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STATISTICAL METHOD BASED ON SIMULTANEOUS DIAGONALISATION FOR POLSAR IMAGES ANALYSIS

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ABSTRACT

In [1], we have proposed a PCA-ICA neural network model for POLSAR image analysis. We propose here a new method that is full based on an algebraic statistical formulation and that is well justified from the mathematical point of view. Its advantage is that it is easy in its implementation that requires certain subroutines of the inverse matrix and the eigenvalues/ eigenvectors decomposition. While the PCA-ICA neural network model is very sensible to both the probabilistic model of the data [2], [3] and the power of the noise that corrupts the input data [1]. In addition, it requires more computation times in its learning process. Thus, the goal of this paper is to arise the power of each method and by this way we try to open new issues in the concern of working out new methods that accumulate the advantages of each method while avoiding their disadvantages.

1. INTRODUCTION

Over the last couple of decades there has been considerable interest in the imaging of the Earth by *POLarimetric Synthetic Aperture Radar* (POLSAR) systems. The images acquired with these systems provide a rich set of data that brings knowledge on the nature of targets and opens the way to new geophysical applications and others [2]. However, the POLSAR images are correlated and corrupted by speckle that appears as a granular signal-dependent noise [3]. Speckle has the characteristics of a non-Gaussian multiplicative noise. It presents challenges to most common models for POLSAR image processing and understanding [2], [3]. Correlation elimination and speckle reduction are necessary for an efficient automatic interpretation of the scene.

We have already proposed in [1] a PCA-ICA neural network for analysing the POLSAR images. With this method, the correlation between the POLSAR images is eliminated and the speckle noise is largely reduced in only the first Independent Component (IC) image. In fact, although the algorithm of the model is well established, the method suffers from the sensibility of the ICA part of the model to the nature of the statistical model of the input data (super-Gaussian or sub-Gaussian) and the power of the noise that corrupts these data [1]. We have used, as input data for the ICA part, only the first principal component (PC) image. The obtained IC image is an image of very high quality and better contrasted than the first PC image. However, when the second and third PC images are also used as input images with the first PC image, the results are not significative and the first IC image becomes less contrasted and more affected by the noise. Note that, in some remote sensing applications such as geo-hazard mitigation in which the radar images are very useful, the second and third PC images are of importance for mapping the thin structures of the observed scene such as the geological structures and the trajectory of the earthquake [4], [5], [6]. This can be justified by the fact that the ICA part of the model is essentially based on the principle of the Infomax algorithm [1]. This algorithm, however, is efficient only in the case where the input data have low noise [7], [8]. Another drawback of the method proposed in [1] is the computation time of the learning process, which is very costly.

The purpose of this paper is to propose a statistical method that performs well for analysing POLSAR images and that presents some advantages in its implementation. The purpose is to overcome the disadvantages of the method proposed in [1] and at the same time we will try to reply, for the moment, to the question how is possible the elaboration of a new method that exploits the advantages of the present method and of that proposed in [1] but also by avoiding their disadvantages. Before detailing the present method in section 3, we give in section 2 the POLSAR images model and the statistics to be used later in the concept of the linear transform matrix. Experiments performed on real POLSAR images, provided by the SIR-C/X system, are given and commented in section 4. In order to prove the effectiveness of the proposed method, the PCA-based method [9] and the PCA-ICA neural network model [1] are used for comparison. We conclude the paper in the last section.

2. STATISTICS TO BE USED

We adopt the same model used in [1]. Let x_i be the content of the pixel in the ith image, s_i the noise-free signal response of the target, and n_i the speckle. Then, we have the following multiplicative model: $x_i = s_i.n_i$. By supposing that the speckle has unity mean, standard deviation of σ_i, and is statistically independent from the observed signal x_i [3], the multiplicative model can be rewritten as: $x_i = s_i + s_i(n_i - 1)$. The term $s_i(n_i - 1)$ represents the zero mean signal-dependent noise and characterizes the speckle noise variation. Now, let X be the stationary random vector of input POLSAR images.

So, the covariance matrix of *X*, Σ_x, can be written as: $\Sigma_x = \Sigma_s + \Sigma_n$, where Σ_s and Σ_n are the covariance matrices of the noise-free signal vector and the signal-dependent noise vector, respectively. These two matrices are used in the conception of the linear transformation matrix of the proposed statistical method.

3. PROPOSED STATISTICAL METHOD FOR POLSAR IMAGES ANALYSIS

The basic idea of the proposed method is inspired from the well-developed aspects of the matrix theories and computations. The method consists of exploiting the simultaneous digitalisation procedure of the signal and multiplicative noise covariance matrices via one orthogonal matrix.
Of course, for making the application coherent with such theories, the model $x_i = s_i + s_i(n_i - 1)$ is adopted for the data as well as the nature of the statistical parameters that enter in the concept of the method must agree the assumptions and the conditions involved by the theories.
The two matrices to be diagonalised are Σ_s and Σ_n, respectively. The orthogonal matrix that diagonalises these two matrices is the linear transform matrix that we look for it. Σ_n becomes an identity matrix, which implies that the variance of the noise in each new image is unity. Thus, the noise becomes uncorrelated with the data in the new images. The signal to noise ratio (SNR) is characterized by the solutions of the generalized symmetric eigenvalue problem fathered by the method. These solutions are the eigenvalues that are the diagonal elements of the covariance matrix of the new images. This means that these images are not correlated and they are ordered according to the values of the corresponding SNR (i.e., according to their qualities).

3.1. Mathematical formulation

The extraction of the new images, via the linear transform matrix noted *A*, consists of maximizing the information carried by the original POLSAR images in a small number of new images that are statistically uncorrelated and in which the SNR and image contrast are improved. The criterion noted *"C"* for determining the matrix *A* can be stated as follows: "*Finding A in order that the matrix* Σ_n *becomes an identity matrix and the matrix* Σ_x *is transformed, at the same time, to a diagonal matrix (i.e., the new images are statistically uncorrelated) whose diagonal elements are ordering in decreasing values (i.e., by ordering the new images in decreasing values of their variances)*". The problem posed according to the criterion *"C"* can be mathematically formulated as follows. The row vectors of the matrix *A* are the vectors, a_i, that maximize the ratio:

$$\lambda_i = \left(a_i^t.\Sigma_X.a_i\right) / \left(a_i^t.\Sigma_n.a_i\right) \quad (1)$$

Taking into account the following two constraints that consist of allowing to make the researched linear transform to be orthogonal and normalised:

$$\begin{cases} a_i^t.\Sigma_n.a_i = 1 \\ and \quad a_i^t.a_j = 0 \quad for \quad i \neq j \end{cases} \quad (2)$$

Equation (1) is equivalent to find the vector a_i such that:

$$\partial\lambda_i / \partial a_i = \partial\left(\left(a_i^t.\Sigma_X.a_i\right)\left(a_i^t.\Sigma_n.a_i\right)^{-1}\right) / \partial a_i = 0 \quad (3)$$

As the two matrices, Σ_x and Σ_n, are real, symmetric and positive-definite, then, from (3) we can deduce that:

$$\begin{aligned} \Sigma_X.a_i - \Sigma_n a_i\left(a_i^t.\Sigma_X.a_i\right)\left(a_i^t.\Sigma_n.a_i\right)^{-1} = 0 \\ \Rightarrow \left(\Sigma_X - \lambda_i.\Sigma_n\right)a_i = 0 \end{aligned} \quad (4)$$

By generalizing equations (1), (2) and (4) for all the row vectors of the matrix *A*, we obtain the following system of equations:

$$\begin{cases} \left(\Sigma_X - \Sigma_n.\Lambda\right)A = 0 \\ A^t.\Sigma_n.A = I \end{cases} \quad (5)$$

where Λ is the diagonal matrix of eigenvalues λ_i, and *I* is the identity matrix. Then, finding the matrix *A* consists of solving the system of equations (5). The matrix *A* is the solution of the generalized symmetric eigenvalues problem given in (4) but with the whitening constraint of the matrix Σ_n. The algorithm proposed hereafter for obtaining the matrix *A* is deduced from the theorem about real symmetric matrices [10], [11], [12], [13]. The theorem insures the existence of the matrix *A* according to the criterion *"C"* mentioned above, and it is stated as follows [11]: *"Let B and C be (nxn) real symmetric matrices. If B is positive definite, there exists a non-singular matrix Q such that* $Q^t.B.Q = I$ *and* $Q^t.C.Q = D$, *where D is a diagonal matrix whose diagonal elements are the roots* λ *of the polynomial equation* $|C - \lambda.B| = 0$*."* As the two covariance matrices, Σ_x and Σ_n, that enter in the resolution of the system of equations (5) are symmetric, real, positive definite matrices, we may identify Σ_n as the matrix *B* and Σ_x as the matrix *C*. Based on this theorem, the algorithm that allows us to obtain the matrix *A* consists of realizing at the same time the whitening operation of the matrix Σ_n and the diagonalisation of the matrix Σ_x. It is the so-called simultaneous diagonalisation of two matrices using one orthogonal matrix, which is the matrix *A*. Then, the matrix *A* exists and it is not singular. However, since Σ_n is estimated from both the filtered and original images, it may occur, for practical reasons, that this matrix becomes a singular matrix [13]. The simultaneous diagonalisation of Σ_n and Σ_x, is still possible [13], [14] but the results require careful interpretation and other algorithms are needed to compute the eigenvalues [13], [14]. So, from the mathematical viewpoint, the method is well justified. This makes easily the translation of the theoretical development of the method to a practical procedure of implementation that performs well with precision the analysis of the POLSAR images.

3.2. Implementation procedure

Based on the above theoretical analysis, the pseudo code of the algorithm that allows us to obtain the matrix *A* can be formulated. We note *Estimate-Fct(.)* and *Decomp-Mat(.)* as the procedures of covariance matrix estimation and spectral decomposition of the matrix, respectively. Note that the procedure of spectral decomposition consists of calculating the orthonormal eigenvectors matrix, noted *EigenVec[.]*, and the corresponding diagonal matrix of the eigenvalues, noted *EigenVal[.]*. So, the procedure of implementation can be stated as follows:

Estimate-Fct (Σ_X) ;
Estimate-Fct (Σ_n) ;
Decomp-Mat (Σ_n) ; $\rightarrow$ *EigenVec[U], EigenVal[V]* ;
Compute : $\Phi := U.V^{-1/2}$; *and* $\Psi := \Phi^t.\Sigma_X.\Phi$;
Decomp-Mat (ψ) ; $\rightarrow$ *EigenVec[E], EigenVal[P]* ;
Compute: $A := (\Phi.E)^t := (U.V^{-1/2}.E)^t$;
Compute the output images from the original images such as: $Y = AX$

3.3. Interpretation

The above algorithm computes the linear transform matrix, *A*, that we look for it. The orthonormalized eigenvector matrix, *U*, and the corresponding matrix of the eigenvalues, *V*, are computed from the spectral decomposition of Σ_n. These two matrices are used to construct the renormalization matrix, $\Phi = U.\ V^{1/2}$, for which we have $\Phi^t.\Sigma_N.\Phi = I$ and $\Phi^t.\Phi = V^{-1}$. It is the requirement that *V* have an inverse which means that Σ_n must be non-singular and, hence, positive definite. The renormalization matrix, Φ, is used to weight the covariance matrix, Σ_x, to give the noise-adjusted data covariance matrix, $\Psi = \Phi^t.\Sigma_X.\Phi$. The spectral decomposition of Ψ is computed in order to determine the linear transform matrix *A*.

The vector of the new images, noted *Z*, is produced from the vector of the original POLSAR images, *X*, such as: $Z = A^t.X$, where *A* is given by $A = (\Phi.E)^t = (U.V^{1/2}.E)^t$ and *E* is the matrix of the orthonormal eigenvectors of Ψ. So, the proposed method can be implemented by a single matrix, *A*, which simultaneously diagonalises Σ_x and Σ_n such as: $A^t.\Sigma_X.A = P$ and $A^t.\Sigma_N.A = I$, where *P* is the diagonal matrix of the eigenvalues of Ψ corresponding to *E*. The eigenvalues of the matrix *P* are then the solutions of the generalized symmetric eigenvalues problem (5). This matrix is the covariance matrix of the new extracted images, which means that these images are statistically uncorrelated. The diagonal elements of *P* characterize the SNR in the new images since the covariance matrix of the signal-dependent noise is equal to an identity matrix. The SNR is then improved in the first few new images in which the variances are important.

4. EXPERIMENTAL RESULTS

A real POLSAR data provided by the SIR-C system are used to evaluate the proposed method. The data were acquired over the Orgeval site (east of Paris, France, 329x329pixels) during summer 1994 [15] and correspond to bands C and L with HH and HV polarizations for each. The four bands are shown in Fig. 1. Classification methods applied to these images will not be effective due to the domination of the speckle noise. The existence of the redundancies between these images is clearly shown in Fig. 1. The extracted new images using the proposed method are given in Fig. 2 beside the first IC image of PCA-ICA neural network model of [1]. It is clear that the IC image is better than the first new image. Most of the information contained in the original images is now concentrated in this image, which is an image of quality. The first new image is also an image of quality but some speckle noise still exists in this image. Second and third new images contain mainly noise more than information. The fourth new image is very noisy and no information can be extracted from it. We quantify the speckle level by computing the contrast ratio (CR), which is the average value of the standard deviation to mean ratios calculated in small homogeneous areas of the observed scene. The reduction of the speckle noise level is quite evident when comparing the first new image of the proposed method with the image of the L band (HV). This image has a CR value of 0.11, while the first new image reduces the speckle noise level to 0.148. However, the IC image is superior to the first new image and its CR value is about 0.39. The SNR values of the original and new images are given in Tables 1 and 2, respectively. The original images have ratios ranging from 4.20 to 18.84. While the SNR value in the first new image is improved to 26.62; this corresponds to a factor of 1.41 compared with the best original image (L band-HV).

In order to prove the effectiveness of the proposed method in data compression and enhancement of POLSAR images, the standard PCA method of POLSAR images in the logarithmic domain is used. Note that this method, which permits us to convert the multiplicative model of the speckle noise to an additive model and hence the possibility to apply the standard PCA method, is often suggested in the literature for data compression and enhancement of POLSAR images. To evaluate the performance of the two methods, we have used a criterion that measures the capability in image compression without an important loss of information. This consists of reconstituting the four original POLSAR images from only the first and the second new images. To quantify the degree of similarity between the original POLSAR images and the reconstituted ones, the mean-square-error (MSE) of the reconstitution process is calculated as: $MSE = \sqrt{E\left((x_i - \hat{x}_i)^t.(x_i - \hat{x}_i)\right)}$, where x_i and $\hat{x}_i$ are the original and the reconstituted images, respectively. Figures 3 and 4 give us the MSE values for each pair of original POLSAR/reconstituted images of the proposed method and the standard PCA method, respectively. For the proposed method, the MSE values vary from 0.50 to 3.25. The average MSE value of the reconstitution process is approximately equal to 2.25, which is an acceptable error and it reflects that the original and reconstituted images are very similar. This means that the proposed method represents data compression by a factor of 2. For the standard PCA method, the reconstitution process presents a less broad interval of the reconstitution errors, from 2.05 to 3.45. The average value of the error of the reconstitution process is equal to 2.50. However, we note that the smallest value, greatest value, and the average value of the error of the standard PCA method are larger than those of the proposed method. This implies that the proposed method is more effective in the data compression with a minimal loss of the original information. This can be justified by the fact that the statistics computed in the logarithmic domain are completely different to those computed from the original data.

5. CONCLUSION

The suggested method for POLSAR image analysis is full based on an algebraic statistical formulation inspired from the well-developed aspects of the matrix theory and computations. It is based on so-called the simultaneous diagonalization procedure of two matrices via one orthogonal matrix. From the mathematical point of view, the method is well justified. This renders easily and clear the implementation procedure. Experiments were performed with the real POLSAR. Comparative study between the proposed method and the standard method of PCA in the logarithmic domain has been shown that the proposed method exceeds largely the performances of the standard method. The results of the proposed method are satisfactory in terms of data compression ability, speckle noise reduction, and image enhancement. However, the comparative study, performed between the proposed method and the PCA-ICA neural network method, shows that, in general, the two methods give acceptable results in terms of scene interpretation and data compression and they resemble each other in this sense. But the PCA-ICA neural network model is superior in terms of image quality because the ICA part of the model consists of treating the speckle effect, which has a non-Gaussian distribution. While the new proposed method used only the covariance matrices of both data and noise and so it can not deal with the non-Gaussian distributions. Note that, for the mapping purpose from the remote sensing data, the use of the high quality images in the classification process is of importance to improve the accuracy of the classification. In the future investigation, we will try to develop a new method that accumulate the advantages of each method while avoiding their disadvantages by exploiting, for example, the concept of the simultaneous diagonalization for diagonalising the higher order statistics of the POLSAR data.

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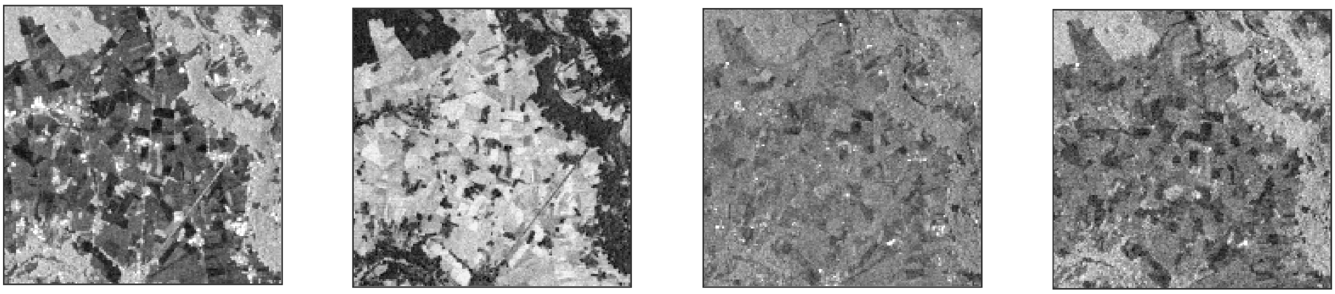
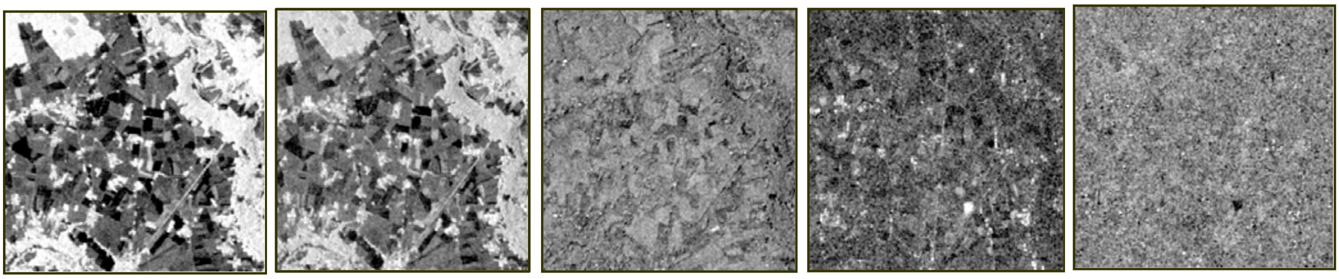


(a) C Band-HH (b) C Band-HV (c) L Band-HH (d) L-Band-HV

Figure 1 - The four original POLSAR images



(a) First IC image (b) First new image (c) Second new image (d) Third new image (e) Fourth new image

Figure 2 - The four new extracted images using the proposed statistical method

Table 1. Signal-to-Noise Ration (SNR) values in the original POLSAR images

Original images	C Band-HH	C Band-HV	L Band-HH	L Band-HV
SNR values	4.205	8.529	12.575	18.840

Table 2. Signal-to-Noise Ration (SNR) values in the extracted images

Extracted images	First image	Second image	Third image	Fourth image
SNR values	26.620	3.978	3.247	2.022

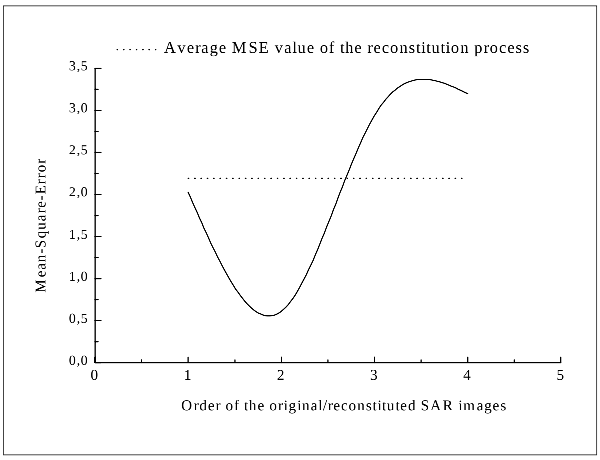


Figure 3 - The Mean-Square-Error (MSE) of the reconstitution process (using the first two extracted images produced by the proposed method)

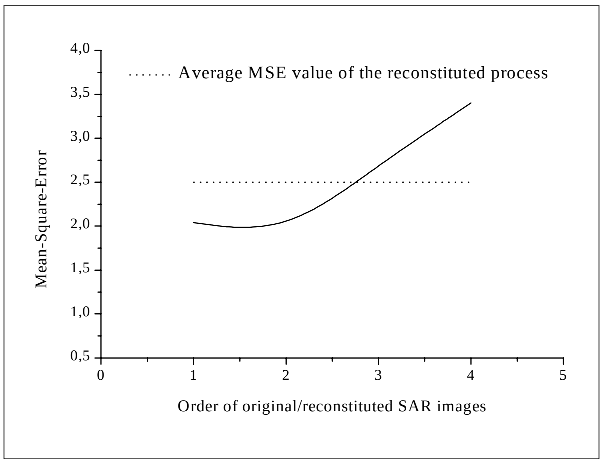


Figure 4 - The Mean-Square-Error (MSE) of the reconstitution process (using the first two PC images produced by the standard PCA method in the logarithmic domain)