BANDWIDTH EFFICIENT SDMA MULTICARRIER SYSTEMS WITH FREQUENCY DOMAIN DIVERSITY

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ABSTRACT

Multiple receiving antennas increase the capacity of wireless medium by admitting multiple users in the SDMA mode and by reducing channel selectivity. Additionally, spectral diversity may be exploited via a frequency spreading of symbols in order to combat frequency selectivity. The structure of a frequency spreading allows to train the channel equalizer / multiuser interference chancellor with no substantial overhead. A multicarrier modulation scheme with frequency spreading is proposed which yields a simple and data efficient detection of a desired user due to its spreading signature only. A numerical study shows this method suitable for broadband wireless applications.

1. INTRODUCTION

Multicarrier modulation is an efficient technique of high-rate transmission over the frequency selective noisy channels, see [1]. Sprectral efficiency of the existing multicarrier systems, such as DMT and OFDM is, however, limited since, one hand, a redundant cyclic prefix is used to handle the ISI, and on the other hand, training symbols are periodically transmitted to ensure a reliable channel acquisition and synchronization in time-varying environments. Several authors have tried to improve bandwidth efficiency of multicarrier systems by using the redundancy introduced by a cyclic prefix [2] or, more interestingly, a redundant precoding of the modulated data, see *e.g.*, [3]. The last contribution is particularly interesting since it allows a simple and data efficient implementation. However, the amount of redundancy imposed by a deep selective fading and a limited data payload lead to a sensitive spectrum efficiency loss.

Alternatively to the spectrum redundancy, the space redundancy may be exploited to mitigate the impact of deep fades. Recently a new multicarrier system has been proposed that achieves a high spectral efficiency due to a simultaneous use of spatial and spectral diversity, see [4]. Spectral diversity is achieved via a frequency domain spreading. The structure of this spreading is also exploited to handle equalization and synchronization tasks in a simple and data efficient way with a vanishingly small wireless overhead.

In this contribution, a multiuser extension of multicarrier systems with spatial diversity and frequency domain spreading is presented. Multiple users are separated in the space-time domain due to a difference of their frequency interleavers. The extraction of each user signal is based on the knowledge of its interleaver only. Moreover, the detection procedure for a certain user is independent on the total number of active users. The algorithm is simple, reliable and data efficient as in the single user case. A theoretical study as well as computer simulations of broadband indoor channels and the underlying physical layer processing, confirm a good fit of the new approach to bandwidth expensive applications such as wireless LANs.

2. SYSTEM DESCRIPTION AND DATA MODEL

The baseband model of the whole transmission system is shown in Fig.1. The standard modulation step consists of converting the sequence of bits $\{b\,[k]\}_{k\in\mathbb{Z}}$ to a sequence of complex-valued symbols $\{v\,[k]\}_{k\in\mathbb{Z}}$. These latter undergo a frequency domain spreading. In particular, a block spreading may be used such that the sequence $\{v\,[k]\}_{k\in\mathbb{Z}}$ is mapped to a sequence $\{s\,[k]\}_{k\in\mathbb{Z}}$ as follows. The consecutive blocks $\underline{s}_q[k]$ of $s[k]$ of size q are obtained by multiplication of the consecutive blocks $\underline{v}_p[k]$ of $v[k]$ of size p by a $q\times p$ spreading matrix C :

$$\underline{s}_q[k] = C\,\underline{v}_p[k],\ \ \underline{s}_q[k] = [\,s[kq+1],\ldots,s[kq+q]\,]^T, \\ \underline{v}_p[k] = [\,v[kp+1],\ldots,v[kp+p]\,]^T. \quad (1)$$

The sequence $\{s\,[k]\}_{k\in\mathbb{Z}}$ is sub-divided into consecutive blocks of the size N (N is a multiple of q) which are sent via interleaver to the IFFT processor. The IFFT output $\{x\,[k]\}_{k\in\mathbb{Z}}$ undergoes a conventional pulse shaping.

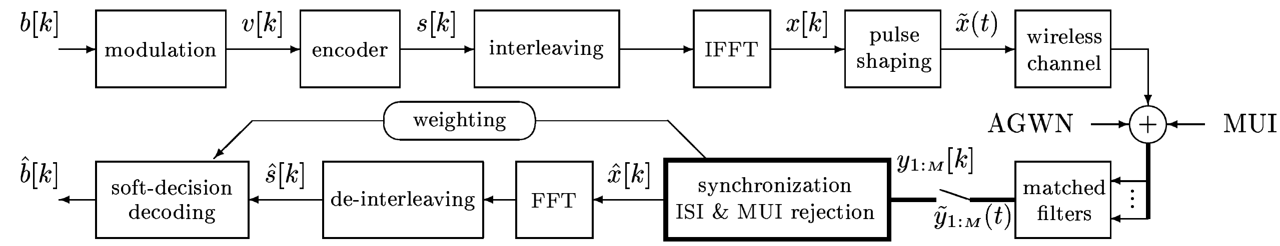

Fig.1. Baseband model of the transmission system

The resulting signal is transmitted over a frequency selective AGWN channel which is shared by $m \leq M$ users. At the receiver, the mixture of signals is collected by M antennas. Each antenna output is subject to bandpass filtering and sampling at the symbol rate. The vector $y[k] = [y_1[k], \ldots, y_M[k]]^T$ of the sampled antenna outputs verifies

$$y[k] = \sum_{l \in \mathbb{Z}} x[l]\, \boldsymbol{h}[k-l] + n[k] + \eta[k], \tag{2}$$

where $n[k]$ and $\eta[k]$ are M-variate vectors of AGWN and multiuser interference (MUI) correspondingly. The $M \times 1$ vectors $\boldsymbol{h}[k] = [\boldsymbol{h}_1[k], \ldots, \boldsymbol{h}_M[k]]^T$ stand for the channel impulse response which depends on the pulse shape, physical channel and the sampling phase. The M-variate series $\{y\,[k]\}_{k \in \mathbb{Z}}$ are forwarded to the ISI and MUI canceler which produces noisy estimates $\hat{x}[k]$ of $x[k]$. After the FFT processing and de-interleaving, the estimates $\hat{s}[k]$ are used for a soft-decision decoding.

The synchronization and ISI/MUI cancelation task is accomplished by a linear $1 \times M$ filter which is given by the *minimum norm* inverse of the user channel within the class of MUI-orthogonal filters. This filter, later referred to as the minimum norm canceler (MNC), provides a maximum output signal-to-noise ratio (SNR) in each subband, under the MUI rejection constraint.

Assume that $q > p$ and define $r_c = p/q$ the *coding rate*. Let us roughly compare the described system with the conventional OFDM system. In this latter, the IFFT processor is fed directly by $\{v\,[k]\}_{k \in \mathbb{Z}}$. Note that the system with spreading yields a bit rate loss of $r_c < 1$. This loss may be alleviated by increasing the constellation size of $v[k]$. This approach implies certain reduction of the free distance and, therefore, requires a higher SNR to reach a target bit error rate (BER) over a Gaussian channel. However, in a Rayleigh or similar fading environments, the decrease of BER in SNR is faster for smaller r_c due to the diversity since $q > p$. When the frequencies fade independently within each spreading block, the slope of BER versus SNR is proportional to the diversity order, $(q/p) = (1/r_c)$, see *e.g.*, [5]. Hence, small target BERs may be achieved at smaller SNR levels due to a frequency domain spreading. Additionally, the low-rank property of the sequence $\{s\,[k]\}_{k \in \mathbb{Z}}$ can be used to estimate the MNC via a sample efficient structure forcing.

3. STRUCTURE FORCING PRINCIPLE

The structure imposed by the encoder may be used to identify up to a constant scalar the MNC. Define by $H\left(e^{\imath 2\pi\nu}\right) = \sum_{k \in \mathbb{Z}} \boldsymbol{H}[k]\, e^{-\imath 2\pi\nu k}$ a $M \times m$ transfer function of a linear time invariant (LTI) channel between m active users and M receiving antennas. Without loss of generality, let the first column of $H\left(e^{\imath 2\pi\nu}\right)$ stand for the user of interest, *i.e.*, $\left(H\left(e^{\imath 2\pi\nu}\right)\right)_{:,1} = \sum_{k \in \mathbb{Z}} \boldsymbol{h}[k]\, e^{-\imath 2\pi\nu k}$. The remaining columns of $H\left(e^{\imath 2\pi\nu}\right)$ contribute to the MUI. Assume that $H\left(e^{\imath 2\pi\nu}\right)$ is left-invertible and denote by $w\left(e^{\imath 2\pi\nu}\right)$ the unit norm MNC: $w\left(e^{\imath 2\pi\nu}\right) H\left(e^{\imath 2\pi\nu}\right) = [\gamma_0, 0, \ldots, 0]$, $\gamma_0 \in \mathbb{C}$, $\int_0^1 \|w\left(e^{\imath 2\pi\nu}\right)\|^2\, d\nu = 1$. Let us explain how to estimate such a MNC.

According to (1), the sequence of encoded blocks $\{\underline{s}_q\,[k]\}_{k \in \mathbb{Z}}$ is a low-rank multivariate series and its range space is given by the column space of C. Define U some unitary basis of the null$\{\ C\ \}$. From (1), we have $U^H\, \underline{s}_q[k] = 0$ for all k. In the absence of noise, ISI and MUI, the $q \times 1$ blocks $\{\hat{\underline{s}}_q\,[k]\}_{k \in \mathbb{Z}}$ after de-interleaving are also low-rank and their range space coincides with the span$\{\ C\ \}$. Hence $U^H \hat{\underline{s}}_q[k] = 0$ in this case. Suppose that this condition does not hold in presence of ISI, MUI or when synchronization is imperfect. In such cases, the MNC may be estimated by forcing the aforementioned condition under the constraint on the filter norm.

A successful application of this estimation principle requires a non-identical spreading structure of different users. One way to ensure such a disparity is to design different spreading matrices. However, much better disparity may be achieved by choosing different frequency domain interleavers. We assume in the sequel that these interleavers are obtained as various random realizations of a *uniform* interleaver, see *e.g.*, [6].

Proposition 1 *Let* $w_N(z) = \sum_{k \in \mathbb{Z}} \boldsymbol{w}_N[k]\ z^{-k}$ *be a sequence of stable LTI systems and* $\hat{x}[k] = \left[\, w_N(z)\, \right] y[k]$.

Define $\gamma_{N,i}(z) = \sum_{k\in\mathbb{Z}} \gamma_{N,i}[k]\, z^{-k}$, $1 \le i \le m$, *the* ***global*** *system between the i-th user and* $\{\hat{x}[k]\}_{k\in\mathbb{Z}}$, *that is,* $\gamma_{N,i}(z) = w_N(z)\,(H(z))_{:,i}$. *Then*

$$\hat{x}[k] = \sum_{n=1}^{m}\sum_{l\in\mathbb{Z}} \gamma_{N,i}[l]\, x_i[k-l] + \hat{n}[k],$$

$$\hat{n}[k] = \sum_{l\in\mathbb{Z}} \boldsymbol{w}_N[l]\, n[k-l],$$

here $x_i[k]$ *is the i-th user signal* ($x[k] \equiv x_1[k]$). *Denote* $\underline{\hat{n}}_q[k] = [\,\hat{n}[kq+1], \ldots, \hat{n}[kq+q]\,]^T$ *and assume that all* $w_N(z)$ *satisfy* $\mathbb{E}\{N^{-1}\sum_{k=1}^{N/q}\|U^H\underline{\hat{n}}_q[k]\|^2\} = c > 0$. *Then* $w_N(z)$ *minimizes* $\mathbb{E}\{N^{-1}\sum_{k=1}^{N/q}\|U^H\underline{\hat{s}}_q[k]\|^2\}$ *as* $N \to \infty$ *iff* $(\sum_{i=2}^{m}|\gamma_{N,i}[0]|^2 + \sum_{i=1}^{m}\sum_{l\neq 0}|\gamma_{N,i}[l]|^2) \xrightarrow{p} 0$.

According to the above proposition, minimization of $\mathbb{E}\{N^{-1}\sum_{k=1}^{N/q}\|U^H\underline{\hat{s}}_q[k]\|^2\}$ under the quadratic constraint $\mathbb{E}\{N^{-1}\sum_{k=1}^{N/q}\|U^H\underline{\hat{n}}_q[k]\|^2\} = c$ yields an asymptotic (w.r.t. N) extraction of the scaled $x[k]$ corrupted by the additive noise: $\hat{x}[k] \approx \gamma_o\, x[k] + \hat{n}[k]$, where $\gamma_o = \lim_{N\to\infty} \gamma_{N,1}$. Hence it ensures synchronization as well as ISI and MUI cancelation at large N.

3.1. Signal fluctuations

Although the constrained quadratic minimization described in the Proposition 1 suggests asymptotic rejection of ISI and MUI at $N \to \infty$, it can not guarantee a reliable level of SNR and signal-to-interference ratio (SIR) at the output of a canceler estimated in such a way. More precisely, the contribution of $x[k]$ at the filter output, specified here by γ_o, is not controlled. In particular, any nonzero $1\times M$ filter $f(z)$ such that $f(z)\,H(z) = 0$ can be scaled so as to satisfy the constraint of the Proposition 1. The resulting filter will be a solution to the constrained quadratic minimization which yields $\gamma_o = 0$. Note that the equation $f(z)\,H(z) = 0$ have nonzero solutions whenever $m < M$.

As explained in [4] in the single user context, the maximum of $|\gamma_o|$ is reached when $w_N(z)$ tends to a minimum norm filter within the class of ISI and MUI cancelers. This condition is not ensured by the procedure suggested in the Proposition 1. In practice, the signal gain $|\gamma_o|$ varies arbitrarily depending upon the additive noise realization. This results in arbitrary variations of the output SNR. Indeed, unlike $|\gamma_o|$, the output noise has a fixed power, due to the constraint $\mathbb{E}\{N^{-1}\sum_{k=1}^{N/q}\|U^H\underline{\hat{n}}_q[k]\|^2\} = c$. Moreover, the output SIR also exhibits arbitrary variations. To show this, let us define an empirical measure of the interference :

$$\hat{\mathcal{I}}_N \triangleq \frac{1}{|\gamma_{N,1}[0]|^2}\sum_{i=1}^{m}\sum_{l_i:\ l_1\neq 0} |\,\gamma_{N,i}[l_i]\,|^2. \tag{3}$$

Note that $\hat{\mathcal{I}}_N$ equals to the inverse of the output SIR where the interference comprises both ISI amd MUI. Although, according to Proposition 1, the numerator of (3) converges to zero as $N \to \infty$, the convergence of $\hat{\mathcal{I}}_N$ is not guaranteed since $|\gamma_{N,1}[0]|$ may be arbitrarily small.

4. ROBUST MNC ESTIMATION

A consistent MNC estimate stems from a similar estimate designed in [4] for the single user case. Let us replace $\|U^H\underline{\hat{s}}_q[k]\|^2$ in the Proposition 1 with the cost

$$\|U^H\underline{\hat{s}}_q[k]\|^2 - \mu\,\|\underline{\hat{s}}_q[k]\|^2. \tag{4}$$

One can show that at $N \to \infty$, (4) converges to

$$(1 - r_c - \mu)\sum_{i=1}^{m}\sum_{l_i:\, l_1\neq 0} |\gamma_{N,i}[l_i]|^2 - \mu\,|\gamma_{N,1}[0]|^2, \tag{5}$$

modulo a term that remains constant under the constraint given in proposition 1. Note that the minimum of (5) over $w_N(z)$ is negative for any $\mu > 0$ since (5) is negative when $w_N(z)$ is replaced by the appropriately scaled $w(z)$ so as to meet the constraint. Indeed, $w(z)(H(z))_{:,1} = \gamma_o$, and therefore (5) equals to $-\mu\,|\gamma_o|^2$. Hence the true minimizer $w_N(z)$ asymptotically yields $(1 - r_c - \mu)\sum_{i=1}^{m}\sum_{l_i:\, l_1\neq 0}|\gamma_{N,i}[l_i]|^2 < \mu|\gamma_{N,1}[0]|^2$, *i.e.*,

$$\hat{\mathcal{I}}_N < \mu\,(1 - r_c - \mu)^{-1},$$

where $\hat{\mathcal{I}}_N$ is defined in (3). By putting $\mu \to 0$, we obtain $\hat{\mathcal{I}}_N \to 0$. The minimizer of (4) is also as good as the MNC in terms of the output SNR. Indeed,

$$(1 - r_c - \mu)\sum_{i=1}^{m}\sum_{l_i:\, l_1\neq 0} |\gamma_{N,i}[l_i]|^2 - \mu\,|\gamma_{N,1}[0]|^2 \le -\mu\,|\gamma_o|^2$$

since $-\mu\,|\gamma_o|^2$ is the value of (5) obtained for $w(z)$. Hence $|\gamma_{N,1}[0]| \ge \gamma_o$, *i.e.*, the signal power at the output of $w_N(z)$ is not less than the signal power at the output of the MNC. Meanwhile, the output noise power is fixed by the constraint. Hence, (4) yields the MNC.

We will estimate a FIR approximation of the MNC. Define $f(z) = z^d\sum_{k=0}^{K}\boldsymbol{f}[k]\,z^{-k}$ a $1\times M$ FIR filter of order K with the reconstruction delay d and the $M(K+1)\times 1$ vector of its coefficients $\boldsymbol{f} = [\boldsymbol{f}[K], \ldots, \boldsymbol{f}[0]]^T$. Denote by $\mathcal{H}_K(\underline{y}_N)$ the $N\times M(K+1)$ *block-Hankel* matrix with $1\times M$ blocks the first block column $[y[1-k_0], y[2-k_0], \ldots, y[N-k_0]]^T$ and the last block row $[y[N-k_0]^T, y[N-k_0+1]^T, \ldots, y[N-k_0+K]^T]$. Here K, k_o are chosen so as to ensure the synchronization and interference cancelation. Let $U_N \triangleq \mathrm{I}_{N/q}\otimes U$, ($\otimes$ is the Kronecker product) and B_N is the *unitary* IFFT basis ordered according to the interleaving. One can show that the vector $\hat{\boldsymbol{w}} = [\hat{\boldsymbol{w}}[K], \ldots, \hat{\boldsymbol{w}}[0]]^T$ of MNC coefficients corresponding to (4) yields

$$\hat{\boldsymbol{w}} = \arg\min_{\|\boldsymbol{f}\|=1} \boldsymbol{f}^H\,[\,\mathcal{H}_K(\underline{y}_N)^H\, Q_\mu\, \mathcal{H}_K(\underline{y}_N)\,]\,\boldsymbol{f},$$

$$Q_\mu = N^{-1}\,(\,B_N\, U_N\, U_N{}^H\, B_N{}^H - \mu\,\mathrm{I}_N\,). \tag{6}$$

Such $\hat{\boldsymbol{w}}$ is given by the minor eigenvector of the $M(K+1) \times M(K+1)$ Hermitian matrix $[\mathcal{H}_K(\underline{y}_N)^H Q_\mu \mathcal{H}_K(\underline{y}_N)]$. The matrix Q_μ depends only on the encoder and interleaver parameters, hence it may be pre-calculated.

A large sample analysis of the (6) reveals a robustness of the estimator w.r.t. μ in the region specified by

$$\mu = \mu_\star \rho^{-2} N^{-\frac{1}{2}} 4 \sqrt{(K+1)(1-r_c)/M}, \quad \mu_\star \approx 1, \quad (7)$$

where ρ^2 is the average SNR per antenna.

5. NUMERICAL STUDY

Let us use (4×2) spreading matrix and QAM-16 symbols $v[k]$ with Gray encoding. The columns of C are given by a subset of the Fourier basis. This semi-unitary matrix maximizes the free distance for big input constellations; it also guarantees the equal energy distribution between different inputs/outputs of each block. After spreading, the block of N_s points of $s[k]$ is completed by N_r *known* training symbols. These symbols are necessary to recover the unknown scalar which is required for the soft-decision decoding; this scalar is not identifiable from the received data only. The following results are presented for $m = 2$ users, $M = 4$ antennas, OFDM symbol sizes $N = 256$ ($N_r = 4$) and $N = 512$ ($N_r = 8$) corresponding to 63 and 126 information bytes per OFDM symbol. The total wireless overhead is less than 2%. A stochastic Rayleigh multipath propagation channel model is used which corresponds to a severe fading in indoor environments in 5.2 and 17 GHz bands.

In Fig.2, the ratio of noise power at the outputs of empirical and true MNC (*i.e.*, the output SNR loss) is plotted versus $10\log_{10}\mu_\star$. The residual interference level versus $10\log_{10}\mu_\star$ is plotted in Fig.3. The asymptotically achievable theoretical interference level, $\mathcal{I}_N$, is plotted by the solid line. The results are averaged over 100 Monte-Carlo trials with independent channel realizations. Note that Fig.2-Fig.3 yield the asymptotically achievable performance at $\mu_\star \approx 1$, *i.e.*, they confirm (7).

We compare the system with spreading versus a conventional OFDM system. To obtain such a reference system, we remove spreading (*i.e.*, $s[k] = v[k]$) and replace QAM-16 by QAM-4. Both systems are equivalent in terms of the raw bit rate (40 Mbps) and the occupied bandwidth (20 MHz). In Fig.4, we plot the BER versus the average SNR per antenna. The theoretical curves (Chernoff bounds) assume a perfect channel knowledge. The empirical BER is plotted for both systems with perfect channel knowledge ($N = \infty$) as well as for the system with spreading and an empirical MNC obtained for the OFDM symbol of sizes $N = 256$ and $N = 512$, with 10% of outage channels. In the latter case, the system with spreading outperforms the perfectly trained reference system at SNR more than 16 dB.

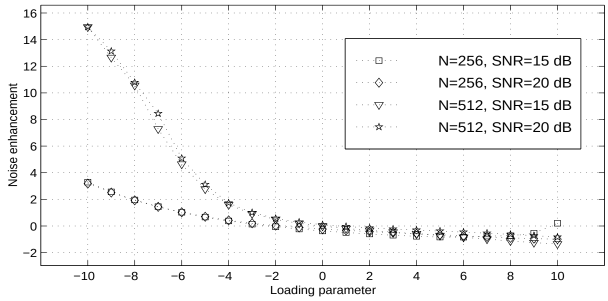

Fig.2. Noise enhancement in dB versus $10\log_{10}\mu_\star$.

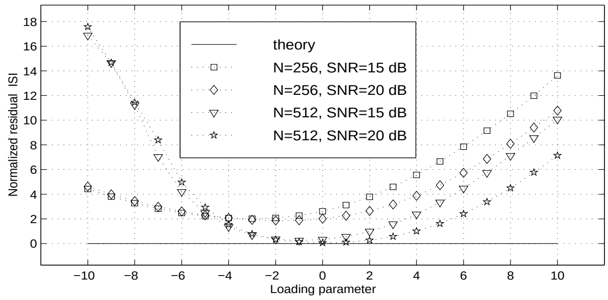

Fig.3. Residual $(\hat{\mathcal{I}}_N/\mathcal{I}_N)$ in dB versus $10\log_{10}\mu_\star$.

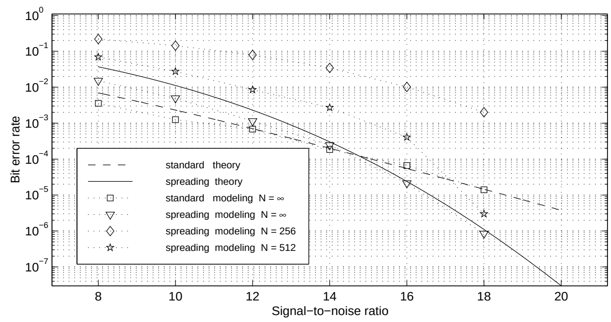

Fig.4. Bit error rate versus the signal-to-noise ratio.

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