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NEW FULL-DIVERSITY HIGH-RATE SPACE-TIME BLOCK CODES BASED ON SELECTIVE POWER SCALING

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ABSTRACT

We design a new rate-$\frac{5}{4}$ full-diversity orthogonal STBC for QPSK and 2 transmit antennas by enlarging the signalling set from the set of quaternions used in the Alamouti [1] code. Selective power scaling of information symbols is used to guarantee full-diversity while maximizing the coding gain and minimizing the transmitted signal peak-to-minimum power ratio. The optimum power scaling factor is derived using two equivalent criteria and shown to outperform schemes based on only constellation rotation while still enjoying a low-complexity ML decoding algorithm. Extensions to the case of 4 transmit antennas are reported in [4].

1. INTRODUCTION

The information-theoretic analyses in [6] showed that multiple antennas at the transmitter and receiver enable very high-data-rate reliable wireless communications. STC introduced in [8], improve the reliability of communication over fading channels by correlation of signals across the different transmit antennas. A characterization of the design criteria of such codes was given in [8, 7]. One class of STC are space-time block codes (STBC) introduced in [1, 2] which are the technical focus of this paper. Orthogonal designs [2] are a class of STBC that achieve maximal diversity at a linear (in constellation size) decoding complexity. Our objective in this paper is to design a new class of full-diversity high-rate (>1) space-time block codes (STBC) by exploiting the inherent algebraic structure in existing orthogonal designs based on quaternions for 2 transmit antennas [1] and quasi-orthogonal designs for 4 transmit antennas [5]. The simplest example of a complex orthogonal design is the 2×2 code

$$\mathscr{Q}(x_1,x_2)\rightarrow\begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}$$

discovered by Alamouti [1] where $(.)^*$ denotes the complex-conjugate transpose. This code achieves rate-1 at full diversity and enjoys low-complexity ML decoding using matched filtering. The correspondence between Alamouti matrices and quaternions means that the set of Alamouti matrices is closed under addition, multiplication and inversion. Consider the set $\tilde{\mathscr{Q}}$ of $\mathbf{x}$ given by 2×2 orthogonal matrices

$$\tilde{\mathscr{Q}}(x_1,x_2)=\begin{bmatrix} x_1 & x_2 \\ x_2^* & -x_1^* \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\mathscr{Q}(x_1,x_2)$$

Then $\mathscr{Q}$ is a multiplicative group, $\tilde{\mathscr{Q}}$ is a coset of $\mathscr{Q}$, and the union $\mathscr{S}=\mathscr{Q}\cup\tilde{\mathscr{Q}}$ is also a multiplicative group. For $\mathbf{E},\mathbf{F}\in\tilde{\mathscr{Q}}\rightarrow\mathbf{EF}\in\mathscr{Q}$ while for $\mathbf{E}\in\mathscr{Q}$ and $\mathbf{F}\in\tilde{\mathscr{Q}}\rightarrow\mathbf{EF}\in\tilde{\mathscr{Q}}$. We will use the expanded set $\mathscr{S}$ to construct new high-rate (>1) full-diversity space-time block code with low complexity decoding and optimized coding gain.

2. PROPOSED CODE FOR 2 TX

2.1 Transmission Scheme

The columns of $\mathscr{Q}$ represent different antennas, the rows represent different time slots, and the entries are the two symbols to be transmitted assuming a quasi-static flat-fading channel. Our code construction is applicable to any M-PSK constellation. However, to sim-

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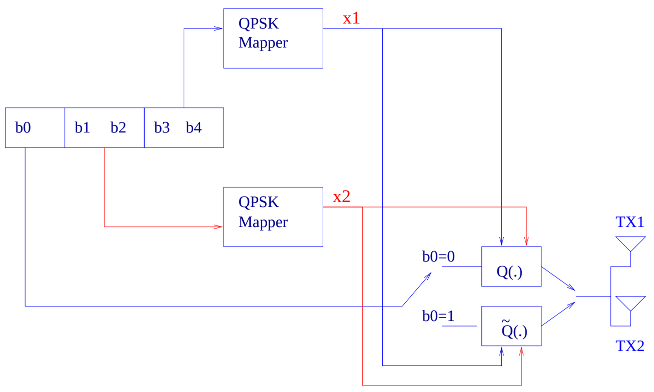

Figure 1: The Block Diagram of Rate-$\frac{5}{4}$ STBC for QPSK Modulation

plify the presentation, we will focus on QPSK modulation. In our proposed scheme, the transmitted space-time signaling matrix is selected from either $\mathscr{Q}$ or $\tilde{\mathscr{Q}}$ according to an additional information bit of 0 or 1, respectively . Hence, the proposed scheme achieves a 25% information rate increase compared to the traditional Alamouti scheme for QPSK modulation without requiring any additional system resources (power or bandwidth). More specifically, we assume that the first information bit (b_0) will alternately choose between $\mathscr{Q}$ or $\tilde{\mathscr{Q}}$ whereas the last four information bits ($b_1 : b_4$) will be first mapped to two QPSK symbols x_1 and x_2 which will be in turn space-time encoded as follows (c.f. Fig.1)

$$(b_0 = 0) \Rightarrow \mathscr{Q}(x_1, x_2), \quad (b_0 = 1) \Rightarrow \tilde{\mathscr{Q}}(x_1, x_2).$$

2.2 Code Design Criteria

Consider two distinct codewords $\mathbf{S}_i, \mathbf{S}_j \in \mathscr{S}$. In order to ensure full spatial diversity, the codeword difference matrix $\mathbf{B} = (\mathbf{S}_i - \mathbf{S}_j)$ between any two distinct codewords in the extended set $\mathscr{S}$ must have full rank [8]. When both codewords $\mathbf{S}_i$ and $\mathbf{S}_j$ belong to $\mathscr{Q}$ or $\tilde{\mathscr{Q}}$, $\mathbf{B}$ will be full rank. However if $\mathbf{S}_i \in \mathscr{Q}$ and $\mathbf{S}_j \in \tilde{\mathscr{Q}}$ (or vice versa), $\mathbf{B}$ loses rank. To restore full-diversity, schemes based on rotations of information symbols have been proposed (see *e.g.* [3, 9]). In this paper, we propose to divide the information symbols in $\tilde{\mathscr{Q}}$ only by a real scalar $K(> 1)$ to guarantee full-diversity, hence the name selective power scaling. For a unit-radius QPSK constellation, this scaling results in an overall signal constellation consisting of two concentric circles of radii 1 and $\frac{1}{K}$.

2.3 Finding the Optimum Power Scaling Factor

2.3.1 Coding Gain/PMPR Criterion

Two important selection criteria for K are maximizing the coding gain (CG) and minimizing the peak-to-minimum power ratio (PMPR) resulting from power scaling. If the QPSK symbols on the outer constellation circle are normalized to unity, and the scaling factor for the inner circle is $\frac{1}{K}$, the PMPR equals K^2. In addition, CG is defined as the minimum product of the nonzero singular values of $\mathbf{B}$ over all distinct codeword pairs. We propose to select K as follows

$$\begin{aligned} K_{opt} &= arg \max_{K>1} \frac{CG}{PMPR} \\ &= arg \max_{K>1} \frac{\min_{\mathbf{S}_i \in \mathscr{Q}, \mathbf{S}_j \in \tilde{\mathscr{Q}}, \mathbf{S}_i \neq \mathbf{S}_j} \det(\mathbf{B}\mathbf{B}^*)}{K^2} \end{aligned}$$

which is shown in [4] to be equivalent to

$$K_{opt} = arg \max_{K>1} \frac{[1 - K^2]}{K^3}$$

Differentiating with respect to K and setting the result to zero yields $K = \sqrt{3}$. Similarly, for any M-PSK constellation, the value of K can be optimized offline as a function of M and the achievable rate in this case is $1 + \frac{1}{2 log_2 M}$.

2.3.2 PEP Criterion

Next, we present another method for approximating the optimal value for K, based on a union bound formulation. At any given SNR, provided it is sufficiently high, the pairwise error probability (PEP) between two codewords $\mathbf{S}_i, \mathbf{S}_j \in \mathscr{S}$ is inversely proportional to the determinant of $\mathbf{B}_{i,j}\mathbf{B}^*_{i,j}$, where $\mathbf{B}_{i,j} = (\mathbf{S}_i - \mathbf{S}_j)$ is the codeword difference [8]. To approximate the overall error probability, we take the union bound approach:

$$P_{\text{union bound}} \propto \frac{1}{|\mathscr{S}|} \sum_{i=1}^{|\mathscr{S}|} \sum_{\substack{j=1 \\ j \neq i}}^{|\mathscr{S}|} \frac{1}{\det(\mathbf{B}_{i,j}\mathbf{B}^*_{i,j})}$$

This quantity is a function of K, and is a measure of the overall error probability for the code. Note that SNR does not appear in this relationship; it is absorbed by the proportionality constant. For optimality, we choose K that minimizes the function, hence minimizing the union bound. For this analysis, we must scale the codewords by the factor $\frac{1}{\sqrt{1+K^2}}$ in order to normalize the total transmit power per time interval to unity. This operation ensures consistency in SNR across all values for K, and hence produces a fair comparison.

The numerical results are given in Figure 4. For QPSK, the optimal K is 1.69 which is very close to the optimum K using the PMPR criterion.

2.4 Low-Complexity Decoding

The output symbols r_1, r_2 received over two consecutive symbol periods can be represented as follows:

$$\underbrace{\begin{bmatrix} r_1 & r_2 \\ -r_2^* & r_1^* \end{bmatrix}}_{\mathbf{R}_{\mathcal{Q}}} = \underbrace{\begin{bmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{bmatrix}}_{\mathbf{H}_{\mathcal{Q}}} \underbrace{\begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}}_{\mathbf{S}_{\mathcal{Q}}} + \underbrace{\begin{bmatrix} z_1 & z_2 \\ -z_2^* & z_1^* \end{bmatrix}}_{\mathbf{Z}_{\mathcal{Q}}},$$

and

$$\underbrace{\begin{bmatrix} r_1 & r_2 \\ r_2^* & -r_1^* \end{bmatrix}}_{\mathbf{R}_{\tilde{\mathcal{Q}}}} = \underbrace{\begin{bmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{bmatrix}}_{\mathbf{H}_{\mathcal{Q}}} \underbrace{\begin{bmatrix} \frac{y_1}{k} & \frac{y_2}{k} \\ (\frac{y_2}{k})^* & -(\frac{y_1}{k})^* \end{bmatrix}}_{\mathbf{S}_{\tilde{\mathcal{Q}}}} + \underbrace{\begin{bmatrix} z_1 & z_2 \\ z_2^* & -z_1^* \end{bmatrix}}_{\mathbf{Z}_{\tilde{\mathcal{Q}}}}$$

where $\mathbf{R}_{\mathcal{Q}}$ and $\mathbf{R}_{\tilde{\mathcal{Q}}}$ are 2×2 complex matrix representations of the received signals corresponding to transmitted codewords in the form of $\mathcal{Q}$ and $\tilde{\mathcal{Q}}$, respectively. The *path gains* from the two transmit antennas to the mobile are h_1, h_2, and the noise samples z_1, z_2 are independent samples of a zero-mean complex Gaussian random variable. The channel matrix $\mathbf{H}_{\mathcal{Q}}$ is a quaternion and we have

$$\mathbf{H}_{\mathcal{Q}}\mathbf{H}_{\mathcal{Q}}^* = (|h_1|^2 + |h_2|^2)\mathbf{I}_2.$$

Now decoding is remarkably simple, provided that the path gains are known at the receiver. Simple matched filtering operations are performed twice to form

$$\begin{aligned} \mathbf{H}_{\mathcal{Q}}^*\mathbf{R}_{\mathcal{Q}} &= (|h_1|^2 + |h_2|^2)\mathbf{S}_{\mathcal{Q}} + \mathbf{H}_{\mathcal{Q}}^*\mathbf{Z}_{\mathcal{Q}}, \text{and} \\ \mathbf{H}_{\mathcal{Q}}^*\mathbf{R}_{\tilde{\mathcal{Q}}} &= (|h_1|^2 + |h_2|^2)\mathbf{S}_{\tilde{\mathcal{Q}}} + \mathbf{H}_{\mathcal{Q}}^*\mathbf{Z}_{\tilde{\mathcal{Q}}} \end{aligned}$$

Observe that the new noise samples remain independent of each other, have zero mean and covariance

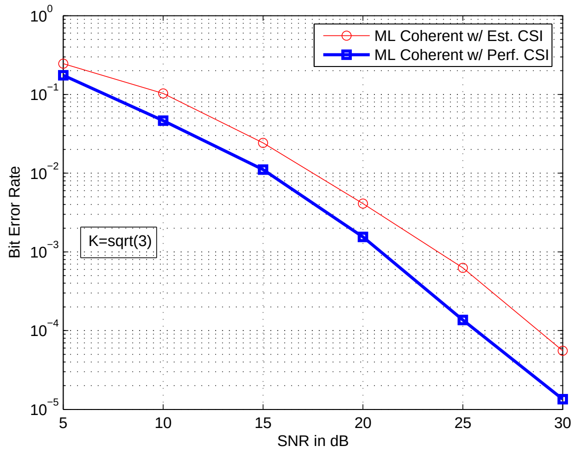

Figure 2: Performance of Proposed Rate-$\frac{5}{4}$ STBC for 2 Tx with ML Decoding under both Perfect and Estimated CSI

$(|h_1|^2 + |h_2|^2)\mathbf{I}_2$. Two candidate solutions, namely, $\hat{\mathbf{S}}_{\mathcal{Q}}$ and $\hat{\mathbf{S}}_{\tilde{\mathcal{Q}}}$ are generated from the outputs of the matched filters and are compared using the metric $\|[r_1\ r_2] - [h_1\ h_2]\ \mathbf{S}\|^2$. The decoding of b_0 follows directly once the decision between $\mathbf{S}_{\mathcal{Q}}$ or $\mathbf{S}_{\tilde{\mathcal{Q}}}$ is made.

3. NUMERICAL RESULTS

In this section we present simulation results on the performance of our proposed STBC both for Rayleigh flat-fading channels and for the IEEE 802.16 WiMAX environment. We assume QPSK modulation and a single antenna at the receiver.

3.1 Flat-Fading Channel

In Figure 2, we consider the resulting performance degradation when the assumption of perfect CSI at the receiver is not satisfied. The coherent ML decoder uses estimated CSI acquired by transmitting a orthogonal 2×2 pilot codeword (Alamouti code) and using simple matched filtering at the receiver to calculate the CSI. We observe a 3 dB SNR penalty at high SNR due to channel estimation error.

In Figure 3, we compare our proposed rate-$\frac{5}{4}$ code with the Alamouti [1] code using the measure of Effective Throughput η defined as $\eta = (1 - FER) * R *$

$\log_2(M)$, where R is the code rate, M is the constellation size, and FER denotes the frame error rate. We assume 20 codewords per frame, and a frame to be in error if any information bit in the frame is decoded incorrectly. A frame in error is not considered for retransmission and is simply discarded from the queue. This Figure shows that at high SNR (where FER ≈ 0), our code achieves a higher throughput level of 2.5 bits per channel use (PCU) whereas the achievable throughput for the Alamouti code is 2 bits PCU. We can observe a cross-over point at an input SNR level of 16 dB. Since it is reasonable to assume that we know the operating input SNR level at the transmitter, we can switch between the Alamouti code and our proposed code to maximize throughput at all SNR levels. Figure 3 depicts the achievable throughput of a rate-$\frac{5}{4}$ code that uses pure rotations (instead of power scaling) to ensure full-diversity and maximize coding gain. More specifically, the symbols in the main diagonal of codewords in $\tilde{\mathscr{Q}}$ are rotated by 45 degrees to maximize coding gain. It demonstrates that optimum power scaling achieves higher throughput than optimum rotation for all SNR.

3.2 WiMAX

In this section, we compare our proposed rate-$\frac{5}{4}$ code with the rate-1 Alamouti code by using different outer codes to match the overall rate. One application that allows such rate tuning is the IEEE 802.16 broadband wireless access standard (WiMAX). We implement the Orthogonal Frequency Division Multiplexing physical layer (OFDM PHY) specification in the standard, and use the widely-used Stanford University Interim (SUI) channel models [10]. Each OFDM symbol contains 192 data subcarriers, and we employ a cyclic prefix of 32 samples. We simultaneously transmit 192 codewords by sending two OFDM symbols per antenna, across two consecutive OFDM symbol durations.

We design each collection of 192 codewords to contain 60 information bytes. For the rate-$\frac{5}{4}$ code ($K = 1.69$), we choose a rate $\frac{1}{2}$ convolutional code to be the outer code. For the Alamouti code, we concatenate a shortened Reed-Solomon (64,60) code with a punctured convolutional code of rate $\frac{2}{3}$. In both cases, the coded bits are frequency-interleaved before forming the 192 codewords. Figure 5 shows the bit error rate for both coding schemes, obtained at a single receive antenna that has perfect CSI. Our $\frac{5}{4}$-rate code provides an improvement over the Alamouti code with a matched rate, at an SNR of 17 dB and above.

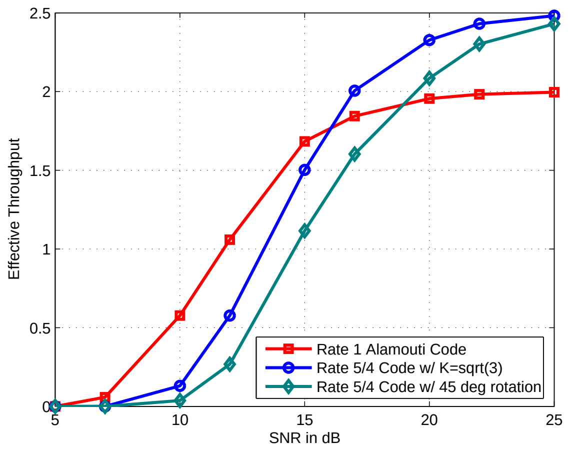

Figure 3: Effective Throughput Comparison between the Alamouti and Proposed Rate-$\frac{5}{4}$ STBC for QPSK and 2 TX

4. CONCLUSIONS

We exploited the algebraic structure of quaternions to design and optimize a novel high-rate, full-diversity STBC for 2 transmit antennas. We introduced the concept of selective power scaling to guarantee full diversity for the designed code. We presented two criteria for optimizing the power scaling factor and demonstrated their equivalence. Extensions to the case of 4 transmit antennas based on the rotated quasi-orthogonal design [5, 9] are reported in [4].

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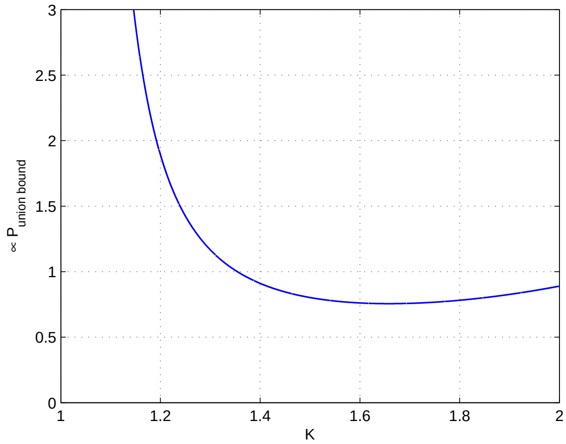

Figure 4: Union Bound Approximation for Different K Values (QPSK constellation)

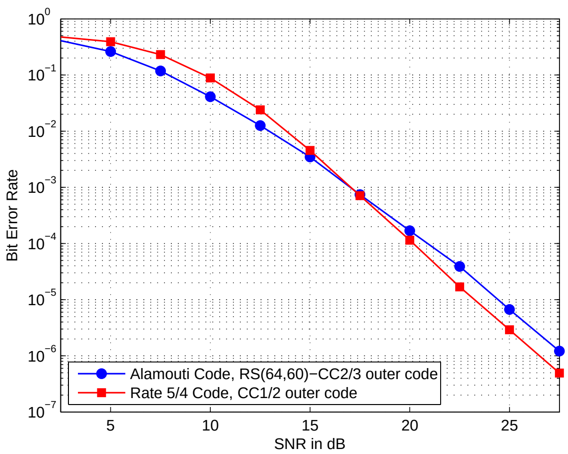

Figure 5: BER Comparison of Proposed Rate-$\frac{5}{4}$ STBC with Alamouti STBC

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