

## MULTI-TIER COOPERATIVE BROADCASTING WITH HIERARCHICAL MODULATIONS

Tairan Wang<sup>1</sup>, Alfonso Cano<sup>2</sup>, Georgios B. Giannakis<sup>1</sup> (contact author) and F. Javier Ramos<sup>2</sup>

<sup>1</sup>Department of ECE, University of Minnesota  
Minneapolis, MN 55455 USA, email: {wang0822, georgios}@umn.edu

<sup>2</sup>Department of TSC, Rey Juan Carlos University  
Fuenlabrada, Madrid 28943 Spain, email: {alfonso.cano, javier.ramos}@urjc.es

### ABSTRACT

We consider broadcasting to multiple destinations with uneven quality receivers. Based on their quality of reception, we group destinations in tiers and transmit using hierarchical modulations. These modulations are known to offer a practical means of achieving variable error protection of the broadcasted information to receivers of variable quality. After the initial broadcasting step, tiers successively re-broadcast part of the information they received from tiers of higher-quality to tiers with lower reception capabilities. This multi-tier cooperative broadcasting strategy can accommodate variable rate and error performance for different tiers but requires complex demodulation steps. To cope with this complexity in demodulation, we derive simplified per-tier detection schemes with performance close to maximum-likelihood and ability to collect the diversity provided as symbols propagate through diversified channels across successive broadcastings. Error performance is analyzed and compared to (non)-cooperative broadcasting strategies. Simulations corroborate our theoretical findings.

### 1. INTRODUCTION

In classical broadcasting scenarios, information is broadcasted by a single source and decoded by different receivers independently. With the proliferation of wireless terminals in sparse broadcast networks, there has been a growing interest towards modalities where besides decoding their own information, certain receiving ends are willing to cooperate with other destinations. With these receivers acting as relays, well-appreciated benefits emerge in terms of resilience against shadowing, enhanced coverage, diversity and rates. For these reasons, cooperative communications have attracted research attention recently from several perspectives [4, 8, 10, 12, 13]. From an information-theoretic point of view, capacity improvements offered by user cooperation and relaying in the degraded broadcast channel (BC) motivate the idea of exploiting user cooperation in broadcasting scenarios [2, 5]. The advantages of cooperative broadcasting (CBC) from a practical perspective, have been also demonstrated in the context of network lifetime maximization and coverage improvements [7, 14].

Paired well with intended receivers of hierarchically unequal quality, hierarchical constellations can achieve variable error protection of the broadcasted information [9, 11]. These modulations map information bits according to their importance onto non-uniformly spaced constellation points. Non-uniformity allows important (a.k.a. basic) bits to be decoded with fewer errors than less important (a.k.a. enhance-

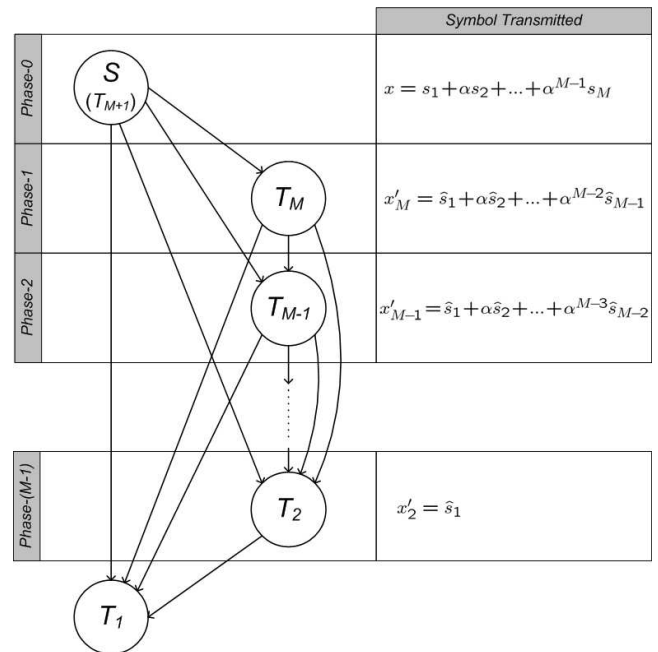


Figure 1: An  $M$ -tier CBC setup.

ment) bits. As such, hierarchical modulations have found applications in broadcasting networks with destinations having unequal quality. These include commercial for e.g., multimedia delivery as well as tactical broadcasting networks [1, 6].

Motivated by these considerations, the present paper deals with cooperative broadcasting based on hierarchical modulations. Consider a source broadcasting information bits mapped to a hierarchical modulation, and suppose that destinations are classified in *tiers* according to their specific reception conditions. One tier may include multiple nodes. Any tier closer (spatially or channel-wise) to the source is able to reliably detect most of the transmitted bits, re-encode part of the bits (the basic information) using a reduced size hierarchical constellation, and act as a relay broadcasting it to other tiers. A tier located farther (spatially or channel-wise) away from the source with poorer reception conditions can combine the symbol received from the original broadcasting with the symbols received from cooperating tiers. This strategy will be shown to offer a convenient means of increasing reliability of the basic information successively broadcasted through the network. We will have cooperating tiers implement modified versions of the decode-and-forward (DF) protocol [4, 12, 13, 16] adapted to multi-tier CBC, which we will abbreviate as DFb since re-encoded symbols are carry-

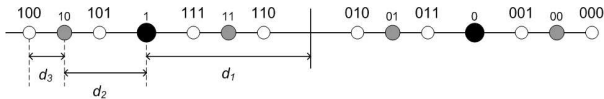


Figure 2: Hierarchical 2/4/8-PAM constellation.

ing less (only basic) information.

To match variable QoS requirements, we will also consider detectors with affordable complexity which efficiently combine heterogeneous constellations broadcasted from source and tiers. To this end, we advocate combiners with adaptive-weights that account for the unequal error probabilities of the received symbols; see also [16] where similar combiners are proposed but not for the CBC setup with hierarchical modulations. If properly designed, such detectors can collect diversity order up to the number of preceding tiers in the hierarchy (full diversity).

Notation: Lower case bold letters will denote column vectors;  $(\cdot)^*$  conjugation;  $(\cdot)^T$  transpose;  $(\cdot)^H$  Hermitian transpose;  $\mathcal{CN}(0, \sigma^2)$  the circular symmetric complex Gaussian distribution with zero mean and variance  $\sigma^2$ ;  $\text{Re}\{\cdot\}$  the real part of a complex number;  $\bar{\gamma} = E\{\gamma\}$  the mean of the random variable  $\gamma$ ;  $\hat{x}$  the estimate of  $x$ ; and  $\text{Diag}(\mathbf{d})$  will denote a diagonal matrix having the elements of vector  $\mathbf{d}$  on its diagonal.

## 2. HIERARCHICAL TRANSMISSIONS

With reference to Figure 1, we consider a source terminal  $S$ , and  $M$  tiers  $\{T_m\}_{m=1}^M$ . We map information-bearing bits at  $S$  to a hierarchical constellation. For simplicity, we will confine ourselves to real-valued hierarchical Pulse-Amplitude-Modulation (PAM) constellations; such constellations can also be seen as one-dimensional counterparts of complex-valued hierarchical Quadrature-Amplitude-Modulation (QAM) constellations. Let us consider a block of  $M$  bits  $i_1, \dots, i_M$ . Bit mappings in hierarchical constellations assign the highest priority bit ( $i_1$ ) to the most significant bit (MSB) position. The bit with second highest priority ( $i_2$ ) is assigned the second most significant position, and so on, until the least priority bit ( $i_M$ ) is assigned the least significant bit (LSB) position. Such construction can be viewed as a nested  $\{2/4/\dots/2^M\}$ -PAM constellation [15]. As an example, for  $M = 3$ , a nested  $\{2/4/8\}$ -PAM constellation is depicted in Figure 2.

A hierarchically modulated symbol  $x$  at  $S$ , can be generically written as

$$x = \pm d_1 \pm d_2 \pm \dots \pm d_M, \quad (1)$$

where  $\pm d_m$  is mapped from bit  $i_m$ , and represents the distance of the constellation point at this level of the hierarchy [c.f. Figure 2]. In order to reduce the amount of parameters that model a hierarchical constellation, we can define a parameter  $\alpha \in [0, 1/2]$  and constrain  $x$  to have the following structure:

$$x = s_1 + \alpha s_2 + \alpha^2 s_3 + \dots + \alpha^{M-1} s_M, \quad (2)$$

where  $s_m = \pm d_1$  is the symbol corresponding to bit  $i_m$  with weight  $\alpha^{m-1}$ .

### 2.1 Broadcasting phases

Similar to [10, 16], we adopt a transmission protocol based on successive broadcasting phases implemented in a time-

division multiplexing fashion. In *phase-0*,  $S$  transmits  $x$  (only once, same for other tiers) and the received symbol at any tier, e.g.,  $T_m$ , can be written as

$$y_{S,m} = h_{S,m}x + n_{S,m}, \quad (3)$$

where  $x$  is given by (2),  $h_{S,m}$  is the flat fading channel coefficient between  $S$  and  $T_m$ , modeled as  $\mathcal{CN}(0, E\{|h_{S,m}|^2\})$ ;  $n_{S,m}$  is the additive white Gaussian noise (AWGN) term, distributed according to  $\mathcal{CN}(0, N_0)$ , and  $N_0$  is the noise energy, which here is assumed to be the same for all  $m \in [1, M]$ .

Each node in the first tier,  $T_M$ , detects source bits  $i_1, \dots, i_M$  and constructs a new hierarchically modulated symbol  $x'_M$  given by [c.f. Figure 1]

$$x'_M = \hat{s}_1 + \alpha \hat{s}_2 + \dots + \alpha^{M-2} \hat{s}_{M-1}. \quad (4)$$

Notice that relative to  $x$  in (2),  $x'_M$  in (4) contains one bit less. In the ensuing *phase-1*,  $x'_M$  is broadcasted from  $T_M$  to the remaining tiers  $T_{M-1}, T_{M-2}, \dots, T_1$ .

Receivers belonging to the next tier,  $T_{M-1}$ , receive information pertaining to  $x$  twice; once from  $S$  during *phase-0*, in the form  $y_{S,M-1} = h_{S,M-1}x + n_{S,M-1}$ ; and second time from  $T_M$  during *phase-1*, in the form  $y_{M,M-1} = h_{M,M-1}x'_M + n_{M,M-1}$ . From  $y_{S,M-1}$  and  $y_{M,M-1}$ , any receiver in  $T_{M-1}$  decodes bits  $i_1, \dots, i_{M-1}$  and builds a new hierarchically modulated symbol,  $x'_{M-1} = \hat{s}_1 + \alpha \hat{s}_2 + \dots + \alpha^{M-3} \hat{s}_{M-2}$  which is broadcasted in *phase-2*.

We can clearly continue this successive broadcasting process to tier  $T_1$ . To this end, let us define  $T_{M+1} := S$  for uniformity in notation. Every node in tier  $T_m$ ,  $m \in [1, M]$  receives a set of  $M - m + 1$  symbols  $\{x'_n\}_{n=m+1}^{M+1}$  in the presence of AWGN, each corresponding to  $M - m + 1$  transmission phases from  $T_{M+1}, T_M, \dots, T_{m+1}$ . We can concatenate all these symbols received by any node belonging to  $T_m$  into an  $(M - m + 1) \times 1$  vector

$$\mathbf{x}_m = \begin{bmatrix} s_1 + \alpha s_2 + \dots + \alpha^{M-1} s_M \\ \hat{s}_1 + \alpha \hat{s}_2 + \dots + \alpha^{M-2} \hat{s}_{M-1} \\ \vdots \\ \hat{s}_1 + \alpha \hat{s}_2 + \dots + \alpha^{m-1} \hat{s}_m \end{bmatrix}. \quad (5)$$

The received symbols can be then correspondingly collected in an  $(M - m + 1) \times 1$  vector  $\mathbf{y}_m$ , which can be written as

$$\mathbf{y}_m = \text{Diag}(\mathbf{h}_m)\mathbf{x}_m + \mathbf{n}_m, \quad (6)$$

where  $\mathbf{h}_m := [h_{M+1,m}, h_{M,m}, \dots, h_{m+1,m}]^T$  and  $h_{n,m}$  is the fading coefficient between  $T_n$  and  $T_m$ ,  $n \in [M+1, m+1]$ , distributed according to  $\mathcal{CN}(0, E\{|h_{n,m}|^2\})$ ; and  $\mathbf{n}_m := [n_{M+1,m}, n_{M,m}, \dots, n_{m+1,m}]^T$  collects all AWGN terms at  $T_m$ , with each entry adhering to  $\mathcal{CN}(0, N_0)$ .

From  $\mathbf{y}_m$ , any receiver in  $T_m$  detects bits  $i_1, \dots, i_m$  and broadcasts a new constellation point

$$x'_m = \hat{s}_1 + \alpha \hat{s}_2 + \dots + \alpha^{m-2} \hat{s}_{m-1} \quad (7)$$

to  $T_{m-1}, T_{m-2}, \dots, T_1$  in *phase-(M - m + 1)*. With this simple broadcasting protocol, we guarantee that information is adaptively broadcasted according to the different detection requirements of each tier.

Vector  $\mathbf{y}_m$  in (6) contains  $M - m + 1$  versions of the broadcasted signal  $x$  which undergo uncorrelated fading realizations; thus, the maximum diversity that can be collected

by any node in tier  $T_m$  is of order  $M - m + 1$ . Challenged by this benchmark, we will next propose optimum and simplified cooperative detection strategies at  $T_m$  which are capable of collecting this order of diversity.

## 2.2 Cooperative Demodulation

For simplicity in exposition, we will henceforth consider only one node per tier. As recognized by [13, 16], when using non-hierarchical constellations, the maximum-likelihood (ML) detector using DF strategy can be quite complicated, and available expressions are tractable only for BPSK constellations. Moreover, our multi-tier cooperative scenario is further complicated by the fact that symbols arriving from different tiers are mapped to different constellations. Our idea is to simplify such a detector using properly weighted signal combiners. In this context, one may be tempted to rely on maximum-ratio-combining (MRC), which yields:

$$\hat{x}_m^{MRC} = \arg \min_{x_m \in \mathcal{A}_{x_m}} \|\mathbf{h}_m^H \mathbf{y}_m - \|\mathbf{h}_m\|^2 x_m\|^2. \quad (8)$$

Unfortunately, MRC is not generally equivalent to ML when at least one copy comes from a cooperating relay because regenerative relay strategies are prone to errors. In fact, MRC maximizes the output signal-to-noise ratio (SNR) of the  $T_n \rightarrow T_m$  links regardless of the errors that may occur when re-encoding symbols at  $T_n$ .

Instead of MRC, our approach will be to seek combiners that maximize the output SNR of the *equivalent* end-to-end path  $T_{M+1} \rightarrow \dots \rightarrow T_n \rightarrow T_m$  whose receive-SNR accounts for per-hop errors. Towards this objective, we propose the following general weighted combiner, which we name cooperative MRC (C-MRC). C-MRC detects bits recursively starting with  $i_1$ . For bit  $i_b$ , it takes the general form:

$$\hat{x}_m^{C-MRC}(i_b) = \arg \min_{x'_b \in \mathcal{A}'_{x'_b}} |(\mathbf{w}_m^b)^H \mathbf{y}_m - (\mathbf{w}_m^b)^H \mathbf{h}_m x'_b|^2, \quad (9)$$

for all  $b = 1, \dots, m$ , where  $x'_b = \hat{x}_m^{C-MRC}(i_{b-1}) + \alpha^{b-1} s_b$  and  $x'_1 = s_1$ . The search now is performed over the set  $\mathcal{A}'_{x'_b}$  with cardinality  $|\mathcal{A}'_{x'_b}| = 2$ . If we define the instantaneous receive-SNR of link  $T_n \rightarrow T_m$  as  $\gamma_{n,m} := |h_{n,m}^2 \mathcal{P}_x / N_0$ ,  $\forall n \in [M + 1, m + 1]$ , where  $\mathcal{P}_x$  denotes the transmit power of  $x$ , then vector  $\mathbf{w}_m^b$  is given by:

$$\mathbf{w}_m^b = [h_{M+1,m}, \frac{\gamma_{eqM,m}^b}{\gamma_{M,m}} h_{M,m}, \dots, \frac{\gamma_{eqm+1,m}^b}{\gamma_{m+1,m}} h_{m+1,m}]^T, \quad (10)$$

where  $\gamma_{eqn,m}^b$  is what we term *equivalent SNR* and can be calculated as follows. Define  $P_{n,m}(i_b)$  to be the bit-error-probability (BEP) for transmitting  $i_b$  from  $T_n$  to  $T_m$ , and  $P_n(i_b)$  to be the BEP of bit  $i_b$  at  $T_n$ . When  $P_n(i_b)$  is known at  $T_m$ , one can calculate the overall BEP of bits  $i_b$  at  $T_m$  sent from  $T_n$ , as [3]

$$P_m(i_b, T_n) = [1 - P_n(i_b)]P_{n,m}(i_b) + [1 - P_{n,m}(i_b)]P_n(i_b). \quad (11)$$

Now,  $\gamma_{eqn,m}^b$  can be understood as representing the SNR of a virtual BPSK-based equivalent link for bit  $i_b$ , which can be calculated by inverting the function

$$P_m(i_b, T_n) = Q\left[\sqrt{2\gamma_{eqn,m}^b}\right]. \quad (12)$$

The right hand side of (12) is just the standard BEP of a BPSK transmission through a channel with instantaneous SNR  $\gamma_{eqn,m}^b$ . Returning to (10), one can now recognize that vector  $\mathbf{w}_m^b$  weighs each entry of  $\mathbf{y}_m$  in order to maximize  $\gamma_{eqn,m}^b$  instead of  $\gamma_{n,m}$  as in (8).

The decoder in (9) is very simple and has general applicability regardless of the underlying constellation. Nevertheless, the calculation of  $P_n(i_b)$ , as in the ML case, becomes complicated when there are multiple heterogeneous constellations arriving at  $T_n$ . To handle this, we look for a simple means of calculating (10) by propagating SNRs in an  $M$ -tier CBC network. As we treated  $\gamma_{eqn,m}^b$  to be an equivalent SNR, we can further approximate it to be independent of  $b$ . This is established in the following lemma:<sup>1</sup>

**Lemma 1** *At high SNR,  $\gamma_{eqn,m}$  can be approximated by*

$$\gamma_{eqn,m} \approx \min \left\{ \gamma_{eqn} \frac{1 + \alpha^2 + \dots + \alpha^{2(n-2)}}{1 + \alpha^2 + \dots + \alpha^{2(n-1)}}, \gamma_{n,m} \right\}, \quad (13)$$

where  $m \in [1, M - 1]$ ,  $n \in [m + 1, M]$ , and

$$\gamma_{eqn} := \sum_{l=n+1}^M \gamma_{eql,n} + \gamma_{M+1,n}. \quad (14)$$

Based on (13) and (14), we can iteratively update  $\gamma_{eqn,m}$ ,  $m \in [1, M - 1]$ , starting with  $\gamma_{eqM} := \gamma_{M+1,M}$  at tier  $M$ , without being necessary to calculate any BEP.

## 3. PERFORMANCE ANALYSIS

We will first derive BEP metrics for  $M = 2$  with 2/4-PAM constellations; pertinent performance for higher  $M$ s will be commented later on. The received signals at  $T_2$  and  $T_1$  from  $T_3$  are denoted as

$$y_{3,1} = h_{3,1}(s_1 + \alpha s_2) + n_{3,1}, \quad (15)$$

$$y_{3,2} = h_{3,2}(s_1 + \alpha s_2) + n_{3,2}. \quad (16)$$

Using the results in [15] and after straightforward changes of variables, we find that the average BEP per fading realization for  $i_1$  is given by

$$P_2(i_1) = \frac{1}{2} \left\{ Q \left[ \frac{(1 + \alpha)\sqrt{2\gamma_{3,2}}}{\sqrt{1 + \alpha^2}} \right] + Q \left[ \frac{(1 - \alpha)\sqrt{2\gamma_{3,2}}}{\sqrt{1 + \alpha^2}} \right] \right\},$$

where  $Q[x] := (1/\sqrt{2\pi}) \int_x^\infty \exp(-t^2/2) dt$ .

As usual, we define the diversity gain (diversity order)  $G_d$ , as the negative exponent in the average BEP when the average SNR tends to infinity, that is  $P^b \approx (G_c \bar{\gamma})^{-G_d}$ , when  $\bar{\gamma} \rightarrow \infty$ , where  $G_c$  denotes the coding gain.

### 3.1 Performance of DFb

Recall that  $T_1$  cares only about bit  $i_1$ ; so  $T_2$  only forwards one BPSK symbol  $x' = \hat{s}_1$ . At  $T_1$ , the entries of the vector  $\mathbf{y}_1 := [y_{3,1}, y_{2,1}]^T$  are:

$$y_{3,1} = h_{3,1}(s_1 + \alpha s_2) + n_{3,1}, \quad (17)$$

$$y_{2,1} = h_{2,1}\hat{s}_1 + n_{2,1}. \quad (18)$$

<sup>1</sup>Omitted due to space limitations, proofs for all the lemmas and propositions in this paper can be found in [17].

Any receiver at tier  $T_1$  combines two received signals in the same constellation with weights  $w_{2,1}$  and  $w_{3,1}$  to obtain

$$y_1 = w_{3,1}y_{3,1} + w_{2,1}y_{2,1}$$

$$= \begin{cases} (w_{3,1}h_{3,1} + w_{2,1}h_{2,1})s_1 + w_{3,1}h_{3,1}\alpha s_2 + n_1, & \hat{s}_1 = s_1, \\ (w_{3,1}h_{3,1} - w_{2,1}h_{2,1})s_1 + w_{3,1}h_{3,1}\alpha s_2 + n_1, & \hat{s}_1 = -s_1, \end{cases}$$

where  $n_1 := w_{2,1}n_{2,1} + w_{3,1}n_{3,1}$ . Because 2/4-PAM is one-dimensional and the complex Gaussian distribution is circularly symmetric, we can take the real part  $y = \text{Re}\{y_1\}$  before detection, which is a real Gaussian random variable with zero mean and variance  $N_0/2$ .

Following the choice in (10), we have  $w_{3,2} = h_{3,2}^*$  and  $w_{2,1} = \frac{\gamma_{eq2,1}}{\gamma_{2,1}}h_{2,1}^*$ . Substituting these two weight coefficients and defining  $\gamma_{eq} := \gamma_{eq2,1}$ , we obtain

$$P_1^{DFb}(i_1|\gamma_{3,2}, \gamma_{3,1}, \gamma_{2,1})$$

$$= \frac{1}{2}[1 - P_2(i_1|\gamma_{3,2})]Q \left[ \frac{\gamma_{3,1}(1-\alpha) + \gamma_{eq}}{\sqrt{\gamma_{3,1} + \gamma_{eq}^2/\gamma_{2,1}}} \sqrt{\frac{2}{1+\alpha^2}} \right]$$

$$+ \frac{1}{2}[1 - P_2(i_1|\gamma_{3,2})]Q \left[ \frac{\gamma_{3,1}(1+\alpha) + \gamma_{eq}}{\sqrt{\gamma_{3,1} + \gamma_{eq}^2/\gamma_{2,1}}} \sqrt{\frac{2}{1+\alpha^2}} \right]$$

$$+ \frac{1}{2}P_2(i_1|\gamma_{3,2})Q \left[ \frac{\gamma_{3,1}(1-\alpha) - \gamma_{eq}}{\sqrt{\gamma_{3,1} + \gamma_{eq}^2/\gamma_{2,1}}} \sqrt{\frac{2}{1+\alpha^2}} \right]$$

$$+ \frac{1}{2}P_2(i_1|\gamma_{3,2})Q \left[ \frac{\gamma_{3,1}(1+\alpha) - \gamma_{eq}}{\sqrt{\gamma_{3,1} + \gamma_{eq}^2/\gamma_{2,1}}} \sqrt{\frac{2}{1+\alpha^2}} \right]. \quad (19)$$

The BEP for  $i_1$  at  $T_1$  using DFb is given by  $P_1^{DFb}(i_1) = E\{P_1^{DFb}(i_1|\gamma_{3,2}, \gamma_{3,1}, \gamma_{2,1})\}$ , and the next proposition provides an upper bound to its error performance.

**Proposition 1** For multi-tier DFb relaying, where  $T_2$  only forwards the basic information to  $T_1$ , full diversity can be achieved for the basic information using C-MRC with 2/4-PAM constellation; i.e.,  $P_1^{DFb}(i_1) \leq \tilde{P}_1^{DFb}(i_1) \stackrel{\bar{\gamma} \rightarrow \infty}{\approx} (k_2\bar{\gamma})^{-2}$ , with  $k_2$  denoting a constant.

When  $T_2$  forwards the entire symbol estimate  $\hat{x} = \hat{s}_1 + \alpha\hat{s}_2$  to  $T_1$ , DFb degrades to the conventional DF. While DFb prevents  $T_2$  from sending the enhancement information to  $T_1$ , it saves power that is used to transmit the basic information bits. For this reason, our simulations will also confirm that DFb exhibits better error performance than DF at  $T_1$ .

Further capitalizing on the results of [16], one can extend the analysis in Proposition 1 to any  $M$ -tier network, that full diversity  $M - m + 1$  can be achieved at  $T_m, \forall m \in [1, M]$ .

#### 4. SIMULATIONS AND NUMERICAL RESULTS

In this section, we compare various schemes on the basis of BEP using numerical results and Monte-Carlo simulations. We assume that the path-loss exponent in all links involved is 3, so the average output SNR of the link  $T_n \rightarrow T_m$  with respect to the link,  $T_{M+1} \rightarrow T_m$ , is

$$\bar{\gamma}_{n,m} = \bar{\gamma}_{M+1,m} \left( \frac{d_{M+1,m}}{d_{n,m}} \right)^3, \quad (20)$$

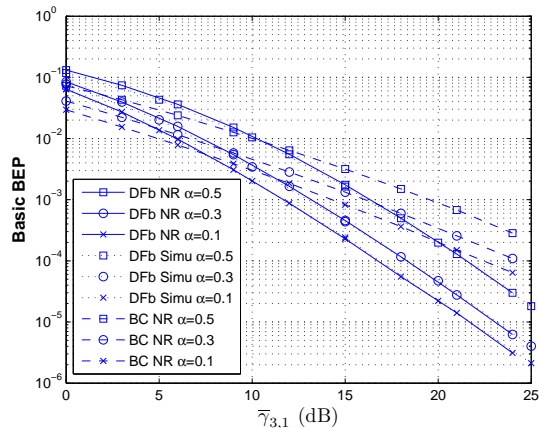


Figure 3: BEP comparison for BC using 2/4-PAM vs. 2-tier CBC using 4/16-QAM with DFb.

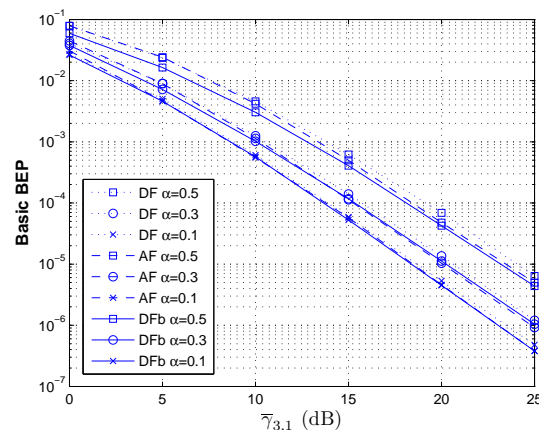


Figure 4: BEP comparison for 2-tier CBC using 2/4-PAM with DFb vs. DF vs. AF.

where  $d_{M+1,m}$  and  $d_{n,m}$  denote the corresponding distances. For a two-tier model where  $T_2$  is equally spaced from  $T_3$  and  $T_1$ , the average SNR setting in (20) becomes  $\bar{\gamma}_{3,2} = \bar{\gamma}_{2,1} = \bar{\gamma}_{3,1} \times 2^3$ . Transmit power will be set to be the same across all terminals.

**Test Case 1 (2-tier CBC):** In Figures 3-4, we compare the probabilities of error at  $T_1$  of different (re-) transmission strategies. Since  $T_1$  only needs the basic information in our multi-tier setting, all error probabilities are henceforth with respect to the MSB  $i_1$ .

In Figure 3, we compare the BEP of such a *conventional* broadcasting (BC) scenario using 2/4-PAM with a 2-tier cooperative broadcasting (CBC) using DFb for successive broadcastings, by both simulations (Simu) and numerical results (NRs) and for different values of  $\alpha$ . To compensate for the 1/2-rate loss due to the time-phases of CBC, we use 4/16-QAM for CBC transmissions; thus all strategies have identical bit rate  $R = 1$  bit per symbol per channel use. We can see that the simulations accurately match numerical results, which corroborates the accuracy of our performance analysis. For any  $\alpha$ , because of its higher diversity, CBC outperforms BC at sufficiently high SNR values. As  $\alpha$  becomes smaller, both BC and CBC improve their performance as expected.

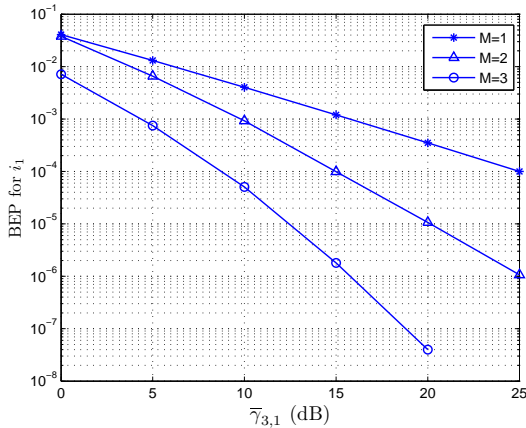


Figure 5: BEP comparison for bit  $i_1$  at  $T_1$  in  $M$ -tier CBC using 2/4-PAM and DFb with  $\alpha = 0.3$ .

In Figure 4, we plot the simulated BEP of different relaying protocols using 2/4-PAM. All three protocols achieve the full diversity gain, which is 2 here. When  $\alpha = 0.1$ , the three protocols perform almost identically. As  $\alpha$  becomes large, DFb outperforms DF and (amplify-and-forward) AF [4, 10], since DFb avoids sending the enhancement information bit. This confirms our thesis that the proposed DFb considerably enhances the advantages of conventional hierarchical bit-mappings offering better protection to MSBs in CBC scenarios.

**Test Case 2 ( $M$ -tier CBC):** Here we validate our full diversity claims for DFb in an  $M$ -tier CBC with constellation parameter  $\alpha = 0.3$ . Figure 5 shows that the BEP slope varies according to the number of tiers, which corroborates that diversity  $M$  is achieved at  $T_1$  for any  $M$ -tier CBC network, as our analysis asserted in Section 3. Moreover, this figure confirms that successive broadcasting strategies bring major performance improvements quantified by both coding and diversity gains for terminals at the edge of a sparse network.

## 5. CONCLUSIONS

This work advocated wedding hierarchical modulations with cooperative broadcasting in multi-tier networks. In the proposed cooperative scenario, we order terminals in tiers according to their reception conditions. Thus, different tiers collect information and successively broadcast part of the information to other terminals differently. Specifically, some tiers collect all broadcasted information (basic and enhancement) and broadcast only the basic information to tiers with worse reception conditions. This offers an adaptive DF protocol, DFb, where successive broadcastings aim to further protect the basic information. We incorporated simple weighted combiners for demodulation to adaptively account for the heterogeneous signals involved in each phase. The simplicity of these demodulators irrespective of the underlying constellation allowed us to assess performance based on the diversity order which has not been quantified even for the ML detectors in cooperative broadcasting.

## Acknowledgments

This work was supported through collaborative participation in the Communications and Networks Consortium sponsored by the U.S. Army Research

Laboratory under the Collaborative Technology Alliance Program, Cooperative Agreement DAAD19-01-2-0011. The U.S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation thereon. The work of the second and fourth authors was supported by the Spanish Government TEC2005-06766-C03-01/TCM.

## REFERENCES

- [1] "DVB-T standard: ETS 300 744, digital broadcasting systems for television, sound and data services; framing structure, channel coding and modulation for digital terrestrial television," *ETSI Draft*, vol. 1.2.1, no. EN300 744, 1999-1.
- [2] R. Dabora and S. D. Servetto, "Broadcast channels with cooperating decoders," *IEEE Trans. on Inform. Theory*, submitted May 2005; downloadable from <http://cn.ece.cornell.edu/publications/papers/20050215a/pp.pdf>.
- [3] M. O. Hasna and M.-S. Alouini, "End-to-end performance of transmission systems with relays over Rayleigh-fading channels," *IEEE Trans. on Wireless Commun.*, vol. 2, no. 6, pp. 1126–1131, Nov. 2003.
- [4] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. on Inform. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.
- [5] Y. Liang and V. V. Veeravalli, "The impact of relaying on the capacity of broadcast channels," in *Proc. of IEEE Int. Symp. on Inform. Theory*, 27 Jun.-2 Jul. 2004, pp. 403–403.
- [6] C. Louis and P. Schier, "Automatic modulation recognition with a hierarchical neural network," in *Military Communications Conference*, vol. 3, Fort Monmouth, NJ, Oct. 2-5 1994, pp. 713–717.
- [7] I. Maric and R. Yates, "Cooperative multihop broadcast for wireless networks," *IEEE J. on Select. Areas in Commun.*, vol. 22, no. 6, pp. 1080–1088, Aug. 2004.
- [8] R. Pabst, B. Walke, D. Schultz, P. Herhold, H. Yanikomeroglu, S. Mukherjee, H. Viswanathan, M. Lott, W. Zirwas, M. Dohler, H. Aghvami, D. Falconer, and G. Fettweis, "Relay-based deployment concepts for wireless and mobile broadband radio," *IEEE Commun. Magazine*, pp. 80–89, Sep. 2004.
- [9] M. B. Pursley and J. M. Shea, "Nonuniform phase-shift-key modulation for multimedia multicast transmission in mobile wireless networks," *IEEE J. on Select. Areas in Commun.*, vol. 17, no. 5, pp. 774–783, May 1999.
- [10] A. Ribeiro, X. Cai, and G. B. Giannakis, "Symbol error probabilities for general cooperative links," *IEEE Trans. on Wireless Commun.*, vol. 4, no. 3, pp. 1264–1273, May 2005.
- [11] A. Seeger, "Broadcast communication on fading channels using hierarchical coded modulation," in *Proc. of Global Telecommunications Conference*, vol. 1, San Francisco, CA, Nov. 2000, pp. 92–97.
- [12] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity - part I: system description," *IEEE Trans. on Commun.*, vol. 51, no. 11, pp. 1927–1938, Nov. 2003.
- [13] —, "User cooperation diversity - part II: implementation aspects and performance analysis," *IEEE Trans. on Commun.*, vol. 51, no. 11, pp. 1939–1948, Nov. 2003.
- [14] B. Sirkeci-Mergen and A. Scaglione, "Coverage analysis of cooperative broadcast in wireless networks," in *Proc. of 5th Workshop on Signal Processing Advances in Wireless Commun.*, Lisbon, Portugal, Jul. 11-14 2004, pp. 16–20.
- [15] P. K. Vitthaladevuni and M.-S. Alouini, "A recursive algorithm for the exact BER computation of generalized hierarchical QAM constellations," *IEEE Trans. on Inform. Theory*, vol. 49, no. 1, pp. 297–307, Jan. 2003.
- [16] T. Wang, A. Cano, and G. B. Giannakis, "Efficient demodulation in cooperative schemes using decode-and-forward relays," in *Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, Oct. 30–Nov. 2 2005.
- [17] T. Wang, A. Cano, G. B. Giannakis, and F. J. Ramos, "Multi-tier cooperative broadcasting with hierarchical modulations," *IEEE Trans. on Wireless Commun.*, submitted Dec. 2005; downloadable from <http://www.ece.umn.edu/users/twang/MTCBC.twacggjr.submitted.pdf>.