plots when the filtered signal $s'(n)$ is used in (4) with $\mu = 0.01$. As is seen from this figure, the cancellation is perfect. Fig. 5 presents the results for the case where the OFDM signal is transmitted through the channel $B(z) = 1 + 1/3z^{-1}$ with $m = 2$. Again, the cancellation is perfect.

6. CONCLUSION

We have presented a method for obtaining the stationary point of the conventional LMS adaptive filter algorithm

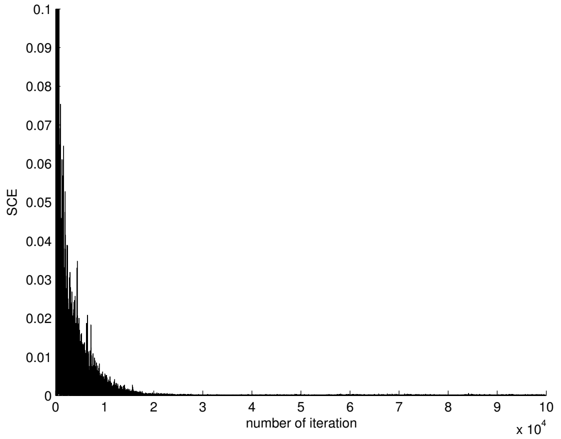

Figure 5: Plots of the cancellation errors for the signal through a multipath channel

for cancellation of the effect of the coupling wave in SFN relay stations. Also, the local stability near this point has been shown. Based on the above findings a new algorithm using the whitening operation has been devised to attain the perfect cancellation. It is a future work to implement this algorithm in an efficient way when the length of the whitening filter is very long.

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