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SEPARATION OF CORRELATED SIGNALS USING SIGNAL CANCELER CONSTRAINTS IN A HYBRID CM ARRAY ARCHITECTURE

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ABSTRACT

In this paper, we present a hybrid implementation of the multistage constant modulus (CM) array for separating correlated signals. Using a cascade architecture of the CM array with a series of adaptive signal cancelers, we derive a parallel set of constrained beamformers. The canceler weights provide estimates of the direction vectors of the captured signals across the cascade stages, which are used in a parallel implementation of the linearly constrained CM (LCCM) array. Since the direction vectors are obtained directly from the canceler weights, the hybrid implementation does not require prior knowledge of the array response matrix. If the source signals are sufficiently separated in angle, then they can be captured individually across the parallel stages. When the sources are correlated, the cascade CM array does not completely cancel the captured signals, and previous versions of the parallel CM array do not always capture different sources across the stages. These problems are handled by the proposed hybrid LCCM architecture based on the signal canceler constraints. Computer simulations for example cochannel scenarios are provided to illustrate some properties of the system.

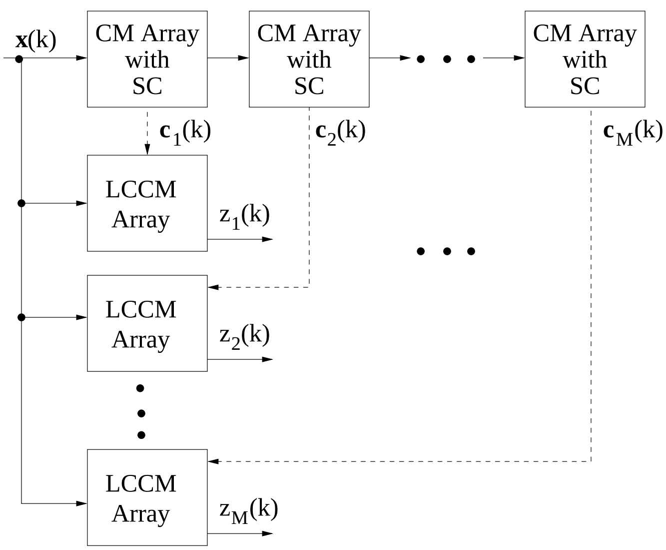

Figure 1: Multistage hybrid CM array with signal canceler constraints copied to the parallel LCCM beamformers.

1. INTRODUCTION

The constant modulus (CM) array is an adaptive beamformer that is designed to blindly separate and extract multiple cochannel signals [1]. It has a linear combiner structure with weights that are adapted by the constant modulus algorithm (CMA) [2]. The CM array generally locks onto the signal with the greatest power at the output of the array [1], and in doing so the beamformer nulls cochannel interference signals. It is possible, however, depending on the initial weight conditions and the relative signal powers, that a null is placed in the direction of a desired signal and an interferer is captured instead. A multistage version of the CM array was proposed to capture multiple cochannel signals; it consists of a cascade of CM array stages [3] with an adaptive signal canceler (SC) included in each stage to remove the captured signal from the input [4]. However, the performance of the cascade structure degrades with an increasing number of stages, and when the sources are correlated, it is not effective because the cancelers remove portions of the cochannel signals according to the degree of cross-correlations.

The linearly constrained CM (LCCM) array employs a constraint based on an estimate of the source direction vector to place a beam in the direction of the desired user [5]. If the angle of arrival (AOA) of the desired signal is not known a priori, it can be estimated using a variety of estimation algorithms (e.g., [6]). However, such algorithms are generally computationally intensive and usually require an estimate of the correlation matrix of the received signal. Moreover, they are sensitive to the cochannel signal cross-correlations and require an accurate estimate of the antenna array response matrix.

When the source signals are correlated, an adaptive beamformer can have difficulty preserving the desired signal while rejecting the interference (refer to [7], [8], [9], [10], and [11] for example prior work on separating correlated sources). As the cascade CM array stage in [12] is adapted, the cumulative transfer function at each stage is computed after convergence and these are used as the initial state for the parallel CM array. In the parallel stages, the users are captured across the stages without signal cancellation. However, it is possible for different parallel stages to lock onto the same signal because they share the same input, so that the adaptation process has to be restarted.

It was shown in [4] that the adaptive SC weights in the cascade CM array are proportional to the direction vectors of the captured signals across the stages. Based on this property, we propose using the SC weight vectors in the constraints of a parallel implementation of several LCCM arrays. As the SC weight vectors are adapted in the cascade CM array, they are simultaneously copied to the parallel LCCM architecture so that there is no delay in adapting the constrained weights of the parallel stages. Since the SC weights are used as direction vector estimates, the implementation does not require knowledge of the antenna response matrix nor the antenna configuration. The signal captured in each stage of the cascade implementation is unique so that the directional constraint for each stage of the parallel LCCM array is also unique. Thus, each parallel stage will capture a different signal and we avoid the problem of multiple stages capturing the same source. We demonstrate that the constraints provided by the SC weights can achieve better performance for correlated sources than if the actual direction vectors are used.

2. COCHANNEL SIGNAL MODEL

Assume that L transmitted (baseband) signals $\{s_l(t)\}$ impinge on M antenna elements, yielding the received signals $\{x_m(t)\}$ which are uniformly sampled. The discrete-time output signals of the antenna array can be written in matrix/vector form as

$$\mathbf{x}(k) = \mathbf{A}\mathbf{s}(k) + \mathbf{n}(k) \tag{1}$$

where $\mathbf{s}(k) \triangleq [s_1(k),\ldots,s_L(k)]^T$, $\mathbf{x}(k) \triangleq [x_1(k),\ldots,x_M(k)]^T$, and $\mathbf{n}(k) \triangleq [n_1(k),\ldots,n_M(k)]^T$ is additive white Gaussian noise (AWGN). The antenna response matrix $\mathbf{A}$ depends on the type of antenna elements and the array configuration used in the receiver. The lth column of $\mathbf{A}$, denoted by $\mathbf{a}_l$, is the direction vector of the lth source. For the case of $L = 2$ sources with correlation coefficient $\rho \triangleq E[s_1(k)s_2^*(k)]/(\sigma_1\sigma_2)$, the correlation matrix $\mathbf{R} \triangleq E[\mathbf{x}(k)\mathbf{x}^H(k)]$ is given by

$$\mathbf{R} = [\mathbf{a}_1\mathbf{a}_2]\begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho^*\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}\begin{bmatrix} \mathbf{a}_1^H \\ \mathbf{a}_2^H \end{bmatrix} + \sigma_n^2\mathbf{I} \tag{2}$$

where the superscript ^{H} denotes complex conjugate transpose, the superscript * denotes complex conjugation, σ_i^2 is the variance of the ith source signal, σ_n^2 is the AWGN variance, and $\mathbf{I}$ is the identity matrix.

3. MULTISTAGE HYBRID RECEIVER

The multistage hybrid LCCM array is shown in Figure 1 where several CM array stages [3] and SCs are arranged in series (as in the original multistage cascade system), and several LCCM array stages are arranged in parallel. The SC weight vector $\mathbf{c}_j(k) \triangleq [c_j(k,1),\ldots,c_j(k,M)]^T$ of the jth stage of the cascade structure is copied to the jth stage of the parallel LCCM array in the form of a directional constraint. Note that it is not necessary to obtain an explicit AOA estimate from the canceler vector; the SC weights can be used directly in the constraint because they are proportional to the direction vector of the captured source at convergence, and hence can be used for an arbitrary antenna array. The received signal $\mathbf{x}(k)$ is weighted by the jth stage LCCM weight vector $\mathbf{h}_j(k)$, yielding the output $z_j(k) = \mathbf{h}_j^H(k)\mathbf{x}(k)$. The LCCM weight vector in Figure 2 has been split into the form of a generalized sidelobe canceler (GSC) [13] as follows:

$$\mathbf{h}_j(k) = \mathbf{u}_j(k) - \mathbf{D}_j(k)\mathbf{v}_j(k) \tag{3}$$

where the weights $\mathbf{u}_j(k) \triangleq [u_j(k,1),\ldots,u_j(k,M)]^T$ satisfy the directional constraint, and the unconstrained adaptive weight vector $\mathbf{v}_j(k) \triangleq [v_j(k,1),\ldots,v_j(k,M-1)]^T$ satisfies the CM condition. The constrained weight vector of the jth stage LCCM array is determined by the SC weights as follows:

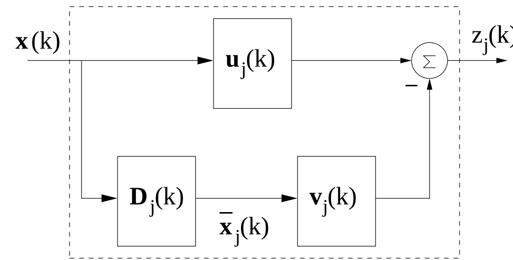

Figure 2: LCCM array weight configuration.

$$\mathbf{u}_j(k) = \mathbf{c}_j(k)/(\mathbf{c}_j^H(k)\mathbf{c}_j(k)), \tag{4}$$

and the rank-reduction matrix $\mathbf{D}_j(k)$ satisfies the condition $\mathbf{D}_j^H(k)\mathbf{c}_j(k) = \mathbf{0}$.

The modified input $\bar{\mathbf{x}}_j(k) \triangleq [\bar{x}_j(k,1),\ldots,\bar{x}_j(k,M-1)]^T$ of the weights $\mathbf{v}_j(k)$ in the jth parallel stage is given by $\bar{\mathbf{x}}_j(k) = \mathbf{D}_j(k)\mathbf{x}(k)$, and the corresponding weight update is

$$\mathbf{v}_j(k+1) = \mathbf{v}_j(k) + \mu(|z_j(k)|^2 - 1)z_j^*(k)\bar{\mathbf{x}}_j(k) \tag{5}$$

where $\mu > 0$ is the step-size parameter. This modified input removes the need for an explicit constraint on the CMA update. Although each stage of the LCCM array operates on the same received signal $\mathbf{x}(k)$ and the adaptive weights of the parallel stages are initialized to the same value, the (modified) input vectors of the adaptive algorithm are different across the stages (unlike previous parallel implementations of the CM array) so that the stages are not likely to capture the same source signal.

4. STEADY-STATE ANALYSIS

Since $\mathbf{h}_1$ is the composite weight vector of the first stage of the parallel LCCM array, $g_{1,j} \triangleq \mathbf{h}_1^H\mathbf{a}_j$ is the gain in the direction of user j. When the beamformer is designed to capture user j, it is desirable that $g_{1,j} = 1$ and $g_{1,i} \approx 0$ for $i \neq j$ so that the interferers are nulled. The directional constraint of the first stage beamformer for capturing user 1 is $\mathbf{h}_1^H\mathbf{a}_1 = 1$, and the output power of the beamformer is

$$\begin{aligned} P &= \mathbf{h}_1^H\mathbf{R}\mathbf{h}_1 \\ &= \mathbf{h}_1^H(\mathbf{a}_1(\sigma_1^2\mathbf{a}_1^H + \rho\sigma_1\sigma_2\mathbf{a}_2^H) \\ &\quad +\mathbf{a}_2(\sigma_2^2\mathbf{a}_2^H + \rho^*\sigma_1\sigma_2\mathbf{a}_1^H) + \sigma_n^2\mathbf{I})\mathbf{h}_1. \end{aligned} \tag{6}$$

To find the optimum weights under the constraint, we add the term $2\lambda(\mathbf{h}_1^H\mathbf{a}_1 - 1)$ where λ is a Lagrange multiplier, and set the partial derivative $\partial P/\partial\mathbf{h}_1$ to the zero vector, yielding

$$\begin{aligned} \mathbf{a}_1(\sigma_1^2 + \rho\sigma_1\sigma_2 g_{1,2}) + \mathbf{a}_2(\sigma_2^2 g_{1,2} + \rho^*\sigma_1\sigma_2) \\ +\sigma_n^2\mathbf{h}_1 + \lambda\mathbf{a}_1 = \mathbf{0} \end{aligned} \tag{7}$$

where $g_{1,2} = \mathbf{a}_2^H\mathbf{h}_1$ is the response of the beamformer in the direction of user 2, and $g_{1,1} = 1$. Since $g_{1,2}$ can be viewed as the gain placed by the beamformer in the direction of the interferer (user 2), we refer to it as the interference gain.

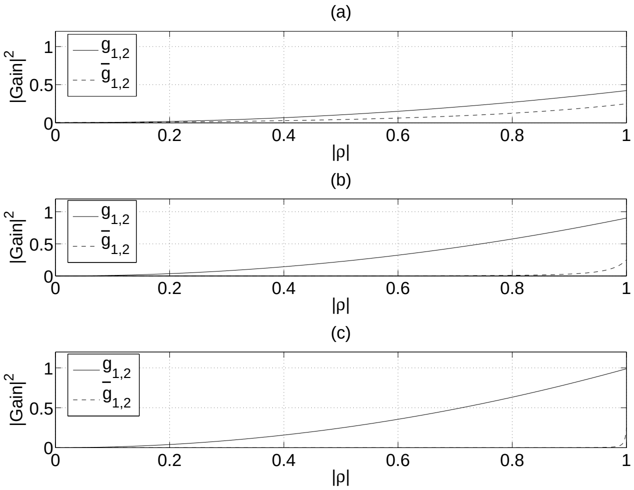

Figure 3: Interference gain for widely-spaced AOAs. (a) $\sigma_n^2 = 1$. (b) $\sigma_n^2 = 0.1$. (c) $\sigma_n^2 = 0.01$.

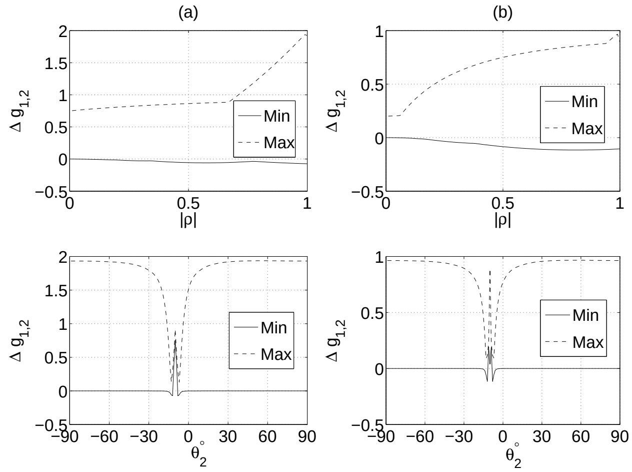

Figure 4: Maximum and minimum values of $\Delta g_{1,2}$. (a) $\sigma_1^2 = 2$, $\sigma_2^2 = 1$. (b) $\sigma_1^2 = 1$, $\sigma_2^2 = 2$.

Substituting $\mathbf{a}_1^H\mathbf{a}_1 = \mathbf{a}_2^H\mathbf{a}_2 = M$ and $G \stackrel{\Delta}{=} \mathbf{a}_1^H\mathbf{a}_2$, the Lagrange multiplier is

$$\lambda = -\frac{M(\sigma_1^2 + \rho\sigma_1\sigma_2 g_{1,2}) + G(\sigma_2^2 g_{1,2} + \rho^*\sigma_1\sigma_2) + \sigma_n^2}{M}, \tag{8}$$

and the interference gain is

$$g_{1,2} = \frac{(|G|^2 - M^2)\rho^*\sigma_1\sigma_2 + G^*\sigma_n^2}{(M^2 - |G|^2)\sigma_2^2 + M\sigma_n^2}. \tag{9}$$

In the ideal case, $g_{1,2}$ should be zero because the beamformer is designed to null user 2 and capture user 1.

It was shown in [12] that for correlated signals, the Wiener solution for the weights in the first stage of the cascade CM array are given by

$$\mathbf{w}_1 = \mathbf{R}^{-1}\mathbf{A}\mathbf{r}_1 \tag{10}$$

where $\mathbf{r}_i$ is the ith column of the correlation matrix $\mathbf{R}$. The corresponding Wiener solution for the SC weights is

$$\mathbf{c}_1 = (1/\sigma_{y_1}^2)\mathbf{A}\mathbf{r}_1 \tag{11}$$

where $\sigma_{y_1}^2 \stackrel{\Delta}{=} E[|y_1(k)|^2] = \mathbf{w}_1^H\mathbf{R}\mathbf{w}_1$, which is no longer an accurate representation of the direction vector of the signal captured in that stage. Instead, it corresponds to a sum of the direction vectors of all the source signals scaled by their respective correlation coefficients. Consider the LCCM array with a modified constraint given by $\bar{\mathbf{h}}_1^H\mathbf{A}\mathbf{r}_j = 1$ where $\bar{\mathbf{h}}_1$ is the corresponding beamformer weight vector. For the two-source scenario, the direction constraint for user 1 is $\bar{\mathbf{h}}_1^H(\sigma_1^2\mathbf{a}_1 + \rho\sigma_1\sigma_2\mathbf{a}_2) = 1$ and the output power is $\bar{P} = \bar{\mathbf{h}}_1^H\mathbf{R}\bar{\mathbf{h}}_1$. To find the optimum constraint weights for user 1, we add the term $2\lambda(\bar{\mathbf{h}}_1^H(\sigma_1^2\mathbf{a}_1 + \rho\sigma_1\sigma_2\mathbf{a}_2) - 1)$ to $\bar{P}$ and set the partial derivative $\partial\bar{P}/\partial\bar{\mathbf{h}}_1$ to the zero vector, yielding

$$\begin{aligned}\mathbf{a}_1 + \mathbf{a}_2\left(\sigma_2^2\bar{g}_{1,2} + \rho^*\sigma_1\sigma_2\frac{1-\rho\bar{g}_{1,2}\sigma_1\sigma_2}{\sigma_1^2}\right)& \\ +\sigma_n^2\bar{\mathbf{h}}_1 + \lambda(\sigma_1^2\mathbf{a}_1 + \rho\sigma_1\sigma_2\mathbf{a}_2) = \mathbf{0}&\end{aligned} \tag{12}$$

where $\mathbf{a}_1^H\bar{\mathbf{h}}_1 = (1 - \rho\sigma_1\sigma_2\bar{g}_{1,2})/\sigma_1^2$ and $\bar{g}_{1,2} \stackrel{\Delta}{=} \mathbf{a}_2^H\bar{\mathbf{h}}_1$ have been substituted. Premultiplying by $\mathbf{a}_2^H$ yields

$$\lambda = -\frac{G^* + M\left(\sigma_2^2\bar{g}_{1,2} + \rho^*\frac{\sigma_2}{\sigma_1} - |\rho|^2\bar{g}_{1,2}\sigma_2^2\right) + \sigma_n^2\bar{g}_{1,2}}{\sigma_1^2 G^* + M\rho^*\sigma_1\sigma_2}. \tag{13}$$

This result leads to the following expression for the interference gain:

$$\begin{aligned}\bar{g}_{1,2} \;=\; & \sigma_n^2(G^* + M\rho^*\sigma_2/\sigma_1) \\ & /\left(\sigma_1^2(M^2 - |G|^2)(1-|\rho|^2)\sigma_2^2\right. \\ & \left.+\sigma_n^2(\sigma_1^2 M + |\rho|^2\sigma_2^2 M + 2\rho G^*\sigma_1\sigma_2)\right).\end{aligned} \tag{14}$$

For correlated sources, $g_{1,2}$ in (9) gives the interference gain when the actual AOA is used in the direction constraint, and $\bar{g}_{1,2}$ in (14) is the interference gain when the canceler weights are used in the direction constraint. Finally, the signal-to-interference-plus-noise ratio (SINR) of the ith user in the jth stage of the parallel LCCM beamformer with the modified constraint is given by

$$\mathrm{SINR}_i^{(j)} = \frac{\sigma_i^2|\bar{g}_{j,i}|^2}{\sum_{p\neq j}\sigma_p^2|\bar{g}_{j,p}|^2 + \bar{\mathbf{h}}_j^H\bar{\mathbf{h}}_j\sigma_n^2}. \tag{15}$$

In order to compare the two interference gains for an arbitrary value of ρ, we present some numerical examples. The receiver is a uniform linear array where the gain of the mth antenna for the lth source is given by $A_{m,l} \stackrel{\Delta}{=} e^{-j(m-1)\phi_l}$. Since the sources are narrowband, the phase angle of the lth source is $\phi_l = 2\pi(d/\lambda_d)\sin(\theta_l)$ where $d = \lambda_d/2$ is the interelement spacing of the antenna array, λ_d is the wavelength, and $\{\theta_l\}$ are the AOAs. Figure 3 shows a plot of the interference gain versus ρ for two unit-power signals with AOAs $\theta_1 = -10^\circ$ and $\theta_2 = 45^\circ$. Observe that $\bar{g}_{1,2}$ is always less than $g_{1,2}$ with improving performance as the noise power decreases. For a given noise power, both $\bar{g}_{1,2}$ and $g_{1,2}$ increase with increasing correlation. These results for correlated sources show that the proposed hybrid beamformer using the modified constraint can perform better than using an estimate of the actual direction vector (based on the AOA) of the desired user.

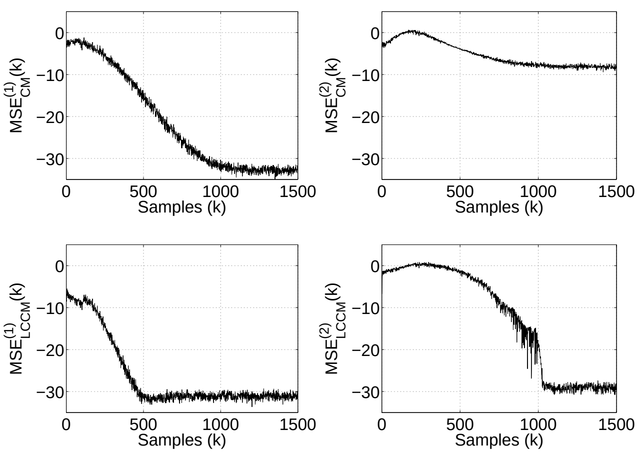

Figure 5: MSE comparison for two correlated sources and two stages: CM array (top) and LCCM array (bottom).

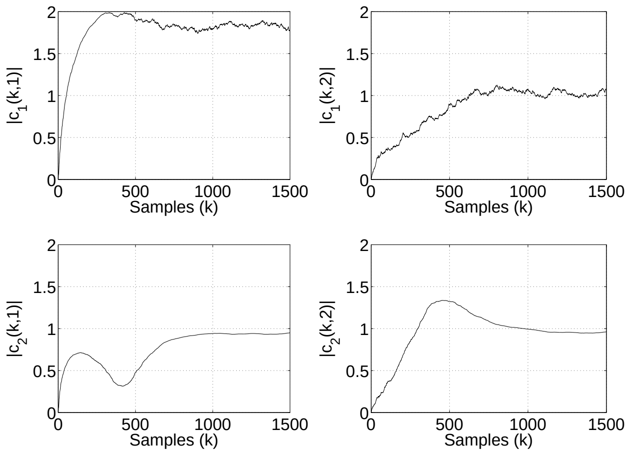

Figure 6: Canceler weights for two correlated sources and two stages.

Fixing $\sigma_n^2 = 0.01$ and the AOA of user 1 at $\theta_1 = 10^\circ$, the AOA of user 2 was varied and the interference gain difference $\Delta g_{1,2} \triangleq |g_{1,2}|^2 - |\bar{g}_{1,2}|^2$ was computed for a range of values of signal powers and ρ. A large value of $\Delta g_{1,2}$ indicates that the proposed beamformer nulls the interferer better than a conventional LCCM beamformer. When $\Delta g_{1,2}$ is less than zero, this corresponds to the situation where the modified LCCM array performs worse than the conventional LCCM array. Since $\Delta g_{1,2}$ is a function of both ρ and θ_2, a three-dimensional plot would be required to fully display its properties. However, in order to simplify the presentation, we examine the two-dimensional plots in Figure 4 which show the maximum and minimum of $\Delta g_{1,2}$, first over the range of $|\rho|$ and then over the range of θ_2. In the plots where $\Delta g_{1,2}$ is plotted versus $|\rho|$, the minimum and maximum values of $\Delta g_{1,2}$ are chosen over all values of θ_2, yielding the worst- and best-case performance, respectively, for each value of $|\rho|$. Similarly, in the plots of $\Delta g_{1,2}$ versus θ_2, the minimum and maximum values over all values of $|\rho|$ specify the worst- and best-case performance, respectively, for each value of θ_2. The results were generated for two scenarios: (a) $\sigma_1^2 = 2$ and $\sigma_2^2 = 1$ (user 1 is stronger) and (b) $\sigma_1^2 = 1$ and $\sigma_2^2 = 2$ (user 2 is stronger). Observe that in both cases, $\min(\Delta g_{1,2})$ can drop below zero so that the performance deteriorates, but only for small $|\theta_2 - \theta_1|$; the magnitude of the difference is less than 0.2 in this example.

5. COMPUTER SIMULATIONS

Computer simulations were performed for $L = 2$ sources with correlation coefficient $\rho_{1,2} = 0.5/\sqrt{2}$ and a receiver with $M = 2$ antenna elements. The source with an AOA of 45° is 3 dB stronger than the other source with an AOA of -10°. The noise variance is $\sigma_n^2 = 0.01$. The received signals from the uniform linear antenna array were sent simultaneously to the cascade multistage CM array and the parallel LCCM array. The SC weights of the cascade structure were copied to the LCCM array to generate the directional constraints. The CM array weights of the cascade structure were initialized to $\mathbf{w}_j(0) = [1, 0]^T$, and each LCCM array was initialized to $\mathbf{v}_j(0) = 1$ (a scalar in this case). Mean-square-error (MSE) curves were generated by averaging over 50 independent runs of the adaptive algorithms, yielding the results shown in Figure 5 (plotted on a log scale). The first stage of the hybrid LCCM array has fast convergence compared with the cascade CM array for the same steady-state MSE performance. The second stage of the cascade CM array has much worse MSE performance compared to that of the hybrid LCCM array, since the MSE performance of the second stage CM array is limited by the degree of correlation as discussed earlier. The second stage of the LCCM array takes longer to converge because the SC weights of the second stage of the cascade system converge slower, as shown in Figure 6. The convergence rate of the second stage of the LCCM array can be improved by increasing the step-size parameter of the second stage canceler, although this leads to greater misadjustment.

Next, we consider a receiver with $M = 3$ antenna elements. The AOAs of the $L = 3$ uncorrelated sources are 45°, -60°, and -10°, and the variances σ_i^2 are 3, 2, and 1, respectively. The noise variance is $\sigma_n^2 = 0.01$, $\mathbf{w}_j(0) = [1, 0, 0]^T$, and $\mathbf{v}_j(0) = [1, 0]^T$. Again, the users were captured using the cascade CM array, and the weights were simultaneously copied to the parallel LCCM implementation. The beampatterns of the LCCM array at convergence for all three stages are shown in Figure 7. In order to view the performance of the hybrid system, we vary the AOA of user 2 and compute the SINR for each user in all stages of the parallel receiver. The output SINR of user 1 is stronger than that of the other two users, and drops as θ_2 approaches 45° as shown in Figure 8. For the second stage, the SINR of user 2 is greater than that of the other users for all AOAs. From Figure 8 we see that similar to the first stage, the SINR of user 2 in the second stage also drops when θ_2 approaches an AOA of the other users. As discussed previously, the SINR of user 1 in the second stage is close to zero so that user 1 does not contribute much interference in the second stage (and thus is outside the range of values shown in the plot). Figure 8 shows that the SINR of user 3 drops below 0 dB when θ_2 approaches $\theta_3 = -10^\circ$. When θ_3 exceeds θ_2, the SINR of user 3 is greater than that of the other users; however, when it is exactly zero, the SINR of user 1 exceeds that of the other two users, and user 1 is captured in the third stage.

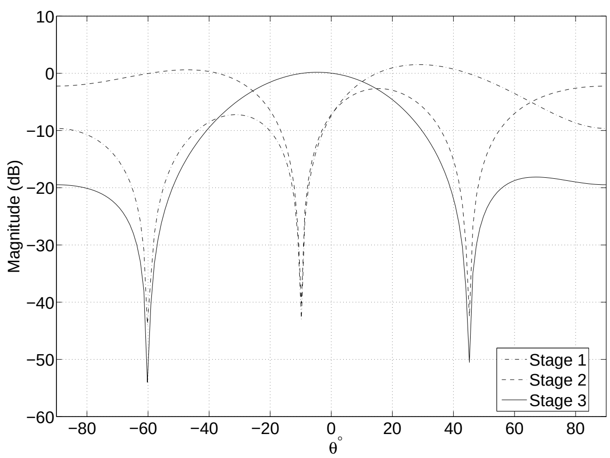

Figure 7: Beampatterns for three stages of the parallel LCCM array.

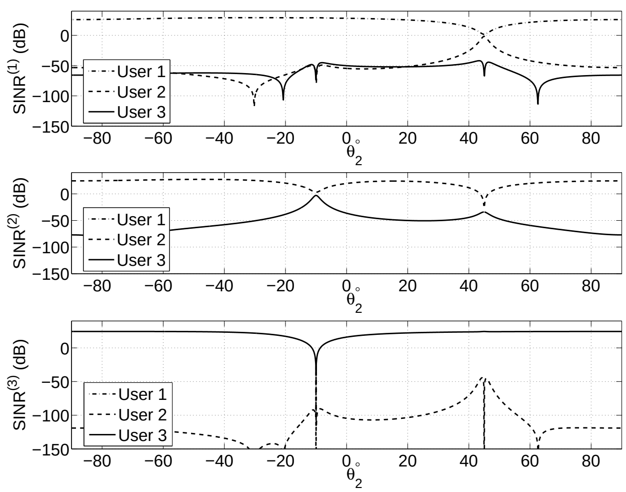

Figure 8: SINRs for three stages of the parallel LCCM array.

6. CONCLUSION

A hybrid implementation of the multistage LCCM array for separating correlated sources was presented. The signal canceler weights from the cascade multistage CM array are used as direction vector estimates for a parallel implementation of the LCCM array. Since the direction vector constraints are obtained directly from the SC weights, the proposed implementation does not require knowledge of the antenna response matrix. Moreover, the hybrid implementation does not capture the same source in multiple stages, unlike previous parallel implementations that require proper initialization (or re-initialization). For correlated sources, it was shown that the modified direction vector constraint obtained from the SC weights can provide better performance than using the actual direction vector. Compared to the cascade implementation, the performance of the hybrid system for correlated sources does not significantly degrade with an increase in the number of stages, has faster convergence, and lower MSE in the higher stages.

7. ACKNOWLEDGMENT

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