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FINITE WORDLENGTH IMPLEMENTATION FOR RECURSIVE DIGITAL FILTERS WITH ERROR FEEDBACK

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ABSTRACT

In this work, we treat a design problem for recursive digital filters described by rational transfer function in discrete space with scaling and error feedback. First, we form the filter design problem using the least-squares criterion and express it as the quadratic form with respect to the filter coefficients. Next, we show the relaxation method using the Lagrange multiplier method in order to search for the good solution. Finally, we show the effectiveness of the proposed method using some numerical examples.

1. INTRODUCTION

Design problem of digital filter has been formulated in the continuous space in many cases. However in the implementation of hardware, the characteristic of the digital filter may be changed due to the finite wordlength of the filter coefficients, also the overflow can occur in the adder of the filter. So we have pay attention to such a finite wordlength effect and should reduce it.

The synthesis of low-sensitivity digital filter structures using coordinate transformation is an effective method to reduce the degradation of filter performance [1, 2, 3]. However this method can be only used when the filter is described by the state-space model. So when the filter is described by rational transfer function, we can not use this method, and have to design the filter coefficients in discrete space to avoid the performance degradation. If we search the filter coefficients in discrete space, it takes enormous time to find the best solution. Hence, the design of digital filter with discrete coefficients is really hard.

In this paper, we treat the problem of designing the discrete coefficients, under the scaling constraint. First, we form the filter design problem using the least-squares criterion and express it as the quadratic form with respect to the numerator and denominator coefficients. Next, we show the relaxation method using the Lagrange multiplier method to search for the good solution. The designed filter is always stable and better than simple roundoff technic. Also, another problem of the finite wordlength implementation is the product roundoff noise. Error feedback(EF) is known as the effective method to reduce the product roundoff noise[4, 6, 5]. So we also apply the branch and bound method to the design problem of EF with discrete coefficients.

Finally, we give two numerical examples to design the digital filters and EF with discrete coefficients under scaling constraint and show the effectiveness of the proposed method.

2. LEAST-SQUARES METHOD

First of all, we summarize the least-squares method proposed in [7]. Let the desired transfer frequency response be $H_d(\omega)$ in $[0, \pi)$. The purpose of that work is to find a rational transfer function

$$H(z) = \frac{N(z)}{D(z)} \tag{1}$$

$$D(z) = 1 + \boldsymbol{q}_1^t(z)\boldsymbol{a} \tag{2}$$

$$N(z) = \boldsymbol{q}_2^t(z)\boldsymbol{b} \tag{3}$$

where $\boldsymbol{a} = [a_1 \dots a_n]^t$, $\boldsymbol{b} = [b_0\ b_1 \dots b_n]^t$, $\boldsymbol{q}_1(z) = [z^{-1} \dots z^{-n}]^t$ and $\boldsymbol{q}_2(z) = [1\ z^{-1} \dots z^{-n}]^t$.

Using $D(z)$, $N(z)$ and $H_d(\omega)$, L_2 weighted function is defined by

$$E(\boldsymbol{a}, \boldsymbol{b}) = \frac{1}{2}\int_0^{\pi} W(\omega) \mid H_d(\omega) - H(e^{j\omega}) \mid^2 d\omega. \tag{4}$$

(4) can be written as

$$E(\boldsymbol{a}, \boldsymbol{b}) = \frac{1}{2}\int_0^{\pi} \frac{W(\omega)}{|D(e^{j\omega})|^2} \mid H_d(\omega)D(e^{j\omega}) - N(e^{j\omega}) \mid^2 d\omega. \tag{5}$$

In (5), $|D(e^{j\omega})|^2$ underneath $W(\omega)$ can be calculated by following function.

$$E(\boldsymbol{a}^{(k)}, \boldsymbol{b}^{(k)}) = \frac{1}{2}\int_0^{\pi} \frac{W(\omega)}{|D_{k-1}(e^{j\omega})|^2} \mid H_d(\omega)D_k(e^{j\omega}) - N_k(e^{j\omega}) \mid^2 d\omega \tag{6}$$

where $D_k(e^{j\omega}) = 1 + \boldsymbol{q}_1^t(\omega)\boldsymbol{a}^{(k)}$ and $N_k(e^{j\omega}) = \boldsymbol{q}_2^t(\omega)\boldsymbol{b}^k$ $(k = 1, 2, \cdots)$. $\boldsymbol{a}^{(k)}$ and $\boldsymbol{b}^{(k)}$ are the coefficients vectors to be determined in the kth iteration. Define

$$W_k(\omega) = \frac{W(\omega)}{|D_{k-1}(e^{j\omega})|^2} \tag{7}$$

then

$$E(\boldsymbol{a}^{(k)}, \boldsymbol{b}^{(k)}) = \frac{1}{2}\int_0^{\pi} W_k(\omega) \mid H_d(\omega)D_k(e^{j\omega}) - N_k(e^{j\omega}) \mid^2 d\omega. \tag{8}$$

Hence $\boldsymbol{a}^{(k)}$ and $\boldsymbol{b}^{(k)}$ can be obtained by using $\boldsymbol{a}^{(k-1)}$. Also, (8) can be written in the quadratic form as

$$E(\boldsymbol{x}^{(k)}) = \frac{1}{2}\boldsymbol{x}^{(k)^t}\boldsymbol{K}\boldsymbol{x}^{(k)} + \boldsymbol{v}^t\boldsymbol{x}^{(k)} + c \tag{9}$$

where $\boldsymbol{x}^{(k)} = [\boldsymbol{a}^{(k)}\ \boldsymbol{b}^{(k)}]^t$, c is a constant independent of $\boldsymbol{a}^{(k)}$ and $\boldsymbol{b}^{(k)}$ and

$$\boldsymbol{K} = \begin{bmatrix} \boldsymbol{K}_{11} & -\boldsymbol{K}_{12} \\ -\boldsymbol{K}_{12}^t & \boldsymbol{K}_{22} \end{bmatrix} \tag{10}$$

$$\boldsymbol{K}_{11} = \triangle \sum_{i=1}^{L} W_k(\omega_i)|H_d(\omega_i)|^2 \boldsymbol{Q}_{11}(\omega_i) \tag{11}$$

$$\boldsymbol{K}_{22} = \triangle \sum_{i=1}^{L} W_k(\omega_i) \boldsymbol{Q}_{22}(\omega_i) \tag{12}$$

$$\boldsymbol{K}_{12} = \triangle \sum_{i=1}^{L} W_k(\omega_i) \boldsymbol{Q}_{12}(\omega_i) \tag{13}$$

$$\boldsymbol{Q}_{11}(\omega) = \begin{bmatrix} 1 & \cos(\omega) & \cdots & \cos[(n-1)\omega] \\ \cos(\omega) & 1 & \cdots & \cos[(n-2)\omega] \\ \vdots & & \ddots & \vdots \\ \cos[(n-1)\omega] & \cos[(n-2)\omega] & \cdots & 1 \end{bmatrix} \tag{14}$$

$$\boldsymbol{Q}_{12}(\omega) = \frac{1}{2}\left[H_d(\omega)\boldsymbol{q}_1(\omega)\overline{\boldsymbol{q}}_2^t(\omega) + \overline{H}_d(\omega)\overline{\boldsymbol{q}}_1(\omega)\boldsymbol{q}_2^t(\omega)\right] \tag{15}$$

$$\boldsymbol{Q}_{22}(\omega) = \begin{bmatrix} 1 & \cos(\omega) & \cdots & \cos(n\omega) \\ \cos(\omega) & & & \\ \vdots & & \boldsymbol{Q}_{11}(\omega) & \\ \cos(n\omega) & & & \end{bmatrix} \tag{16}$$

$$\boldsymbol{v} = \begin{bmatrix} \boldsymbol{v}_1 \\ -\boldsymbol{v}_2 \end{bmatrix} \tag{17}$$

$$\boldsymbol{v}_1 = \triangle \sum_{i=1}^{L} W_k(\omega_i)|H_d(\omega_i)|^2 \tilde{\boldsymbol{q}}_1(\omega_i) \tag{18}$$

$$\boldsymbol{v}_2 = \triangle \sum_{i=1}^{l} W_k(\omega_i) \tilde{\boldsymbol{q}}_2(\omega_i) \tag{19}$$

$$\tilde{\boldsymbol{q}}_1(\omega) = [\cos(\omega), \cos(2\omega), \ldots, \cos(n\omega)]^t \tag{20}$$

$$\tilde{\boldsymbol{q}}_2(\omega) = \frac{1}{2}[H_d(\omega)\overline{\boldsymbol{q}}_2(\omega) + \overline{H}_d(\omega)\boldsymbol{q}_2(\omega)]. \tag{21}$$

$\triangle$ is the increment for numerical integral of (9). Here $D_k(e^{j\omega})$ must be stable and the stable solution can be obtained to minimize (9) under the following constraint

$$\mathrm{Re}[D_k(e^{j\omega})] > 0 \quad \omega \in [0, \pi] \tag{22}$$

or

$$\mathrm{Re}[\boldsymbol{B}\boldsymbol{x}^{(k)}] \leq (1-\delta)\boldsymbol{e}_{2n+1} \tag{23}$$

where

$$\boldsymbol{B} = -\begin{bmatrix} \boldsymbol{q}_1^t(e^{j\Omega_1}) & \\ \vdots & \boldsymbol{0} \\ \boldsymbol{q}_1^t(e^{j\Omega_M}) & \end{bmatrix}_{M\times(2n+1)} \qquad \boldsymbol{e}_{2n+1} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}_{M\times 1} \tag{24}$$

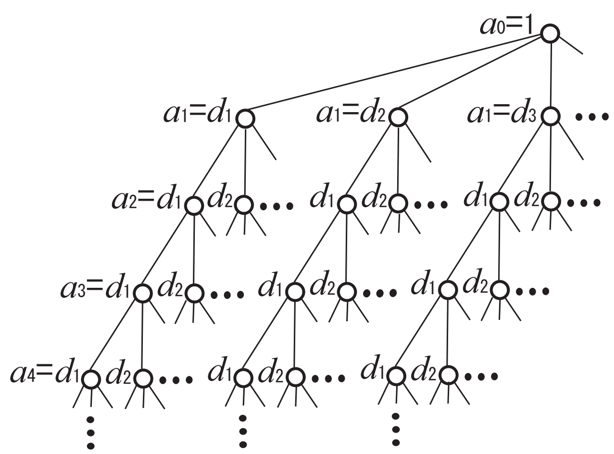

Figure 1: Example of the tree structure where $a_0, a_1 \cdots$ are the filter coefficients, and $d_1, d_2 \cdots$ are their candidates.

and δ is a positive small number. Hence the filter design problem is expressed as

$$\min E(\boldsymbol{x}) \tag{25}$$

$$\text{subject to } \mathrm{Re}[\boldsymbol{B}\boldsymbol{x}^{(k)}] \leq (1-\delta)\boldsymbol{e}_{2n+1}. \tag{26}$$

Thus $\boldsymbol{x}$ (continuous values) can be obtained by calculating the iteration (25) under the constraint (26).

In this paper, we find the discrete coefficients which minimize (9) under scaling constraint using $\boldsymbol{K}$ and $\boldsymbol{v}$.

3. FINITE WORDLENGTH DESIGN

3.1 Lower bound estimation principle

Consider when the filter order is $n = 12$, and the wordlength of the sign, integer part and decimal part of the coefficients are 1, 3 and 5 bits, respectively. Then each coefficients can take $2^9 - 1 = 511$ values. Hence the number of combination for discrete coefficients is $(2^9 - 1)^{(2n+1)} = 511^{25} = 5.13 \times 10^{67}$. So it seems very hard to search all combination if using the tree structure as shown in Fig.1.

The branch and bound method is based on the lower bound estimation principle by the Lagrange multiplier method and that can reduce the calculation costs efficiently. The procedure is summarized as follows.

Assume the temporary solution $\overline{\boldsymbol{x}}_P$ is already obtained. Let the discrete coefficients vector be $\overline{\boldsymbol{x}}$ and divide it into two sets as

$$\overline{\boldsymbol{x}} = \{\boldsymbol{x}_1, \boldsymbol{x}_2\} \tag{27}$$

where $\boldsymbol{x}_1 = (x_1, \cdots x_i)$ and $\boldsymbol{x}_2 = (x_{i+1}, \cdots x_{2n+1})$. Hence $\boldsymbol{x}_i$ indicates the tree with depth i.

Next, we write the relaxation solution of $\boldsymbol{x}_2$ as follows.

$$\overline{\boldsymbol{x}}' = \{\boldsymbol{x}_1, \boldsymbol{x}_2^*\} \tag{28}$$

Then if

$$E(\overline{\boldsymbol{x}}_P) \leq E(\overline{\boldsymbol{x}}') \tag{29}$$

holds, the set $\boldsymbol{x}_2$ in $\boldsymbol{x}_1$ which beats $\overline{\boldsymbol{x}}_P$ does not exit. So then it is not necessary to search $\boldsymbol{x}_2$ in $\boldsymbol{x}_1$ and it leads to reduction of search space.

3.2 Lagrange multiplier method

Relaxation means to calculate $\boldsymbol{x}_2^*$ which minimizes (9) with continuous values.

Define

$$\boldsymbol{x}^t \boldsymbol{S} = \boldsymbol{x}_1^t \tag{30}$$

where

$$\begin{aligned}\boldsymbol{S} =&(\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_i)\\ \boldsymbol{s}_1 =&(1,0,0,\ldots,0)^t\\ \boldsymbol{x}_1 =&(x_1, x_2, \ldots, x_i)^t.\end{aligned}$$

(30) indicates some of coefficients are discrete. The relaxation solution can be calculated by using the Lagrange multiplier method.

By applying the Lagrange multiplier method to the problem that minimizes (9) under constraint (30), we have

$$J(\boldsymbol{x}, \boldsymbol{\lambda}) = \frac{1}{2}\boldsymbol{x}^t \boldsymbol{K}\boldsymbol{x} + \boldsymbol{v}^t\boldsymbol{x} + c - (\boldsymbol{x}^t\boldsymbol{S} - \boldsymbol{x}_1^t)\boldsymbol{\lambda}. \tag{31}$$

Differentiating $J(\boldsymbol{x}, \boldsymbol{\lambda})$ with respect to $\boldsymbol{x}$ and $\boldsymbol{\lambda}$, and equating the resulting expression to zero yields

$$\frac{\partial J(\boldsymbol{x}, \boldsymbol{\lambda})}{\partial \boldsymbol{x}} = \boldsymbol{K}\boldsymbol{x} + \boldsymbol{v} - \boldsymbol{S}\boldsymbol{\lambda} = \boldsymbol{0} \tag{32}$$

$$\frac{\partial J(\boldsymbol{x}, \boldsymbol{\lambda})}{\partial \boldsymbol{\lambda}} = -\boldsymbol{x}^t\boldsymbol{S} + \boldsymbol{x}_1^t = \boldsymbol{0}. \tag{33}$$

From (32) and (33), we have

$$\boldsymbol{\lambda} = (\boldsymbol{S}^t\boldsymbol{K}^{-1}\boldsymbol{S})^{-1}(\boldsymbol{x}_1 + \boldsymbol{S}^t\boldsymbol{K}^{-1}\boldsymbol{v}). \tag{34}$$

Hence $\boldsymbol{x}$ can be obtained as

$$\boldsymbol{x} = \boldsymbol{K}^{-1}(\boldsymbol{S}(\boldsymbol{S}^t\boldsymbol{K}^{-1}\boldsymbol{S})^{-1}(\boldsymbol{x}_1 + \boldsymbol{S}^t\boldsymbol{K}^{-1}\boldsymbol{v}) - \boldsymbol{v}). \tag{35}$$

Thus we can get the relaxation solution by setting $\boldsymbol{S} \in \{0, \pm 1\}$ and $\boldsymbol{p}$. For example, $\boldsymbol{S}$ and $\boldsymbol{x}_1$ corresponding to $x_1 = 1,\ x_2 = 0.5$ and $x_3 = 1.25$ are as follows

$$\boldsymbol{S} = \begin{bmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix} \tag{36}$$

$$\boldsymbol{x}_1 =(1,\ 0.5,\ 1.25)^T. \tag{37}$$

3.3 Branch and bound method

Let a tree of the filter coefficients be A, and define the search function $P_i = S(A)$ which divides A into subtrees P_i. In this paper, we use the depth-first search algorithm as the search function $S(A)$ as follows.

$S(A)$: We select the tree P_i that is located in the deepest branch among the sets of sub-tree A. The d_i's search priorities $s_p(d_i)$ are $s_p(d_1) > s_p(d_2) \cdots > s_p(d_m)$ where $d_1 < d_2 < \cdots < d_m$.

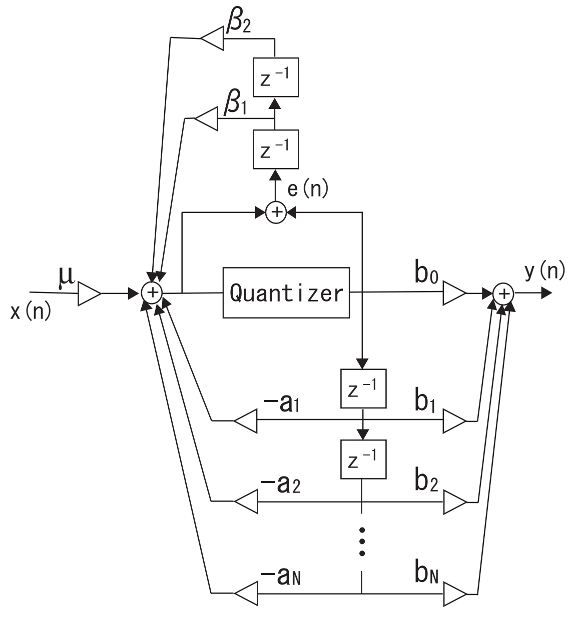

Figure 2: Recursive digital filter with the scaling coefficient s and error feedback where β_1 and β_2 are the EF coefficients, and $x(n)$, $y(n)$ and $e(n)$ are the filter input, filter output, and product roundoff noise, respectively.

The procedure of the branch and bound method based on the lower bound estimation principle is summarize as follows.

A: The set of sub-trees that have already been generated but not divided and finalized.

E_P: The noise gain of the optimum solution.

O: The set of the optimum solution(s).

STEP 1: $A = \{P_0\}$, $E_P = E(\overline{\boldsymbol{x}}_P)$ and $O = \{\overline{\boldsymbol{x}}_O\}$.

STEP 2: $A = \phi$? If yes, go to STEP 8. If not, $P_i = S(A)$ and go to STEP 3.

STEP 3: Are all coefficients discrete? If yes, go to STEP 4. If not, calculate the relaxation solution by (35) and go to STEP 5.

STEP 4: If there is the solutions $\overline{\boldsymbol{x}}$ satisfying $E(\overline{\boldsymbol{x}}) < E_P$, then renew the temporary solution and go to STEP 7. If not, go to STEP 5.

STEP 5: $E(\overline{\boldsymbol{x}}') \geq E_P$ or $E(\overline{\boldsymbol{x}}) \geq E_P$? If yes, go to STEP 7, if not, go to STEP 6.

STEP 6: Generate the j sub-trees under P_i as $P_{i1},\ P_{i2},\ \cdots P_{ij}$. $A = A \cup \{P_{i1},\ P_{i2},\ \cdots P_{ij}\} - \{P_i\}$ and go to STEP 2.

STEP 7: $A = A - \{P_i\}$ and go to STEP 2.

STEP 8: End

3.4 Scaling

Scaling is a technic to prevent the overflow using the scaling coefficient. Fig. 2 shows the proposed structure. Here the scaling coefficient μ is selected to prevent the

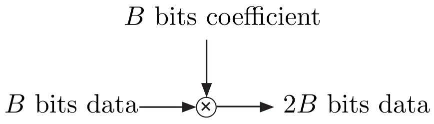

Figure 3: Product of B bits data and B bits coefficient.

overflow at the adder. Let the impluse response from the filter input to the adder be $f(\xi)$, we decide the scaling coefficient μ by using L_2-norm [8] as

$$\mu = \left\{ \frac{1}{\left[\sum_{\xi=0}^{\infty} f^2(\xi) \right]^{\frac{1}{2}}} \right\}_{k-\text{bits}} \tag{38}$$

where μ is k bit word-length. In general, the filter coefficients b_i must be modified to b_i/μ, then the word-length of the multiplication results of b_i and $1/\mu$ may be longer than that of original b_i. In this paper, we propose the new method to get the finite word-length coefficients under scaling constraint. First we design the finite word-length coefficients and the scaling coefficients (k-bits) by using the designed coefficients and (38). Next, we modify the desired frequency response and to search for the filter coefficients based on the lower bound estimation method. The algorithm is summarized as follows

STEP 1: Calculate $\boldsymbol{K}$ and $\boldsymbol{v}$ by using (9).

STEP 2: Design the finite wordlength coefficients based on the lower bound estimation method.

STEP 3: Calculate the scaling coefficient and round it to k-bits.

STEP 4: Modify the desired frequency response as $H'_d(\omega) = \frac{1}{\mu} H_d(\omega)$.

STEP 5: Recalculate the filter coefficient with desired response $H'_d(\omega)$.

4. ERROR FEEDBACK

As shown in Fig. 3, the product of B bits data and B bits coefficient becomes the $2B$ bits result and that must be rounded back to B bit. Then the product round-off noise is generated. With EF network [4] as shown in Fig.2, the product roundoff noise can be canceled. Note that the frequency responses of the filters are never changed by the EF. According to [4], the formulation of EF is summarized as follows.

The transfer function of the EF is given by

$$B(z) = \sum_{k=0}^{N} \beta_k z^{-k} \tag{39}$$

where $\beta_0 = 1$ and $N \leq n$. Then the transfer function from the error source to the filter output is

$$H_e(z) = B(z)G(z) \tag{40}$$

where $G(z) = 1/D(z)$ in Direct form I and $G(z) = N(z)/D(z)$ in Direct form II, respectively.

The gain of the roundoff noise can be expressed as

$$I = \frac{1}{2\pi j} \oint H_e(z) H_e(z^{-1}) z^{-1} dz \tag{41}$$

or

$$I = \frac{1}{\pi} \int_0^{\pi} |B(e^{j\omega})|^2 |G(e^{j\omega})|^2 d\omega. \tag{42}$$

Note that I is the normalized noise gain. The general noise gain is $\sigma_{out}^2 = I \times 2^{-2B}/12$ where B indicates the wordlength of the filter. For convenience, let us denote

$$Q(\omega) = |G(e^{j\omega})|^2 \tag{43}$$

and let the autocorrelation coefficients of the error signal be

$$\begin{aligned} q_k &= \frac{1}{\pi} \int_0^{\pi} \cos k\omega Q(\omega) d\omega \\ &= \sum_{\varsigma=0}^{\infty} g(\varsigma) g(k+\varsigma) \end{aligned} \tag{44}$$

where $g(\varsigma)$ is the impulse response corresponding to $G(z)$. Since q_k is symmetric ($q_{-k} = q_k$), (42) can be expressed as

$$I(\boldsymbol{w}) = \boldsymbol{w}^T \boldsymbol{R} \boldsymbol{w} \tag{45}$$

where

$$\boldsymbol{w} = (\beta_0 \ \beta_1 \ \cdots \ \beta_N)^T \tag{46}$$

$$\boldsymbol{R} = \begin{pmatrix} q_0 & q_1 & \cdots & q_N \\ q_1 & q_0 & \cdots & q_{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ q_N & q_{N-1} & \cdots & q_0 \end{pmatrix}. \tag{47}$$

As in the section 3.1, the vector minimizing (45) under constraint $\boldsymbol{w}^t \boldsymbol{S} = \boldsymbol{w}_1^t$ is obtained by [5]

$$\boldsymbol{w} = \boldsymbol{R}^{-1} \boldsymbol{S} (\boldsymbol{S}^T \boldsymbol{R}^{-1} \boldsymbol{S})^{-1} \boldsymbol{w}_1. \tag{48}$$

From (48), we can calculate the relaxation solution as well as (35). As the same procedure of the filter design, we can find the discrete EF coefficients by the lower bound estimation method.

5. EXAMPLES

We design the following low-pass filter and high-pass filters with the filter order $n = 12$.

Low-pass filter:

$$H_d(\omega) = \begin{cases} e^{-j12\omega} & \omega \leq 0.525\pi \\ 0 & \omega > 0.525\pi \end{cases} \tag{49}$$

High-pass filter:

$$H_d(\omega) = \begin{cases} e^{-j12\omega} & \omega \geq 0.525\pi \\ 0 & \omega < 0.525\pi \end{cases} \tag{50}$$

Let the word-length of the sign, integer part and decimal part of the coefficients be 1, 3 and 5 bits, respectively. We use the simple rounded coefficients as a first temporary solution.

Because the calculation cost is required to search all candidates of each filter coefficients, the search range is selected within ± 1 of each continuous coefficients.

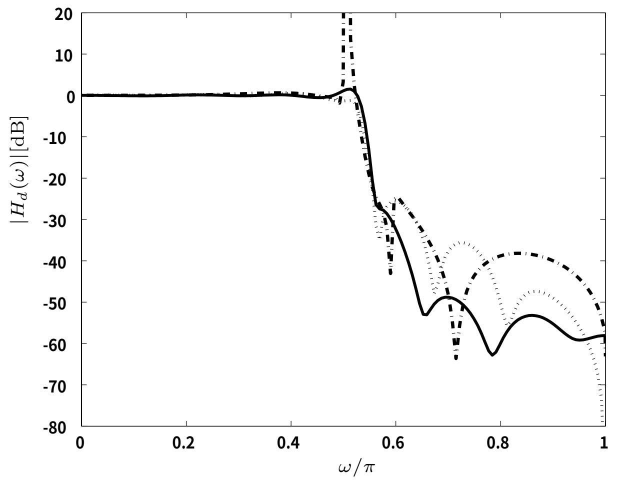

Figure 4: Low-pass filter.

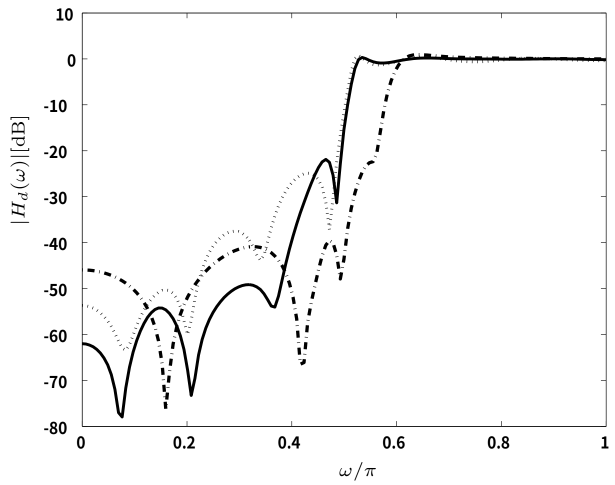

Figure 5: High-pass filter.

Figs. 4 and 5 show the results where the scaling coefficients are $s = 0.15625$ and $s = 0.12500$, respectively. The computation times are 9.09×10^4 [s] and 2.71×10^5 [s], respectively, when using a computer with 1.83 GHz CPU and 512 MB memory. In Figs. 4 and 5, the solid line indicates the frequency response by the proposed method, and the dotted and chained lines are the case of the simple rounded coefficients designed by the iteration of (25) under the constraint (26), and the results calculated by *'yulewalk'* function of MATLAB[1], respectively. Note that all poles of the filters designed by the proposed method are inside the unit circle, thus the designed filters are both stable. From Figs. 4 and 5, we can see the proposed method beats the other methods which have the simple rounded coefficients.

Next let us consider the product roundoff noise. The gains of the product roundoff noise without the EF are 30.3552 (Low-pass filter) and 22.2271 (High-pass filter), respectively. We add the 6th order EF network to the designed filters. With the EF, the noise gains are 0.5745 and 0.9949 where the word-length of the sign, integer part and decimal part of the EF coefficients are 1, 4 and 3 bits, respectively. Unlike the case of filter design, we search all candidates of each EF coefficients. It follows that the optimality of the discrete EF can be guaranteed.

6. CONCLUSION

In this work, we have proposed the design method for recursive digital filters with finite wordlength coefficients under scaling constraint to avoid the coefficients quantization noise. We search the good solution by using the branch and bound method based on the lower bound estimation principle. Additionally, with the lower bound estimation method, we can also design the EF with discrete coefficients for reduction of the product roundoff noise. It follows that we can reduce the product roundoff noise as well as the coefficients quantization noise without overflow. Finally, we have confirmed the effectiveness of the proposed method using the numerical examples.

[1]MATLAB® is registered trademark of The MathWorks, Inc.

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