RECEIVER WINDOW DESIGNS FOR RADIO FREQUENCY INTERFERENCE SUPPRESSION

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ABSTRACT

The DMT (discrete multitone) based VDSL (very high speed digital subscriber line)system is susceptible to interference due to radio frequency transmission. It is known that windowing at the receiver can reduce radio frequency interference (RFI). In this paper, we formulate the interference of individual tones and minimize the total interference. The optimal window can be obtained in a closed form. The proposed windows have faster roll-off in low frequency. As a result, fewer tones will be dominated by RFI. Simulations will be given to demonstrate the usefulness of the proposed design. We also see that not knowing the statistics of the interference source leads to only a minor degradation. Therefore, we can obtain very good suppression effect with interference-independent windows, which has the advantage that the window need not be redesigned when the interference changes.

1. INTRODUCTION

The very high speed digital subscriber line (VDSL) transmission system shares its spectrum with different types of radio transmission, for example, amplitude-modulation stations and amateur radio. These radio signals can be coupled into telephone wires and interfere with the VDSL signal at the receiving side. These types of noise in a VDSL transmission system is known as RFI ingress [1]. The magnitude of RFI ingress depends on the cable structure, e.g. shielding, twist, and physical orientation of the cable. The tones corresponding to the RFI frequency bands can be turned off to reduce interference to radio transmission (ingress) [2][3]. These tones usually have too large interference to carry any bits. One approach of reducing RFI effect is by applying window at the receiver. One method that minimizes the error of the windowed signal at the DFT output is given in [4]. A frequency-domain or time-domain windowing followed by decision feedback equalizer [5], and a combination of raised-cosine window and per tone equalizer (PTEQ) are proposed to suppress RFI interference [6]. However, the channel information is required in these designs. Another work is to design the channel-independent window by minimizing the sidelobe energy [7]. This work would introduce ISI (inter symbol interference) and post processing is required to cancel ISI.

In this paper, we formulate the interference of individual tones at the receiver outputs. The total interference of the tones used for transmission is given as a function of the window coefficients. The problem can be solved by differentiating the interference function with respect to window coefficients. When compared with well known window (e.g. Hanning or Blackman), our proposed windows have faster roll-off in low frequency. As a result, fewer tones will be dominated by RFI. We will consider both the case when the statistics of the interference is available to the receiver and the case when it is not. In the simulation, we will see that not knowing the statistics leads to only a minor performance degradation. Interference-independent windows have the advantage that the windows need not be updated when the statistics of interference changes.

The paper is organized as follows. In section 2, we will find the equivalent filter bank (FB) representation for the convenience of analysis. In section 3, we will design the windows for both the case when the statistics of the RFI interference is available at the receiver and the case when the statistics of RFI is not available. In section 4, we will compare the performance of the receiving windows by simulations.

2. FILTERBANK REPRESENTATION

In this section, we derive the filterbank representation of the receiver with windowing. The representation will be useful in formulation th interference of individual tones. Fig. 1 shows a typical DMT receiver. After the removal of cyclic extension, M-pt DFT is performed. Assume the DMT system has block size M with cyclic prefix (CP) length P. The transmitted block size is $N = P + M$. Suppose the channel order is l_c with $l_c < P$ and we have $L = P - l_c$ samples of cyclic prefix not affected by the channel. Therefore

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there are $M + L$ samples free from inter-block interference for each block. To apply windows, the receiver takes the $M + L$ samples, multiplies the first L samples by window coefficients $w(n)$, $n = 0, 1, \cdots, L - 1$ and multiplies the last L samples by $1 - w(n)$. Assume $\mathbf{w} = [w_0 \cdots w_{L-1}]^T$ is the $L \times 1$ window coefficient vector. Fig. 2 shows the equivalent matrix representation of the DMT receiver with windowing. The matrix $\mathbf{B}$ in Fig. 2 is given by

$$\mathbf{B} = \begin{pmatrix} 0 & 0 & I_{M-L} & 0 \\ 0_{L\times(P-L)} & \mathbf{C} & 0 & \mathbf{D} \end{pmatrix}, \qquad (1)$$

where

$$\mathbf{C} = \begin{pmatrix} w_0 & 0 & \cdots & 0 \\ 0 & w_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & & w_{L-1} \end{pmatrix} \qquad (2)$$

$$\mathbf{D} = \mathbf{I}_L - \mathbf{C} \qquad (3)$$

The structure in Fig. 2 has an equivalent filter bank representation [8] as shown in Fig. 3. The M receiving filters $H_i(z)$ for $i = 0, 1, \cdots, M - 1$ are related to $\mathbf{B}$ and $\mathbf{W}$ by

$$\begin{pmatrix} H_0(z) \\ H_1(z) \\ \vdots \\ H_{M-1}(z) \end{pmatrix} = \mathbf{WB} \begin{pmatrix} 1 \\ z \\ \vdots \\ z^{N-1} \end{pmatrix}. \qquad (4)$$

Using the expression of $\mathbf{B}$ in (1), we can verify that the coefficients of the first receiving filter$h_0(n)$ are given by

$$h_0(n) = \begin{cases} w_{-n-P+L}, & -(P-1) \le n \le -(P-L) \\ 1, & -(N-L-1) \le n \le -P \\ 1 - w_{-n-N+L}, & -(N-1) \le n \le -(N-L) \\ 0, & \text{otherwise.} \end{cases} \qquad (5)$$

We can further verify that $H_i(z)$ is related to $H_0(z)$ by

$$H_i(z) = W^{-iP} H_0(zW^i), \qquad (6)$$

where $W = e^{-j\frac{2\pi}{M}}$. The i-th receiving filter is essentially a shifted version of $H_0(z)$.

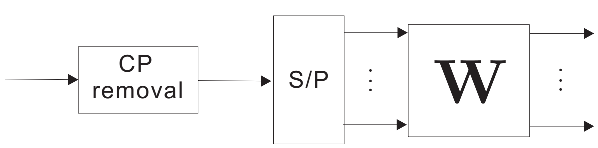

Figure 1: Traditional DMT receiver.

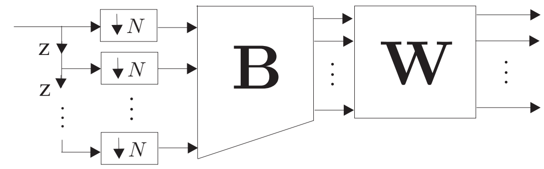

Figure 2: Matrix representation of a DMT receiver with windowing.

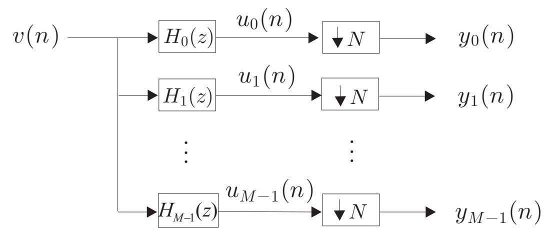

Figure 3: Filter bank representation of a DMT receiver with windowing.

3. OPTIMUM WINDOW DESIGN

In this section we formulate the interference of individual tones. We show that the optimal window coefficients can be obtained by using first order necessary condition [9]. We will consider two cases. In the first case, the statistics of RFI is assumed to be available to the receiver. In the second case, we assume the statistics of RFI frequency is not known to the receiver.

Informed receivers:

The RFI is known to be a narrow band noise. For the duration of one VDSL symbol, it can be considered as sinusoids. We assume that RFI interference occurs at frequency ω_l with amplitude α_l and phase θ_l, $l = 0, \cdots, R - 1$. Thus we can model the interference as

$$v(n) = \sum_{l=0}^{R-1} \alpha_l \cos(\omega_l n + \theta_l). \qquad (7)$$

To analyze the effect of interference, we apply the interference-only signal $v(n)$ to the receiver (Fig. 3). The output of the i-th receiving filter is

$$u_i(n) = \frac{1}{2} \sum_{l=0}^{R-1} \alpha_l [c_{l,i} e^{j(\omega_l n + \theta_l)} + c'_{l,i} e^{-j(\omega_l n + \theta_l)}], \qquad (8)$$

where $c_{l,i} = H_i(e^{j\omega_l})$ and $c'_{l,i} = H_i(e^{-j\omega_l})$. The interference at the i-th receiver output is $y_i(n) = u_i(Nn)$, which has the same amplitude as $u_i(n)$. To suppress the total interference, we can minimize

$$J = \sum_{l=0}^{R-1} \sum_{i=0, i \in U}^{M-1} \alpha_l^2 [|c_{l,i}|^2 + |c_{l,i}^{'}|^2] \tag{9}$$

where U is the set of tones that are used for transmission.

$$\begin{aligned} c_{l,i} &= W^{-iP} H_0(e^{j(\omega_l - 2\pi i/M)}) \\ c_{l,i}^{'} &= W^{-iP} H_0(e^{-j(\omega_l - 2\pi i/M)}) \end{aligned} \tag{10}$$

Window design for informed receivers:
With the above expressions, the objection function in (9) can be written as

$$\begin{aligned} J = \sum_{l=0}^{R-1} \sum_{i=0, i \in U}^{M-1} \alpha_l^2 \quad & [|H_0(e^{j(\omega_l - 2\pi i/M)})|^2 + \\ & |H_0(e^{-j(\omega_l - 2\pi i/M)})|^2] \end{aligned} \tag{11}$$

From (5) we can verify that $H_0(e^{j(\omega_l - 2\pi i/M)})$ can be given in terms of the window coefficients as

$$H_0(e^{j(\omega_l - 2\pi i/M)}) = b_{l,i} + \mathbf{a}_{l,i}^{\dagger} \mathbf{w}, \tag{12}$$

where the notation '†' denotes Hermitian, $b_{l,i}$ is a scalar and $\mathbf{a}_{l,i}$ is a $L \times 1$ column vectors given respectively by

$$\begin{aligned} b_{l,i} = & \sum_{l=P}^{P+M-1} e^{j(\omega_l - 2\pi i/M)l}, \\ [\mathbf{a}_{l,i}]_m = & \quad e^{j(\omega_l - 2\pi i/M)(P-L+m)} \\ & -e^{j(\omega_l - 2\pi i/M)(N-L+m)}. \end{aligned} \tag{13}$$

Similarly, we can verify that $H_0(e^{-j(\omega_l + 2\pi i/M)})$ can be expressed by

$$H_0(e^{-j(\omega_l + 2\pi i/M)}) = b_{l,i}' + \mathbf{a'}_{l,i}^{\dagger} \mathbf{w}, \tag{14}$$

where $b_{l,i}'$ and $\mathbf{a'}_{l,i}$ are respectively

$$\begin{aligned} b_{l,i}' = & \sum_{l=P}^{P+M-1} e^{-j(\omega_l + 2\pi i/M)l}, \\ [\mathbf{a'}_{l,i}]_m = & \quad e^{-j(\omega_l + 2\pi i/M)(P-L+m)} \\ & -e^{-j(\omega_l + 2\pi i/M)(N-L+m)}. \end{aligned} \tag{15}$$

Using (11) to (15), we can derive,

$$\begin{aligned} J = \quad & \mathbf{w}^T(\mathbf{A}^\dagger \mathbf{A} + \mathbf{A'}^\dagger \mathbf{A'})\mathbf{w} + \mathbf{w}^T(\mathbf{A}^\dagger \mathbf{b} + \mathbf{A'}^\dagger \mathbf{b'}) \\ & +(\mathbf{b}^\dagger \mathbf{A} + \mathbf{b'}^\dagger \mathbf{A'})\mathbf{w} + ||\mathbf{b}||^2 + ||\mathbf{b'}||^2. \end{aligned} \tag{16}$$

The matrices $\mathbf{A}$ and $\mathbf{A'}$ are of dimensions $RM \times L$, and $\mathbf{b}$ and $\mathbf{b}^{'}$ are $RM \times 1$ column vectors given by

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}_0 \\ \vdots \\ \mathbf{A}_{R-1} \end{pmatrix}, \mathbf{A'} = \begin{pmatrix} \mathbf{A}_0' \\ \vdots \\ \mathbf{A}_{R-1}' \end{pmatrix}, \tag{17}$$

$$\mathbf{b} = \begin{pmatrix} \mathbf{b}_0 \\ \vdots \\ \mathbf{b}_{R-1} \end{pmatrix}, \mathbf{b'} = \begin{pmatrix} \mathbf{b}_0' \\ \vdots \\ \mathbf{b}_{R-1}' \end{pmatrix}, \tag{18}$$

where $\mathbf{A}_l$ and $\mathbf{A}_\mathbf{l}'$ are $M \times L$ matrices, and $\mathbf{b}_l$ and $\mathbf{b}_\mathbf{l}'$ are $M \times 1$ column vectors given by

$$\mathbf{A}_l = \alpha_l \begin{pmatrix} \mathbf{a}_{l,0} \\ \vdots \\ \mathbf{a}_{l,M-1} \end{pmatrix}, \mathbf{A}_l' = \alpha_l \begin{pmatrix} \mathbf{a}_{l,0}' \\ \vdots \\ \mathbf{a}_{l,M-1}' \end{pmatrix}, \tag{19}$$

$$\mathbf{b}_l = \alpha_l \begin{pmatrix} b_{l,0} \\ \vdots \\ b_{l,M-1} \end{pmatrix}, \mathbf{b}_l' = \alpha_l \begin{pmatrix} b_{l,0}' \\ \vdots \\ b_{l,M-1}' \end{pmatrix}. \tag{20}$$

From (16), we can obtain the solution of the optimal window $\mathbf{w}$ that minimizes the total interference by differentiating J with respect to $\mathbf{w}$. Using first order necessary condition [9], we have

$$\mathbf{w} = -[Re(\mathbf{A}^\dagger \mathbf{A} + \mathbf{A}^{'\dagger} \mathbf{A}^{'})]^{-T} [Re(\mathbf{b}^\dagger \mathbf{A} + \mathbf{b}^{'\dagger} \mathbf{A}^{'})]^T. \tag{21}$$

where notation $Re(X)$ denote the real part of X.

Uninformed receivers:
We now consider the case when the statistics of RFI interference are not available to the receiver. In this case, the frequency and amplitude of RFI are not known. We can minimize the total interference by minimizing the following objective function

$$J_1 = \int_{\omega_s}^{\pi} |H_0(e^{j\omega})|^2 d\omega, \tag{22}$$

where ω_s is the stopband bandedge. The objective function J_1 is the stopband energy of $H_0(e^{j\omega})$.

Window design for uninformed receivers:
From (5) we can write $H_0(e^{j\omega})$ as

$$H_0(e^{j\omega}) = \mathbf{s}^\dagger \mathbf{g}, \tag{23}$$

where $\mathbf{s}$ and $\mathbf{g}$ are $(M+L) \times 1$ column vectors given by

$$\mathbf{s} = \begin{pmatrix} e^{-j\omega(P-L)} \\ e^{-j\omega(P-L+1)} \\ \cdots \\ e^{-j\omega(P+M+1)} \end{pmatrix}, \mathbf{g} = \begin{pmatrix} \mathbf{w} \\ \mathbf{1}_{M-L} \\ \mathbf{1}_{L} - \mathbf{w} \end{pmatrix}. \tag{24}$$

The notation 1_n denotes an $n \times 1$ column vector whose elements are equal to one. Then the stopband energy J_1 can be rewritten as

$$J_1 = \int_{\omega_s}^{\pi} (\mathbf{g}^\dagger \mathbf{s}\mathbf{s}^\dagger \mathbf{g}) d\omega = \mathbf{g}^\dagger \mathbf{Q} \mathbf{g}, \tag{25}$$

where

$$\mathbf{Q} = \int_{\omega_s}^{\pi} \mathbf{s}\mathbf{s}^{\dagger} d\omega. \qquad (26)$$

The elements of Q are given by

$$[\mathbf{Q}]_{mn} = \begin{cases} \frac{-sin(m-n)\omega_s}{(m-n),} & m \neq n, \\ 1 - \omega_s, & m = n. \end{cases} \qquad (27)$$

On the other hand, we can write $\mathbf{g}$ as

$$\mathbf{g} = \mathbf{d} + \mathbf{E}\mathbf{w} \qquad (28)$$

where $\mathbf{d}^T = [\ \mathbf{0} \quad \mathbf{1}_M^T\ ]$, and $\mathbf{E}^T = [\ \mathbf{I}_L \quad \mathbf{0} \quad -\mathbf{I}_L\ ]$. Using $\mathbf{g} = \mathbf{d} + \mathbf{E}\mathbf{w}$ and the first order necessary condition [9], we can obtain the following optimal solution $\mathbf{w}$ that minimizes the stopband energy

$$\mathbf{w} = -(\mathbf{E}^T\mathbf{Q}\mathbf{E})^{-T}(\mathbf{E}^T\mathbf{Q}^T\mathbf{d}) \qquad (29)$$

4. SIMULATION

In this section, we will evaluate the proposed window design technique. The DFT size $M = 1024$, cyclic prefix $P = 80$, and window length $L = 20$. The channel is VDSL loop1 of 4500ft, and RFI noise is simulated in differential mode with strength -55dBm and distance 6000ft [1]. We assume the RFI noise are of frequencies $1.8MHz$ and $2.0MHz$ and design windows for informed receiver and uninformed receiver. The resulting two windows are called respectively w_0 (informed) and w_1 (uninformed).

To evaluate the performance of the two windows, we apply an interference only signal to the receiver. In this case, the DFT output y_k contains only interference. Fig. 4 shows the RFI interference $\sigma_{y_k}^2$ at the DFT outputs. It shows that major interference occurs in the tones close to the RFI frequencies and the proposed windows have faster roll-off near the RFI source. Although Hanning and Blackman window have faster roll-off below the tone number 390, RFI is too small to be the dominated noise. Fig. 5 zooms in the part of Fig. 4 for the tones of indices from 350 to 512. Fig. 5 shows that, for the proposed windows, the RFI in neighborhood tones is smaller than that with other windows. As a result, the total interference of the proposed windows will be smaller; the total interference is given by

$$\Phi = \sum_{k=0}^{M/2-1} \sigma_{y_k}^2. \qquad (30)$$

As a measure of performance, we calculate percentage of improvement with respect to the rectangular window,

$$\frac{(\Phi_{rec} - \Phi)}{\Phi_{rec}} \times 100\%. \qquad (31)$$

Windows	Percentage improvement
w_0	83.9%
w_1	83.3%
Hanning	42.9%
Blackman	39.9%

Table 1: Percentages of improvement of the windowed DMT system in terms of RFI with respect to the conventional DMT system

Loop	w_0	w_1	Hanning	Blackman
VDSL1L	23.74	23.52	23.48	23.37
VDSL2L	22.80	22.78	22.59	22.56
VDSL3L	20.86	20.74	20.68	20.60
VDSL4L	12.04	11.95	11.90	11.94
VDSL5	33.14	33.09	32.76	32.74
VDSL6	24.94	24.90	24.75	24.64
VDSL7	18.98	18.80	18.73	18.65

Table 2: Bit rate (Mbits/sec) on VDSL loops

Table.1 shows the percentages of improvement of the windowed DMT system for the proposed windows, Hanning window, and Blackman window [10]. It shows that the proposed windows perform better than, Hanning window, and Blackman window. It also shows that designing receiving window without knowing the statistics of the RFI source leads to only a minor performance degradation.

In a second experiment, AWGN (additive white gaussian noise) channel noise is added. Fig. 6 shows the SINR (signal to interference ratio) of individual tones. Fig. 7 zooms in the part of Fig. 6 for the tones of indices from 350 to 512. From Fig. 7 we see that the SINRs of the proposed windows are higher than those of the other windows near the RFI source frequency. That is, we can transmit more bits in the neighboring tones by using the proposed windows. Table 2 shows the bit rate for seven VDSL loops [1] where VDSL loop 1 to 4 are of length $4500ft$. The proposed windows have higher bit rates for all test loops.

5. CONCLUSION

We have proposed a window design method to suppress the effect of RFI in DMT systems. We consider both the case when the receiver knows the statistics of the interference (informed receiver) and the case when the statistics are not available to the receiver (uninformed receiver). The proposed windows have faster roll-off in low frequency. Therefore, fewer tones will be dominated by RFI than in the case of Hanning and Blackman window. Windows designed for uninformed receiver (interference-independent window) has

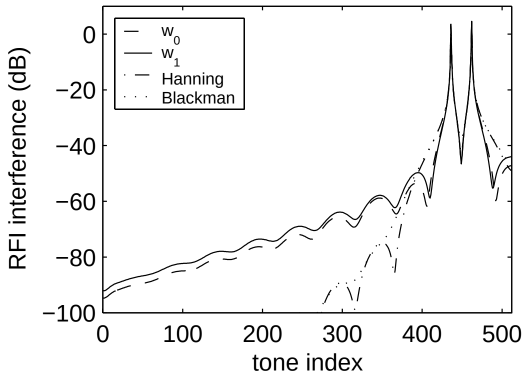

Figure 4: RFI interference of the DMT system with windowing.

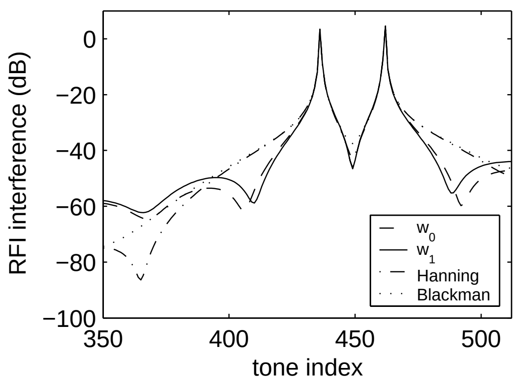

Figure 5: Zoom in of Fig. 4 for tones of indices from 350 to 500.

the advantage that the window coefficients need not be updated when the statistics of the RFI interference changes. In both cases, the optimal windows have closed form solution that can be obtained by using first order necessary condition. In simulations we show that the proposed windows have better performance than rectangular window, Hanning window, and Blackman window. We also shows not knowing the statistics of the RFI source leads to only a minor performance degradation.

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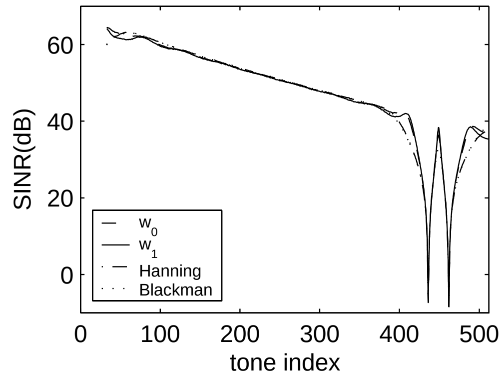

Figure 6: SINR of the DMT system with windowing.

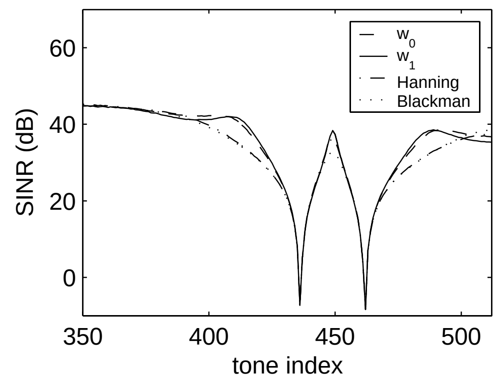

Figure 7: Zoom in of Fig. 6 for tones of indices from 350 to 500.

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